<table>
<thead>
<tr>
<th>Geologica Macedonica</th>
<th>Год.</th>
<th>стр.</th>
<th>1–80</th>
<th>Штип</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geologica Macedonica</td>
<td>Vol.</td>
<td>pp.</td>
<td></td>
<td>Štip</td>
<td></td>
</tr>
</tbody>
</table>

**GEOLOGICA MACEDONICA**

Published by: — Издава:

The "Goce Delčev" University, Faculty of Natural and Technical Sciences, Štip, Republic of Macedonia

Универзитет „Гоце Делчев“, Факултет за природни и технички науки, Штип, Република Македонија

**ADVISORY BOARD**

David Alderton (UK), Tadej Dolenc (R. Slovenia), Ivan Zagorchev (R. Bulgaria), acad. Stevan Karamata (Serbia), Dragoljub Stefanović (Serbia), Todor Serafimovski (R. Macedonia), Wolfgang Todt (Germany), acad. Nikolay S. Bortnikov (Russia), Clark Burchfiel (USA), Thierry Auge (France) and Vlado Bermanec (Croatia)

**ИЗДАВАЧКИ СОВЕТ**

Дејвид Олдертон (В. Британија), Тадеј Доленец (Р. Словенија), Иван Загорчев (Р. Бугарска), акад. Стефан Карамата (Србија), Драгољуб Стефановиќ (Србија), Тодор Серафимовски (Р. Македонија), Волфганг Тод (Германија), Николай С. Бортиков (Русија), Клаrk Burchfiel (САД), Thierry Auge (Франција) и Владо Берманец (Хрватска)

**EDITORIAL BOARD**

**УРЕДУВАЧКИ ОДБОР**

**Editor in Chief**

Prof. Todor Serafimovski, Ph. D.

**Editor**

Prof. Blažo Boev, Ph. D.

Members of the Editorial Board

Prof. Nikola Dumurđanov, Ph. D.

Prof. Vančo Ćifliganec, Ph. D.

Prof. Risto Stojanov, Ph. D.

Prof. Todor Delipetrov, Ph. D.

**Language editor**

Marijana Kroteva

(English)

Georgi Georgievski, Ph. D.

(Macedonian)

**Technical editor**

Blagoja Bogatinoski

Proof-reader

Alena Georgievska

**Address**

GEOLOGICA MACEDONICA

EDITORIAL BOARD

Faculty of Natural and Technical Sciences

P. O. Box 96

MK-2000 Štip, Republic of Macedonia

Tel. ++ 389 032 550 575

E-mail: todor.serafimovski@ugd.edu.mk

400 copies

Published yearly

Printed by:

2nd August – Štip

Price: 500 den.

The edition was published in December 2009

**Photo on the cover:**

Artesian well, Medzitlija Village, Bitola, Republic of Macedonia,
СОДРЖИНА

Орце Спасовски
Хемиски и геохемиски карактеристики на главните минерали од наоѓалиштето Митрашинци (источна Македонија) .......................................................... 1–7

Тена Шијакова-Иванова, Блажо Боев, Зоран Панов, Дејан Павлов
Минералошко хемиски карактеристики на мермерот од наоѓалиштето Бела Пола ........................................................................................................ 9–16

Горан Тасев, Тодор Серафимовски
REE во некои терциерни вулкански комплекси во Република Македонија .................. 17–25

Милихате Алиу, Роберт, Шаји, Трајче Стафилов
Дистрибуција на кадмиум во почвите во регионот на К. Митровица, Косово .................... 27–34

Биљана Балабанова, Трајче Стафилов, Катерина Бачева, Роберт Шаї
Загадување на воздухот со бакар во околната на рудникот и флотацијата за бакар „Бучим“, Република Македонија, со примена на биомониторинг со мовови и лишаи.... 35–41

Трајче Стафилов, Роберт Шаї, Блажо Боев, Јулијана Цветковиќ, Душко Мукаетов, Марјан Андреевски, Сонја Лепиткова
Дистрибуција на карбонати во орizzовите почви во Кочанско Поле ........................................ 43–53

Настја Роган, Тодор Серафимовски, Горан Тасев, Тадеј Доленец, Матеј Доленец
Дистрибуција на Pb и Zn и нивната форма орizzовите почви во Кочанско Поле (Македонија) ................................................................. 55–62

Дељо Каракашев, Тена Шијакова-Иванова, Елизабета Каракашева, Зоран Панов
Стабилност на карпестите маси пробени со повеќекратно лизгање на површини ........... 63–72

Војо Мирчовски, Александар Кекиќ, Орце Спасовски, Владо Мирчовски
Карсниот воносник на планината Галичица и можности за водоснабдување на Охрид со подземна вода ...................................................................................................................................... 73–77

Упатство за авторите .......................................................................................................................................................................................... 79–80
# TABLE OF CONTENTS

**Orce Spasovski**  
Chemical and geochemical characteristics of the major minerals in the ore deposit Mitrašinci (Eastern Macedonia) ................................................................. 1–7

**Tena Šijakova-Ivanova, Blažo Boev, Zoran Panov, Dejan Pavlov**  
Mineralogical and chemical characteristics of marble of Bela Pola deposite ......................... 9–15

**Goran Tasev, Todor Serafimovski,**  
REE in some tertiary volcanic complexes in the Republic of Macedonia ............................ 17–25

**Milihate Aliu, Robert Šajn, Trajče Stafilov**  
Distribution of cadmium in surface soils in K. Mitrovica region, Kosovo ............................ 27–34

**Biljana Balabanova, Trajče Stafilov, Katerina Bačeva, Robert Šajn**  
Atmospheric pollution with copper around the copper mine and flotation, Bučim, Republic of Macedonia, using biomonitoring moss and lichen technique  ......................... 35–41

**Trajče Stafilov, Robert Šajn, Blažo Boev, Julijana Cvetković, Duško Mukaetov,**  
**Marjan Andreevski, Sonja Lepitkova**  
Distribution of cobalt in soil from Kavadarc i and the environs .............................................. 43–53

**Nastja Rogan, Todor Serafimovski, Goran Tasev, Tadej Dolenc, Matej Dolenc**  
Distribution of Pb and Zn and their chemical speciations in the paddy soils from the Kočani field (Macedonia) ........................................................................................................ 55–62

**Deljo Karakašev, Tena Šijakova-Ivanova, Elizabeta Karakaševa, Zoran Panov**  
Stability analysis of rock wedges with multiple sliding surfaces ............................................. 63–72

**Vojo Mirčovski, Aleksandar Kekić, Orce Spasovski, Vlado Mirčovski**  
Karst aquifer in Mt Galicica and possibilities for water supply to Ohrid with ground water ....... 73–77

**Instructions to authors** ........................................................................................................ 79–80
STABILITY ANALYSIS OF ROCK WEDGES WITH MULTIPLE SLIDING SURFACES

Deljo Karakašev¹, Tena Šijakova-Ivanova¹, Elizabeta Karakaševa¹, Zoran Panov²

¹Faculty of Natural and Technical Sciences, Department of Hydrogeology and Ingineering Geology, “Goce Delčev” University, Goce Delčev 89, MK-2000, Štip, Republic of Macedonia
²Mechanical Faculty, University, Goce Delčev 89, MK-2000, Štip, Republic of Macedonia
deljo.karakasev@ugd.edu.mk

Abstract: Although wedge and plane sliding stability analyses are well established in the geotechnical literature, certain geologic environments produce blocks which cannot be adequately modelled as either wedges or plane slides. An example is blocks forming in cylindrically folded sedimentary rocks, where the surface of sliding is neither a single plane nor a double plane but is curved. This type of block may be idealized as a prismatic block with multiple sliding planes, all with parallel lines of intersection. If the sliding planes number three or more, the distribution of normal forces and hence the factor of safety is indeterminate. A new analytical model for sliding stability analysis is described in which the distribution of normal forces on the contact planes is chosen to minimize the potential energy of the system. The classic wedge and plane solutions are shown to be special cases of this more general model, which allows determination of the safety factor for any shape of prismatic contact surface. An example from block part of Bregalnica river with a curved sliding surface is described and the factor of safety compared with the standard wedge analysis. It is shown that with three or more contact planes, the safety factor may be significantly lower than that calculated from the wedge model, which provides an upper limit on stability.

Key words: rock slope stability; wedge slides

INTRODUCTION

Stability analyses for wedge and plane failures are now well established in the geotechnical literature. Some of the early work on the subject is due to Londe et al. (1969, 1970), John (1968), Wittke (1965, 1990) and Goodman (1976, 1989). Other developments and useful summaries are given by Hoek and Bray (1981), Goodman and Shi (1985), Giani (1992), Warburton (1993), Einstein (1993) and Watts (1994), among others. The requirements for I sliding may be summarized as follows. Plane failures can occur when the strike of a discontinuity plane such as bedding is approximately parallel to the strike of the slope face and the weak plane daylight in the free face at a dip angle greater than the friction angle. Wedge failures can occur for a block defined by two planes whose line of intersection daylight in the free face and plunges sufficiently steeply that the destabilizing forces exceed the shear resistance. Typical geometries for wedge and plane failures are depicted in Fig. 1.

The important factors in the solution of stability problems are the shear strength, dip and dip direction of the discontinuity planes, the geometry of the slope and the loading conditions.

Fig. 1. Typical geometries for rock slope translational failures: (a) plane failure; (b) wedge failure

The system of forces governing plane slides is statically determined and the frictional shear resistance can be determined by resolution of forces. Wedge slides are rendered statically determinate by making an assumption about shear stresses in the contact planes, namely that shear stresses vanish in the plane perpendicular to the potential sliding direction. Although this assumption is rarely discussed in the literature (Chan and Einstein, 1981), it is implicit in the classic wedge analysis (e.g., Hoek and Bray, 1981). With the above-mentioned assumption, and if strengths and water pressures, etc., are known, the factor of safety for plane and wedge slides can be obtained directly by limiting equilibrium methods, either by direct cal-
culation or by graphical methods based on stereographic projection. With three or more sliding planes, however (e.g., Fig. 2), the distribution of normal forces is statically indeterminate and the net frictional resistance to sliding, and hence the factor of safety, cannot be uniquely determined. In this paper (see also Ureta, 1994; Mauldon and Ureta, 1994, 1995) we describe an energy method for determination of the factor of safety against sliding failure for blocks with multiple sliding planes that form a cylindrical surface. Such a block is referred to as a prismatic block.

**PRISMATIC BLOCKS**

A prismatic block is a rock block bounded by $n$ contact planes, where the lines of intersection of the contact planes are all parallel. Fig. 2 shows a special case of a prismatic block with three planes of contact with the rock mass. Also, in Fig. 2, a Cartesian coordinate system is defined with the $X$ axis parallel to the line of intersection (the potential sliding direction) and $Y$ horizontal. The contact planes of a prismatic block are cozonal (to use crystallographic terminology, e.g., Bloss, 1961), with zone axis parallel to the line of intersection, so that the normals of the contact planes fall on a great circle when plotted in stereographic projection (Fig. 3). As the number of planes ($n$) approaches infinity ($n \to \infty$), the contact surface approaches a curved cylindrical shape. Thus a block defined by cylindrically folded bedding or foliation is a limiting case of a prismatic block. Plane and wedge failures, with one and two contact planes, and zero and one lines of intersection respectively, are special cases of prismatic blocks.

**Statement of problem**

As discussed above, if $n \leq 2$ the distribution of normal forces on the contact plane(s) is statically determinate. If $n \geq 3$, however, the normal forces on the planes of contact between a prismatic block and the rock mass are statically indeterminate and the stability conditions cannot be determined directly by limiting equilibrium methods. In this paper we describe an energy method to determine the contact forces and to evaluate the factor of safety against sliding of prismatic rock blocks as a function of shear strength, block geometry, and loading conditions. The block is assumed to be supported at each face by a series of normal springs, each with stiffness $k_i$ and each spring subject to a “no tension” condition. The magnitude of the stiffness $k_i$ at each contact face is assumed to be proportional to the planar contact area per unit length. We assume an elastic, conservative system to obtain the distribution of normal forces that minimizes the potential energy of the system.

**Assumptions**

The stability analysis is based on a simplified model of rock mass geometry and strength, with the following assumptions:
- Contact faces of the prismatic block are planar.
- The displacement of the block is purely translational.
- Frictional shear stresses act parallel to the sliding direction only. Note that this assumption is standard in limiting equilibrium analysis of plane and wedge slides.
The block is undeformable, except for elastic contacts at the bounding faces.

The block is acted on by an active resultant $\vec{R}$ which includes self-weight, and may in addition include hydraulic forces, seismic forces, or supporting forces due to anchors or bolts. The effect of moments is not considered in the analysis.

The normal stiffness of each contact plane is proportional to its surface area.

### Analytical model

The total potential energy of a system consisting of a prismatic rock block supported by $n$ contact planes is given by:

$$V = \sum_{i=1}^{n} V_i - W_r$$

where $V_i$ is the elastic potential energy associated with each discontinuity surface $i = 1, 2, \ldots, n$, and $W_r$ is the work done by the active forces acting on the block. If self-weight is the only active force, the work done by the active forces is the negative of the change in gravitational potential energy. We assume that the block itself is undeformable, but that elastic deformation occurs at the contacts between the block and the rock mass. We assume that the block-rock mass contacts behave like linear springs, with stiffness proportional to surface area and we establish a datum for the gravitational potential energy such that the contact springs initially have zero extension.

The first task will be to find the equilibrium distribution of forces in the $YZ$ plane (the plane perpendicular to the potential sliding direction), such that the total potential energy is a minimum. We assume that the equilibrium position in the $YZ$ plane of the block results from a small translational displacement of the block in the $YZ$ plane (i.e., perpendicular to the sliding direction) under the action of the active forces. Elastic strain energy in the spring contacts is stored as a result of this displacement. We denote this small displacement by a vector $\vec{s}$ at an angle $\theta_s$ and with magnitude $s$ (see Fig. 4). Then, for unit normal $\hat{n}_i$ and stiffness $k_i$ corresponding to each contact plane, the elastic potential energy $V_i$ at each contact face is given by

$$V_i = \begin{cases} 
1/2k_i(\hat{n}_i \cdot \vec{s})^2 & \text{if } (\hat{n}_i \cdot \vec{s}) > 0 \\
0 & \text{if } (\hat{n}_i \cdot \vec{s}) \leq 0 
\end{cases}$$

The latter condition means that we do not admit tensile contact forces. In order to ensure no tension, it is convenient to introduce an index set $A$, defined for any displacement $s$ by

$$A(\theta) = \{i : \hat{n}_i \cdot \vec{s} > 0, \quad i = 1, 2, \ldots, n\}$$

This index set $A$ is a function of the angle of displacement $\theta_s$ and is essentially a list of contact planes for which the block face maintains a positive normal contact force with the rock mass. Now the potential energy of the system can be written as

$$V = \sum_{i \in A} 1/2k_i(\hat{n}_i \cdot \vec{s})^2 - R_N \cdot \vec{s}$$

where $R_N$ is the component of the active resultant force acting in the plane perpendicular to the potential sliding direction (Fig. 5). Expanding the above, and noting that the $n_i$ are unit vectors, we obtain

$$V = \sum_{i \in A} 1/2k_i s^2 \cos^2(\theta_s - \theta_i) - R_N \cdot s \cdot \cos(\theta_s - \theta_r)$$

where $R_N$ and $s$ are magnitudes. In the above expression, $\theta_i$ and $\theta_r$ give the direction of each unit normal $\hat{n}_i$ and the resultant force vector, respectively, measured clockwise from the positive $Y$ axis.
Equilibrium condition

With the assumption of no rotation, the block displacement in the YZ plane has two degrees of freedom, \( \theta_s \) and \( s \). The equilibrium displacement is one for what the total potential energy \( V \) of the system is stationary. Thus, for equilibrium we have the requirements:

\[
\frac{\partial V}{\partial \theta_s} = 0 \quad \text{and} \quad \frac{\partial V}{\partial s} = 0
\]  

(6)

where we note that, although the set \( A \) changes with \( s \), the function \( V \) is piecewise continuous.

Differentiating, we obtain,

\[
\frac{\partial V}{\partial \theta_s} = \sum_{i \in A} (-k_i) s^2 \cos(\theta_s - \theta_i) \sin(\theta_s - \theta_i) + R_N s \sin(\theta_s - \theta_r) = 0
\]

(7)

and

\[
\frac{\partial V}{\partial s} = \sum_{i \in A} k_i s \cos^2(\theta_s - \theta_i) - R_N \cos(\theta_s - \theta_r) = 0
\]

(8)

Assuming for the present that the stiffness \( k_i \) is known for each contact plane, we now have two Equations (7 and 8), and two unknowns, \( \theta_s \) and \( s \). We let \( \theta_s^* \) and \( s^* \) denote, respectively, the values of \( \theta_s \), and \( s \) which satisfy Equations 7 and 8.

Then, solving for \( s^* \), we obtain,

\[
s^* = \frac{R_N \sin(\theta_s^* - \theta_r)}{\sum_{i \in A} k_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i)}
\]

(9)

and

\[
s^* = \frac{R_N \cos(\theta_s^* - \theta_r)}{\sum_{i \in A} k_i \cos^2(\theta_s^* - \theta_i)}
\]

(10)

Equating Equations 9 and 10, we obtain,

\[
\frac{R_N \sin(\theta_s^* - \theta_r)}{\sum_{i \in A} k_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i)} = \frac{R_N \cos(\theta_s^* - \theta_r)}{\sum_{i \in A} k_i \cos^2(\theta_s^* - \theta_i)}
\]

(11)

which simplifies to

\[
\tan(\theta_s^* - \theta_r) = \frac{\sum_{i \in A} k_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} k_i \cos^2(\theta_s^* - \theta_i)}
\]

(12)

The value of \( \theta_s^* \), which satisfies Equation 12, is the direction of the small displacement of the block, perpendicular to the potential sliding direction, such that the potential energy of the system is minimized. However, the equation includes the unknown spring constants \( k_i \).

Spring constant \( k_i \)

In the elastic model we assume that the spring constant \( k_i \) for each contact plane is proportional to the contact area and therefore, due to the prismatic shape, to the length \( L \) of a planar contact in the YZ plane. This assumption is reasonable given the behaviour of springs in parallel: two parallel springs each with a stiffness \( k \) yield an effective stiffness of \( 2k \). If the spring constants are replaced in Equation 12 by the product \( cL_i \), where the constant \( c \) is the unknown constant of proportionality, and \( L_i \) is the length of plane \( i \) perpendicular to \( R_r \) we obtain from which we obtain the value of \( \theta_s^* \).

\[
\tan(\theta_s^* - \theta_r) = \frac{\sum_{i \in A} cL_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} cL_i \cos^2(\theta_s^* - \theta_i)}
\]

(13)

Since the \( c \) is constant, it can be cancelled, yielding

\[
\tan(\theta_s^* - \theta_r) = \frac{\sum_{i \in A} cL_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} cL_i \cos^2(\theta_s^* - \theta_i)}
\]

(14)

We then use a numerical routine to solve the equation

\[
\sum_{i \in A} cL_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i) = \frac{\sum_{i \in A} cL_i \cos^2(\theta_s^* - \theta_i)}{\sum_{i \in A} cL_i \cos^2(\theta_s^* - \theta_i)} - \tan(\theta_s^* - \theta_r) = 0
\]

(15)

The data required for determination of \( \theta_s^* \) are the angles \( \theta_i \), and lengths \( L_i \), of the \( n \) planes of discontinuity and the direction of the resultant force, \( \theta_r \).

With \( \theta_s^* \) known, the magnitude of the displacement, \( s^* \), for minimum potential energy can
be obtained by substituting the value of $\theta_s^*$ into Equation 9 or 10. For example, replacing $k_i$, by $(c/L_i)$ in Equation 10, we have,

$$ s^* = \frac{R_N}{c} \frac{\cos(\theta_s^* - \theta_r)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} $$  \hfill (16)

**Distribution of normal forces**

The magnitude of the normal force for each discontinuous surface due to the small displacement, $s$ is computed as:

$$ N_i = k_i (\hat{n}_i \cdot \hat{s}^*) = c L_i (\hat{n}_i \cdot \hat{s}^*) $$  \hfill (17)

or

$$ N_i = c s^* L_i \cos(\theta_s^* - \theta_i) $$  \hfill (18)

The friction force can be determined for each contact plane by

$$ F_i = \tan \phi_i [c s^* L_i \cos(\theta_s^* - \theta_i)] $$  \hfill (19)

where $\phi_i$ is the angle of friction on plane $i$. If we assume the friction angle is the same for each plane, noting that this assumption, although convenient, is by no means necessary, we arrive at

$$ \sum_{i=1}^{n} F_i = \tan \phi c s^* \sum_{i \in A} L_i \cos(\theta_s^* - \theta_i) $$  \hfill (20)

for the total frictional resistance. Substituting $s^*$ from Equation 16:

$$ \sum_{i=1}^{n} F_i = \tan \phi \left[ \frac{R_N}{c} \frac{\cos(\theta_s^* - \theta_r)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} \right] \cdot \sum_{i \in A} L_i \cos(\theta_s^* - \theta_i) $$  \hfill (21)

which simplifies to

$$ \sum_{i=1}^{n} F_i = R_N \tan \phi \cos(\theta_s^* - \theta_r) \frac{\sum_{i \in A} L_i \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} $$  \hfill (22)

where $A$ is the index set defined previously. Equation 22 represents the total resisting force against sliding for the contact faces of a prismatic rock block.

**FACTOR OF SAFETY AGAINST SLIDING**

The safety factor is defined in the usual way as:

$$ FS = \frac{\text{Resisting force}}{\text{Driving force}} $$  \hfill (23)

where the numerator is the summation of all friction forces, and the denominator is $R_T$, the component of the active resultant force acting parallel to the potential sliding direction. The safety factor is therefore given by:

$$ FS = \frac{R_N \tan \phi \cos(\theta_s^* - \theta_r)}{R_T} \frac{\sum_{i \in A} L_i \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} $$  \hfill (24)

Replacing $R_N$ and $R_T$ by $(R \sin \delta)$ and $(R \cos \delta)$, respectively, the magnitude of $R$ cancels out, so that we obtain,

$$ FS = \tan \theta \tan \delta \frac{\sum_{i \in A} L_i \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} $$  \hfill (25)

Applications

Having developed an analytical model for the factor of safety against sliding of an arbitrary prismatic block, we now apply the model to some particular cases: a two-plane wedge, a three-plane wedge and an example from Bregalnica river involving a block with a curved sliding surface.

The two plane prismatic block is the classic wedge; we use this to verify the energy-based stability model. Figure 6 shows three wedges with dip/dip direction of the bounding planes as given.

Replacing $R_N$ and $R_T$ by $(R \sin \delta)$ and $(R \cos \delta)$, respectively, the magnitude of $R$ cancels out, so that we obtain,

$$ FS = \frac{\sum_{i \in A} L_i \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} $$  \hfill (25)

where $A$ is the index set defined previously. Equation 22 represents the total resisting force against sliding for the contact faces of a prismatic rock block.

Geologica Macedonica, 23, 63–71 (2009)
solutions lends credence to the model described in this paper. In fact, the agreement may be made arbitrarily close by increasing the number of iterations in the numerical solution to Equation 15.

![Fig. 6. Stability analysis of three wedges; comparison of results between energy-based model and analytical solution](image)

<table>
<thead>
<tr>
<th>Plane 1: 40/235</th>
<th>Plane 1: 40/250</th>
<th>Plane 1: 20/255</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane 2: 50/85</td>
<td>Plane 2: 30/100</td>
<td>Plane 2: 50/125</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wedge 1</th>
<th>Wedge 2</th>
<th>Wedge 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε_F (model)</td>
<td>1.9495834</td>
<td>2.46302</td>
</tr>
<tr>
<td>ε_F (analytical)</td>
<td>1.9495835</td>
<td>2.46303</td>
</tr>
</tbody>
</table>

We now consider prismatic block with three contact planes. The distribution of contact forces for such a block is statically indeterminate and therefore the stability cannot be determined by standard limiting equilibrium methods. The model described in this paper determines the configuration of contact forces corresponding to minimum potential energy of the system, from which the factor of safety may be calculated. The initial data are in Table 1, where \( f \) is the dip and \( a \) the dip direction of each discontinuity plane.

<table>
<thead>
<tr>
<th>Plane</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>45°</td>
<td>23°</td>
<td>45°</td>
<td></td>
</tr>
<tr>
<td>( a^\ast )</td>
<td>120°</td>
<td>185°</td>
<td>250°</td>
</tr>
</tbody>
</table>

The friction angle on each plane is 20°. Fig. 7 graphs the factor of safety of the block against the length of the middle plane relative to the left and right planes. The main result is that there is a reduction in the factor of safety when the length ratio \( L_2/L_1 \) increases. Upper and lower bounds for the factor of safety for the wedge (\( L_2 = 0 \)) and plane (\( L_2 = \infty \)), calculated from the equations in Hoek and Bray (1981), are shown as horizontal lines. The factor of safety for the three-plane prismatic block is shown to range between these limiting values depending on the length of the middle plane. Figure 8 shows the change in the factor of safety versus the length ratio for a range of friction angles.

![Fig. 7. Factor of safety versus length ratio \( L_2/L_1 \) where \( L_1 \) and \( L_2 \) are the lengths of the end and middle planes, respectively. Upper and lower bounds correspond to the wedge and plane cases](image)

![Fig. 8. Reduction of the factor of safety with change in the length ratio \( L_2/L_1 \), for a range of friction angles](image)
EXAMPLE FROM FLOW TO BREGALNICA RIVER

The analytical model described above was used in a sliding stability investigation of a flow to the Bregalnica river (Bregalnica), in the east part of Macedonia. We analyzed the rock block shown in Fig. 9, in which the horizontal dimension is approximately 4 m. A line drawing of the block and the potential sliding surface is shown in Fig. 10.

The first step in the stability analysis is to discretize the sliding surface into a series of planar segments, as shown in Fig. 9. For each segment, the length $L$, and the angle $\theta$, (measured clockwise from the $Y$ direction) corresponding to each planar segment is measured. The discretized failure surface must be a true profile, i.e. a profile in the plane perpendicular to the sliding direction (in this case, the fold axis). The plunge of the line of intersection was determined by standard structural geologic methods (e.g., Ramsay, 1967), to be 32°.

Two separate analyses were carried out, with the results presented in Table 2. For the Case 1 analysis we treated the block as a standard wedge bounded by the limiting planes in Fig. 11 and determined the factor of safety from the equation given by Hoek and Bray (1981). For Case 2 we analyzed the prismatic block with four contact planes and the geometry in Fig. 11 by the methods of this paper. As shown in Table 2, the factor of safety according to Case 1 was 1.14, whereas that determined from Case 2 was 1.0. Although there is evidence of previous translational failures in this particular road cut, we do not consider this an accurate back analysis. The friction angle used in the analysis is an estimated value and several of the other parameters, in particular the water conditions, are poorly known. What is clear, however, is that the factor of safety for the prismatic block with four sliding surfaces is significantly lower than that obtained from the wedge analysis. Thus treating this type of block as a wedge is, in general, unconservative.

<table>
<thead>
<tr>
<th>Plane No.</th>
<th>$L$ [cm]</th>
<th>$\theta$ (°)</th>
<th>$\phi$ (°)</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: Wedge analysis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>240</td>
<td>37.5</td>
<td>27</td>
<td>1.14</td>
</tr>
<tr>
<td>2</td>
<td>117</td>
<td>126</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>Case 2: Prismatic block analysis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>193</td>
<td>37.5</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>36</td>
<td>90</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>106</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>55</td>
<td>126</td>
<td>27</td>
<td>1.0</td>
</tr>
</tbody>
</table>

A friction angle of $27^\circ$ was used in the analysis in accordance with friction angles for shale and siltstone reported by Hoek and Bray (1981). The applied load was assumed to include self-weight only and hydrostatic pressures on the sliding surface were assumed to be zero.
CONCLUSIONS

In this paper we have described an energy-based approach to the determination of sliding stability of rock wedges with \( n \) cozonal sliding surfaces. Traditional methods of stability analysis of plane \((n = 1)\) and wedge \((n = 2)\) systems are based on limiting equilibrium of statically determinate systems. If the number \( n \) of contact planes is three or more, the distribution of normal forces among these planes is statically indeterminate.

We determine the distribution of normal forces corresponding to minimum potential energy of the system. Since we assume that effective normal stiffness is proportional to contact surface area, elastic moduli are not required. Thus we construct a general model for sliding stability analysis of blocks with any number of cozonal sliding surfaces. Since this is a general model, it includes plane and wedge failures as special cases, and indeed our results agree with the standard formulae for plane and wedge slides.

Field situations which give rise to \( n \) cozonal sliding surfaces with \( n > 3 \) include wedges bounded by joint planes and internally subdivided by a third plane such as bedding, yielding blocks with three potential sliding surfaces such as that of Fig. 10, and blocks bounded by cylindrical folds such as that of Fig. 11. One of the practical implications of this work is that there may be a significant reduction of the factor of safety when three or more discontinuity planes form the sliding surface, as compared with the wedge case. This is important because many practitioners would analyze such blocks as wedges, and in doing so would arrive at an overly high factor of safety.

Future research plans on this topic will be directed towards development of a computer model for sliding stability analysis for digitized potential failure surfaces. This will open the way to rock slope stability analysis based on photogrammetric data and to a single coherent model which incorporates plane and wedge slides.

APPENDIX

Definition of terms

1. \( s \) is the magnitude of a small translation of the prismatic block, directed into the rock mass, in the direction perpendicular to the potential sliding direction.
2. \( \theta \) is the angle defining the direction of the small translation.
3. \( k_i \) is the normal stiffness of plane \( i \).
4. \( n_i \) is the unit normal vector for each contact plane, directed out of the block.
5. \( \theta \) is the angle of each plane measured from the positive \( X \) axis.
6. \( L \) is the length of each contact plane in the \( YZ \) plane.
7. \( \theta \) is the angle defining the direction of the resultant force.
8. \( R \) is the resultant of the active forces acting on the rock block. \( R \) has components \( RN \) and \( RT \), perpendicular to and parallel to the sliding direction respectively.
9. \( I \) is the line of intersection between the contact planes.
10. \( i \) is a unit vector in the direction of the line of intersection \( I \).

The terms defined in (3)...(10) are initially known from the loading conditions or the block geometry.

The variables (1) and (2), which determine the displacement vectors \( s \), are initially unknown; their values are determined such that the potential energy of the system is minimized.

REFERENCES


Резиме

СТАБИЛНОСТ НА КАРПЕСТИТЕ МАСИ ПРОБИЕНИ СО ПОВЕЌЕКРАТНО ЛИЗГАЊЕ НА ПОВРШИНИ

Дељо Каракашев1, Тена Шијакова-Иванова1, Елизабета Каракашева1, Зоран Панов2

1Универзитет „Гоце Делчев“, Факултет за природни и ензинички науки, Институт за геолошкa, Гоце Делчев 89,МК-2000 Шиbи, Република Македонијa
2Машински факултети, МК-Виница, Република Македонијa
deljo.karakasev@ugd.edu.mk

Ключни зборови: стабилност на падини; лизгави површини

И покраj тоa што стабилностa на некоj падини и лизгави повrшини е дефинирана во геотехниката и во стручна линтура, сепак во природата и во нашата околина во коja живееме секогаш не е можно лесно да се дефинира и моделира. Тие површини можат да создаваат цилиндрични геоморфолошки седиментни структури во кои jасно можат да се издвоjат базични делови од цилиндрав коеj се дуплираат. Новите анализи и модели на ваквите структури се дадени во овоj труд. Моделите се направени наjпрвин како тие да се нормални со помош на пресметувањe на силите коj деjствуваат на системот и инициираната потенцијална енергиja, за потоа преку овие параметри постепено да се дефинира повеќекратното лизгањe на структурите спомнати погоре, со што се дефинира и коjфициентот на стабилностa на истражуванот терен. На пример, земен е профил по течението на река Брегалница и направени се анализи коj се прикажани во овоj труд.

Geologica Macedonica, 23, 63–71 (2009)