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STABILITY ANALYSIS OF ROCK WEDGES WITH MULTIPLE SLIDING SURFACES

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A b s t r a c t: Although wedge and plane sliding stability analyses are well established in the geotcchnical literature, certain geologic environments produce blocks which cannot be adequately modelled as either wedges or plane slides. An example is blocks forming in cylindrically folded sedimentary rocks, where the surface of sliding is neither a single plane nor a double plane but is curved. This type of block may be idealized as a prismatic block with multiple sliding planes, all with parallel lines of intersection. If the sliding planes number three or more, the distribution of normal forces and hence the factor of safety is indeterminate. A new analytical model for sliding stability analysis is described in which the distribution of normal forces on the contact planes is chosen to minimize the potential energy of the system. The classic wedge and plane solutions are shown to be special cases of this more general model, which allows determination of the safely factor for any shape of prismatic contact surface. An example from block part of Bregalnica river with a curved sliding surface is described and the factor of safety compared with the standard wedge analysis. It is shown that with three or more contact planes, the safety factor may be significantly lower than that calculated from the wedge model, which provides an upper limit on stability.

Key words: rock slope stability; wedge slides

INTRODUCTION

Stability analyses for wedge and plane failures are now well established in the geotechnical literature. Some of the early work on the subject is due to Londe et al. (1969,1970), John (1968), Wittke (1965, 1990) and Goodman (1976, 1989). Other developments and useful summaries are given by Hoek and Bray (1981), Goodman and Shi (1985), Giani (1992), Warburton (1993), Einstein (1993) and Watts (1994), among others. The requirements for *I* sliding may be summarized as follows. Plane failures can occur when the strike of a discontinuity plane such as bedding is approximately parallel to the strike of the slope face and the weak plane daylights in the free face at a dip angle greater than the friction angle. Wedge failures can occur for a block defined by two planes whose line of intersection daylights in the free face and plunges sufficiently steeply that the destabilizing forces exceed the shear resistance. Typical geometries for wedge and plane failures are depicted in Fig. 1.

The important factors in the solution of stability problems are the shear strength, dip and dip direction of the discontinuity planes, the geometry of the slope and the loading conditions.

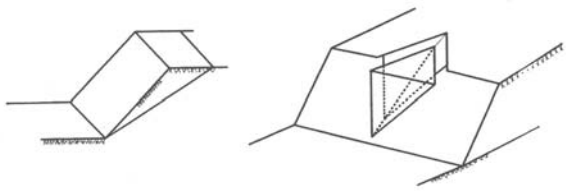

Fig. 1. Typical geometries for rock slope translational failures: (a) plane failure; (b) wedge failure

The system of forces governing plane slides is statically determined and the frictional shear resistance can be determined by resolution of forces. Wedge slides are rendered statically determinate by making an assumption about shear stresses in the contact planes, namely that shear stresses vanish in the plane perpendicular to the potential sliding direction. Although this assumption is rarely discussed in the literature (Chan and Einstein, 1981), it is implicit in the classic wedge analysis (e.g., Hoek and Bray, 1981). With the abovementioned assumption, and if strengths and water pressures, etc., are known, the factor of safety for plane and wedge slides can be obtained directly by limiting equilibrium methods, either by direct calculation or by graphical methods based on stereographic projection. With three or more sliding planes, however (e.g., Fig. 2), the distribution of normal forces is statically indeterminate and the net frictional resistance to sliding, and hence the factor of safety, cannot be uniquely determined. In this paper (see also Ureta, 1994; Mauldon and Ureta, 1994, 1995) we describe an energy method for determination of the factor of safety against sliding failure for blocks with multiple sliding planes that form a cylindrical surface. Such a block is referred to as a prismatic block.

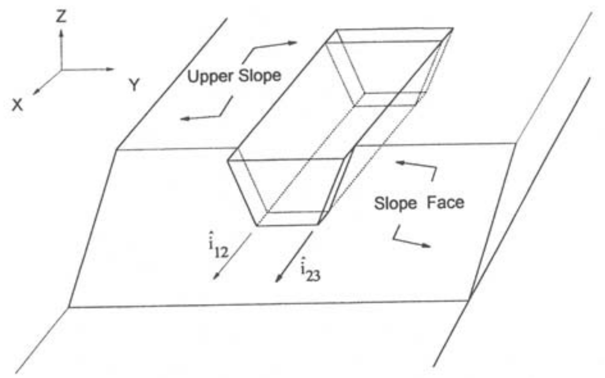

Fig. 2. Prismatic rock block with three contact planes; coordinate system defined

PRISMATIC BLOCKS

A prismatic block is a rock block bounded by n contact planes, where the lines of intersection of the contact planes are all parallel. Fig. 2 shows a special case of a prismatic block with three planes of contact with the rock mass. Also, in Fig. 2, a Cartesian coordinate system is defined with the X axis parallel to the line of intersection (the potential sliding direction) and Y horizontal. The contact planes of a prismatic block are cozonal (to use crystallographic terminology, e.g., Bloss, 1961), with zone axis parallel to the line of intersection, so that the normals of the contact planes fall on a great circle when plotted in stereographic projection (Fig. 3). As the number of planes (n) approaches infinity ($n \to \infty$), the contact surface approaches a curved cylindrical shape. Thus a block defined by cylindrically folded bedding or foliation is a limiting case of a prismatic block. Plane and wedge failures, with one and two contact planes, and zero and one lines of intersection respectively, are special cases of prismatic blocks.

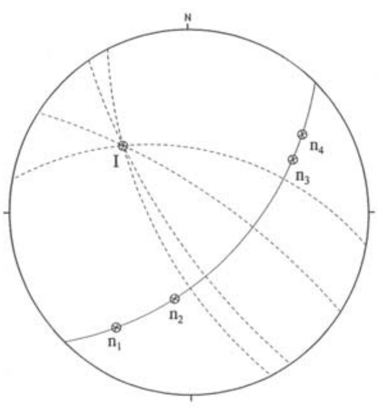

Fig. 3. Stereographic projection showing geometry of a prismatic block. Contact plane normals fall on a great circle. The lines of intersection of contact planes are parallel to the zone axis I

Statement of problem

As discussed above, if $n \leq 2$ the distribution of normal forces on the contact plane(s) is statically determinate. If $n \geq 3$, however, the normal forces on the planes of contact between a prismatic block and the rock mass are statically indeterminate and the stability conditions cannot be determined directly by limiting equilibrium methods. In this paper we describe an energy method to determine the contact forces and to evaluate the factor of safety against sliding of prismatic rock blocks as a function of shear strength, block geometry, and loading conditions. The block is assumed to be supported at each face by a series of normal springs, each with stiffness k_i and each spring subject to a "no tension" condition. The magnitude of the stiffness k_i at each contact face is assumed to be proportional to the planar contact area per unit length. We assume an elastic, conservative system to obtain the distribution of normal forces that minimizes the potential energy of the system

Assumptions

The stability analysis is based on a simplified model of rock mass geometry and strength, with the following assumptions:

– Contact faces of the prismatic block are planar.

– The displacement of the block is purely translational.

– Frictional shear stresses act parallel to the sliding direction only. Note that this assumption is standard in limiting equilibrium analysis of plane and wedge slides.

– The block is undeformable, except for elastic contacts at the bounding faces.

– The block is acted on by an active resultant $\vec{R}$ which includes self-weight, and may in addition include hydraulic forces, seismic forces, or supporting forces due to anchors or bolts. The effect of moments is not considered in the analysis.

– The normal stiffness of each contact plane is proportional to its surface area.

Analytical model

The total potential energy of a system consisting of a prismatic rock block supported by n contact planes is given by:

$$V = \sum_{i=1}^{n} V_i - W_r \tag{1}$$

where V_i , is the elastic potential energy associated with each discontinuity surface i = 1, 2,..., n, and W_r is the work done by the active forces acting on the block. If self-weight is the only active force, the work done by the active forces is the negative of the change in gravitational potential energy. We assume that the block itself is undeformable, but that elastic deformation occurs at the contacts between the block and the rock mass. We assume that the block-rock mass contacts behave like linear springs, with stiffness proportional to surface area and we establish a datum for the gravitational potential energy such that the contact springs initially have zero extension.

The first task will be to find the equilibrium distribution of forces in the YZ plane (the plane perpendicular to the potential sliding direction), such that the total potential energy is a minimum. We assume that the equilibrium position in the YZ plane of the block results from a small translational displacement of the block in the YZ plane (i.e., perpendicular to the sliding direction) under the action of the active forces. Elastic strain energy in the spring contacts is stored as a result of this displacement. We denote this small displacement by a vector $\vec{s}$ at an angle θ_s and with magnitude s (see Fig. 4). Then, for unit normal $\widehat{n_i}$ and stiffness k_i corresponding to each contact plane, the elastic potential energy V_i at each contact face is given by

$$V_i = \begin{cases} 1/2k_i(\hat{n}_i \cdot \vec{s})^2 & \text{if} \quad (\hat{n}_i \cdot \vec{s}) > 0 \\ 0 & \text{if} \quad (\hat{n}_i \cdot \vec{s}) \le 0 \end{cases} \tag{2}$$

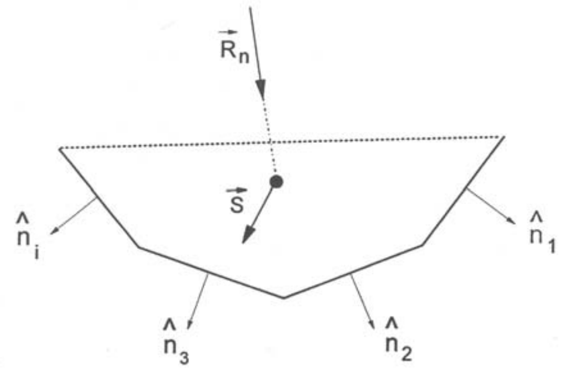

Fig. 4. Prismatic block in the YZ plane showing unit normal vectors n_i, normal component R_n of active resultant and displacement vector S

The latter condition means that we do not admit tensile contact forces. In order to ensure no tension, it is convenient to introduce an index set A, denned for any displacement s by

$$A = (\theta_s) = \{i : \hat{n}_i \cdot \vec{s} > 0, \quad i = 1, 2, ..., n\} \tag{3}$$

This index set A is a function of the angle of displacement θ_s and is essentially a list of contact planes for which the block face maintains a positive normal contact force with the rock mass. Now the potential energy of the system can be written as

$$V = \sum_{i \in A} 1/2k_i(\hat{n}_i \cdot \vec{s})^2 - \vec{R}_N \cdot \vec{s} \tag{4}$$

where $\vec{R}_N$ is the component of the active resultant force acting in the plane perpendicular to the potential sliding direction (Fig. 5). Expanding the above, and noting that the n_i are unit vectors, we obtain,

$$V = \sum_{i \in A} 1/2k_i s^2 \cos^2(\theta_s - \theta_i) - R_N \cdot s \cdot \cos(\theta_s - \theta_r) \tag{5}$$

where R_N and s are magnitudes. In the above expression, θ_i and θ_r give the direction of each unit normal $\hat{n}_i$ and the resultant force vector, respectively, measured clockwise from the positive Y axis.

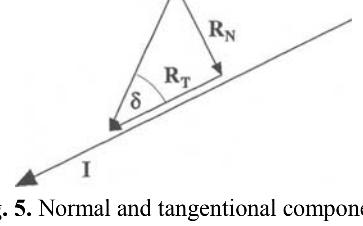

Fig. 5. Normal and tangentional components of active resultant

Equilibrium condition

With the assumption of no rotation, the block displacement in the *YZ* plane has two degrees of freedom, θ_s and s. The equilibrium displacement is one for what the total potential energy V of the system is stationary. Thus, for equilibrium we have the requirements:

$$\frac{\partial V}{\partial \theta_s} = 0 \quad \text{and} \quad \frac{\partial V}{\partial s} = 0 \tag{6}$$

where we note that, although the set A changes with θ_s, the function V is piecewise continuous. Differentiating, we obtain,

$$\frac{\partial V}{\partial \theta_s} = \sum_{i \in A} (-k_i) s^2 \cos(\theta_s - \theta_i) \sin(\theta_s - \theta_i + \\ + R_N s \cdot \sin(\theta_s - \theta_r) = 0 \tag{7}$$

and

$$\frac{\partial V}{\partial s} = \sum_{i \in A} k_i \cdot s \cdot \cos^2(\theta_s - \theta_i) - R_N \cos(\theta_s - \theta_r) = 0 \tag{8}$$

Assuming for the present that the stiffness k_j is known for each contact plane, we now have two Equations (7 and 8), and two unknowns, θ_s and s. We let θ_s^* and s^* denote, respectively, the values of θ_s, and s which satisfy Equations 7 and 8. Then, solving for s^*, we obtain,

$$s^* = \frac{R_N \sin(\theta_s^* - \theta_r)}{\sum\limits_{i \in A} k_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i)} \tag{9}$$

and

$$s^* = \frac{R_N \cos(\theta_s^* - \theta_r)}{\sum_{i \in A} k_i \cos^2(\theta_s^* - \theta_i)} \tag{10}$$

Equating Equations 9 and 10, we obtain,

$$\frac{R_N \sin(\theta_s^* - \theta_r)}{\sum_{i \in A} k_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i)} = \\ = \frac{R_N \cos(\theta_s^* - \theta_r)}{\sum_{i \in A} k_i \cos(\theta_s^* - \theta_i)} \tag{11}$$

which simplifies to

$$\tan(\theta_s^* - \theta_r) = \frac{\sum_{i \in A} k_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} k_i \cos^2(\theta_s^* - \theta_i)} \tag{12}$$

The value of θ_s^* which satisfies Equation 12 is the direction of the small displacement of the block, perpendicular to the potential sliding direction, such that the potential energy of the system is minimized. However, the equation includes the unknown spring constants k_i.

Spring constant k_i

In the elastic model we assume that the spring constant k_i for each contact plane is proportional to the contact area and therefore, due to the prismatic shape, to the length L of a planar contact in the *YZ* plane. This assumption is reasonable given the behaviour of springs in parallel: two parallel springs each with a stiffness k yield an effective stiffness of $2k$. If the spring constants are replaced in Equation 12 by the product c, L_i, where the constant c is the unknown constant of proportionality, and L_i, is the length of plane i perpendicular to R_T we obtain from which we obtain the value of θ_s^*.

$$\tan(\theta_s^* - \theta_r) = \frac{\sum_{i \in A} cL_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} cL_i \cos^2(\theta_s^* - \theta_i)} \tag{13}$$

Since the c is constant, it can be cancelled, yielding

$$\tan(\theta_s^* - \theta_r) = \frac{\sum_{i \in A} cL_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} \tag{14}$$

We then use a numerical routine to solve the equation

$$\frac{\sum_{i \in A} cL_i \sin(\theta_s^* - \theta_i) \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} - \tan(\theta_s^* - \theta_r) = 0 \tag{15}$$

The data required for determination of θ_s^* are the angles θ_i, and lengths L_i, of the n planes of discontinuity and the direction of the resultant force, θ_r.

With θ_s^* known, the magnitude of the displacement, s^*, for minimum potential energy can be obtained by substituting the value of θ_s^* into Equation 9 or 10. For example, replacing k_i, by (c) (L_i) in Equation 10, we have,

$$s^* = \frac{R_N}{c} \frac{\cos(\theta_s^* - \theta_r)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} \tag{16}$$

Distribution of normal forces

The magnitude of the normal force for each discontinuous surface due to the small displacement, s is computed as:

$$N_i = k_i(\hat{n}_i \cdot \vec{s}^*) = cL_i(\hat{n}_i \cdot \vec{s}^*) \tag{17}$$

or

$$N_i = cs^* L_i \cos(\theta_s^* - \theta_i) \tag{18}$$

The friction force can be determined for each contact plane by

$$F_i = \tan\phi_i [cs^* L_i \cos(\theta_s^* - \theta_i)] \tag{19}$$

where ϕ_i is the angle of friction on plane i. If we assume the friction angle is the same for each plane, noting that this assumption, although convenient, is by no means necessary, we arrive at

$$\sum_{i=1}^{n} F_i = \tan\phi cs^* \sum_{i \in A} L_i \cos(\theta_s^* - \theta_i) \tag{20}$$

for the total frictional resistance. Substituting s^* from Equation 16:

$$\sum_{i=1}^{n} F_i = \tan\phi \left[\frac{R_N}{c} \frac{\cos(\theta_s^* - \theta_r)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} \right] \cdot \sum_{i \in A} L_i \cos(\theta_s^* - \theta_i) \tag{21}$$

which simplifies to

$$\sum_{i=1}^{n} F_i = R_N \tan\phi \cos(\theta_s^* - \theta_r) \frac{\sum_{i \in A} L_i \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} \tag{22}$$

where A is the index set defined previously. Equation 22 represents the total resisting force against sliding for the contact faces of a prismatic rock block.

FACTOR OF SAFETY AGAINST SLIDING

The safety factor is defined in the usual way as:

$$FS = \frac{\text{Resisting force}}{\text{Driving force}} \tag{23}$$

where the numerator is the summation of all friction forces, and the denominator is R_T, the component of the active resultant force acting parallel to the potential sliding direction. The safety factor is therefore given by:

$$FS = \frac{R_N}{R_T} \tan\phi \cos(\theta_s^* - \theta_r) \frac{\sum_{i \in A} L_i \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} \tag{24}$$

Replacing R_N and R_T by ($R \sin\delta$) and ($R \cos\delta$), respectively, the magnitude of R cancels out, so that we obtain,

$$FS = \tan\theta \tan\delta \cos \frac{\sum_{i \in A} L_i \cos(\theta_s^* - \theta_i)}{\sum_{i \in A} L_i \cos^2(\theta_s^* - \theta_i)} \tag{25}$$

Applications

Having developed an analytical model for the factor of safety against sliding of an arbitrary prismatic block, we now apply the model to some particular cases: a two-plane wedge, a three-plane wedge and an example from Bregalnica river involving a block with a curved sliding surface.

The two plane prismatic block is the classic wedge; we use this to verify the energy-based stability model. Figure 6 shows three wedges with dip/dip direction of the bounding planes as given.

In each case the view is up the line of intersection of the two planes, i.e., opposite to the potential sliding direction. Loading is gravitational and the friction angle in each case is assumed to be 20°. On successive rows in the table (Fig. 6) are given the equilibrium angle of the displacement in the YZ plane (which is noticeably different for the three cases even though the loading is the same), the factor of safety as determined from the new energy-based model and the factor of safety determined from the well-known analytical solution for wedge slides. The good agreement between the solutions lends credence to the model described in this paper. In fact, the agreement may be made arbitrarily close by increasing the number of iterations in the numerical solution to Equation 15.

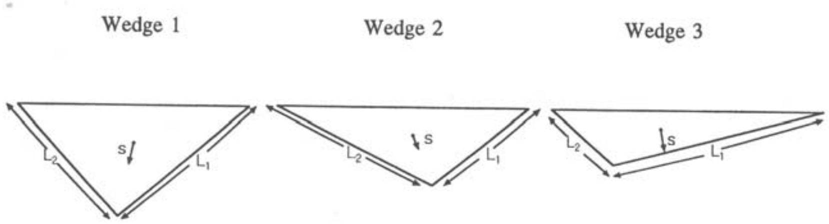

Fig. 6. Stability analysis of three wedges; comparison of results between energy-based model and analytical solution

Plane 1: 40/235	Plane 1: 40/250	Plane 1: 20/255
Plane 2: 50/85	Plane 2: 30/100	Plane 2: 50/125

	Wedge 1	Wedge 2	Wedge 3
e_s^*	101.6°	67.7°	84.9°
F.S. (model)	1.9495834	2.46302	1.18806
F.S. (analytical)	1.9495835	2.46303	1.18807

We now consider prismatic block with three contact planes. The distribution of contact forces for such a block is statically indeterminate and therefore the stability cannot be determined by standard limiting equilibrium methods. The model described in this paper determines the configuration of contact forces corresponding to minimum potential energy of the system, from which the factor of safety may be calculated. The initial data are in Table 1, where f is the dip and a the dip direction of each discontinuity plane.

Table 1

Data for analysis of three-plane prismatic block

Plane	1	2	3
	45°	23°	45°
a"	120°	185°	250°

The friction angle on each plane is 20°. Fig. 7 graphs the factor of safety of the block against the length of the middle plane relative to the left and right planes. The main result is that there is a reduction in the factor of safety when the length ratio L_2/L, increases. Upper and lower bounds for the factor of safety for the wedge ($L_2 = 0$) and plane ($L_2 = {}^{\circ\circ}$), calculated from the equations in Hoek and Bray (1981), are shown as horizontal lines. The factor of safety for the three-plane prismatic block is shown to range between these limiting values depending on the length of the middle plane. Figure 8 shows the change in the factor of safety versus the length ratio for a range of friction angles

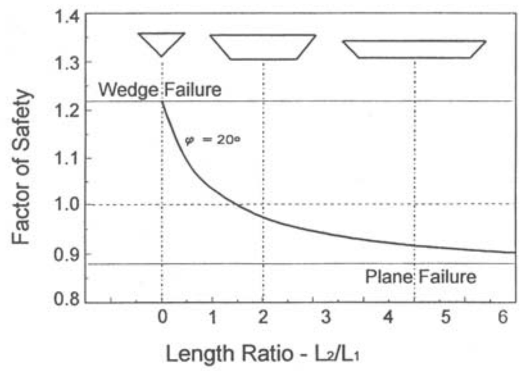

Fig. 7. Factor of safety versus length ratio L_2/L_1 where L_1 and L_2 are the lengths of the end and middle planes, respectively. Upper and lower bounds correspond to the wedge and plane cases

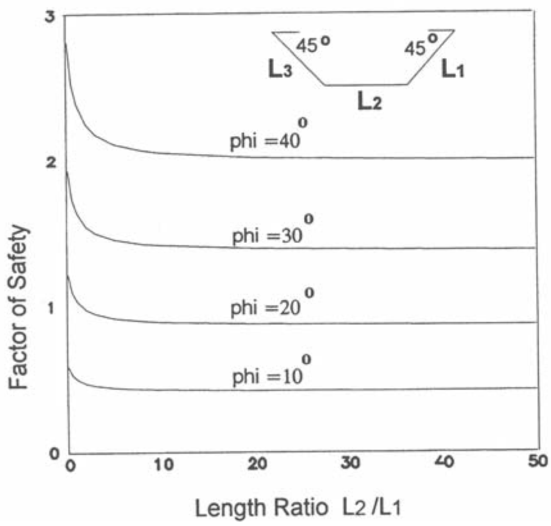

Fig. 8. Reduction of the factor of safety with change in the length ratio L_2/L, for a range of friction angles

EXAMPLE FROM FLOW TO BREGALNICA RIVER

The analytical model described above was used in a sliding stability investigation of a flow to the Bregalnica river (Bregalnica), in to the east part of Macedonia. We analyzed the rock block shown in Fig. 9, in which the horizontal dimension is approximately 4 m. A line drawing of the block and the potential sliding surface is shown in Fig. 10.

The first step in the stability analysis is to discretize the sliding surface into a series of planar segments, as shown in Fig. 9. For each segment, the length *L*, and the angle 8, (measured clockwise from the *Y* direction) corresponding to each planar segment is measured. The discretized failure surface must be a true profile, i.e. a profile in the plane perpendicular to the sliding direction (in this case, the fold axis). The plunge of the line of intersection was determined by standard structural geologic methods (e.g., Ramsay, 1967), to be 32°.

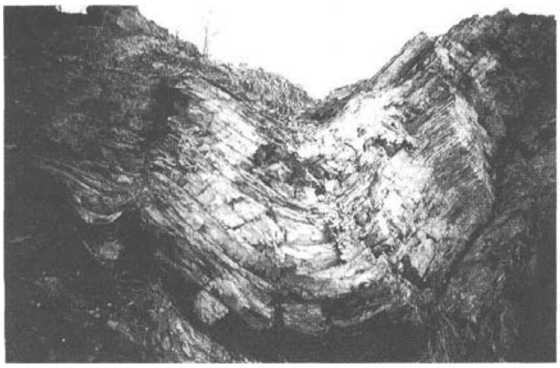

Fig. 9. Block bounded by curved sliding surface. Sevier Shale formation, Blount County, Bregalnica river. Horizontal dimension: 3 m

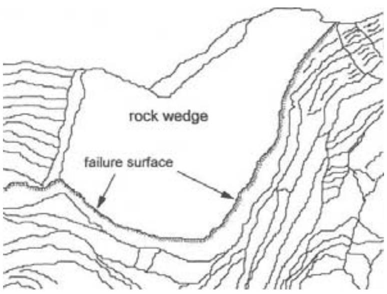

Fig.10. Line drawing of block showing potential sliding surface

A friction angle of 27° was used in the analysis in accordance with friction angles for shale and siltstone reported by Hoek and Bray (1981). The applied load was assumed to include self-weight only and hydrostatic pressures on the sliding surface were assumed to be zero.

Two separate analyses were carried out, with the results presented in Table 2. For the Case 1 analysis we treated the block as a standard wedge bounded by the limiting planes in Fig. 11 and determined the factor of safety from the equation given by Hoek and Bray (1981). For Case 2 we analyzed the prismatic block with four contact planes and the geometry in Fig. 11 by the methods of this paper. As shown in Table 2, the factor of safety according to Case 1 was 1.14, whereas that determined from Case 2 was 1.0. Although there is evidence of previous translational failures in this particular road cut, we do not consider this an accurate back analysis. The friction angle used in the analysis is an estimated value and several of the other parameters, in particular the water conditions, are poorly known. What is clear, however, is that the factor of safety for the prismatic block with four sliding surfaces is significantly lower than that obtained from the wedge analysis. Thus treating this type of block as a wedge is, in general, unconservative.

Table 2

Comparison of safety factor obtained from the wedge analysis (FS = 1.14) with safety factor obtained from the prismatic block analysis (FS = 1.00) for the block shown in Fig. 9

Fold axis: 32°, Trend 30°				
Plane No.	*L* [cm]	θ (°)	ϕ (°)	*FS*
Case 1: Wedge analysis				
1	240	37.5	27	
2	117	126	27	1.14
Case 2: Prismatic block analysis				
1	193	37.5	27	
2	36	90	27	
3	36	106	27	
4	55	126	27	1.0

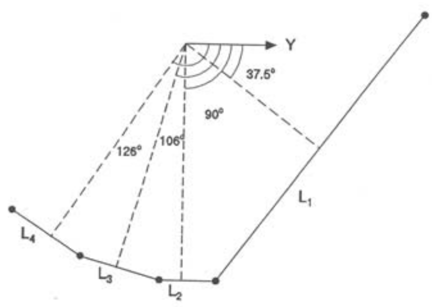

Fig. 11. Discretization of curved failure surface into planar segments

CONCLUSIONS

In this paper we have described an energy-based approach to the determination of sliding stability of rock wedges with *n* cozonal sliding surfaces. Traditional methods of stability analysis of plane (n = l) and wedge (n = 2) systems are based on limiting equilibrium of statically determinate systems. If the number *n* of contact planes is three or more, the distribution of normal forces among these planes is statically indeterminate.

We determine the distribution of normal forces corresponding to minimum potential energy of the system. Since we assume that effective normal stiffness is proportional to contact surface area, elastic moduli are not required. Thus we construct a general model for sliding stability analysis of blocks with any number of cozonal sliding surfaces. Since this is a general model, it includes plane and wedge failures as special cases, and indeed our results agree with the standard formulae for plane and wedge slides.

Field situations which give rise to *n* cozonal sliding surfaces with $n > 3$ include wedges bounded by joint planes and internally subdivided by a third plane such as bedding, yielding blocks with three potential sliding surfaces such as that of Fig. 10, and blocks bounded by cylindrical folds such as that of Fig. 11. One of the practical implications of this work is that there may be a significant reduction of the factor of safety when three or more discontinuity planes form the sliding surface, as compared with the wedge case. This is important because many practitioners would analyze such blocks as wedges, and in doing so would arrive at an overly high factor of safety.

Future research plans on this topic will be directed towards development of a computer model for sliding stability analysis for digitized potential failure surfaces. This will open the way to rock slope stability analysis based on photogrammetric data and to a single coherent model which incorporates plane and wedge slides.

APPENDIX

Definition of terms

(1) 5 is the magnitude of a small translation of the prismatic block, directed into the rock mass, in the direction perpendicular to the potential sliding direction.

(2) 0^ is the angle defining the direction of the small translation.

(3) k_t is the normal stiffness of plane *i*.

(4) n_i is the unit normal vector for each contact plane, directed out of the block.

(5) 9 is the angle of each plane measured from the positive *X* axis.

(6) *L* is the length of each contact plane in the *YZ* plane.

(7) 0_r is the angle defining the direction of the resultant force.

(8) *R* is the resultant of the active forces acting on the rock block. *R* has components *RN* and *RT*, perpendicular to and parallel to the sliding direction respectively.

(9) *I* is the line of intersection between the contact planes.

(10) *i* is a unit vector in the direction of the line of intersection *I*.

The terms defined in (3)...(10) are initially known from the loading conditions or the block geometry.

The variables (1) and (2), which determine the displacement vectors s, are initially unknown; their values are determined such that the potential energy of the system is minimized.

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Р е з и м е

СТАБИЛНОСТ НА КАРПЕСТИТЕ МАСИ ПРОБИЕНИ СО ПОВЕЌЕКРАТНО ЛИЗГАЊЕ НА ПОВРШИНИ

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Клучни зборови: стабилност на падини; лизгави површини

И покрај тоа што стабилноста на некои падини и лизгави површини е дефинирана во геотехниката и во струча литература, сепак во природата и во нашата околина во која живееме секогаш не е можно лесно да се дефинира и моделира. Тие површини можат да создаваат цилинрични геоморфолошки седиментни структури во кои јасно можат да се издвојат базични делови од цилиндарот кои се дуплираат. Новите анализи и модели на ваквите структури се дадени во овој труд. Моделите се направени најпрвин како тие да се нормални со помош на пресметување на силите кои дејствуваат на системот и иницираната потенцијална енергија, за потоа преку овие параметри постепено да се дефинира повеќекратното лизгање на структурите спомнати погоре, со што се дефинира и коефициентот на стабилност на истражуваниот терен. На пример, земен е профил по течението на река Брегалница и направени се анализи кои се прикажани во овој труд.