266
GEOME 2
Manuscript received: April 7, 2014
Accepted: June 16, 2014

Geologica Macedonica, Vol. **28**, No. 1, pp. 1–6 (2014)
In print: ISSN 0352 – 1206
On line: ISSN 1857 – 8586
UDC: 550.348.436.098.3-047.58

Original scientific paper

THEORETICAL MODEL FOR DEFINING SEISMIC ENERGY

Todor Delipetrov, Blagica Doneva, Marjan Delipetrev

Faculty of Natural and Technical Sciences, "Goce Delčev" University,
P.O.Box 201, MK 2000 Štip, Republic of Macedonia
todor.delipetrov@ugd.edu.mk

A b s t r a c t: The paper presents theoretical aspect in defining the energy of certain earthquake. The energy, as a physical quantity, is basic parameter with strong influence on all other characteristics of the earthquake. Because of the complex earthquake process, processes in the focus, as well as propagation of seismic waves in the ground, small number of researchers deals with space modelling of the earthquake. Larger is the number of those which, based on the numerous studied earthquakes, determined empirical formulas for determination of seismic energy, usually as a magnitude of a given earthquake. Given theoretical model can be a good base for defining the model of certain seismic area. Defining such type of model requires deeper knowledge of several scientific disciplines, with detailed studying of the seismic regime and geology of the terrain.

Key words: seismicity; energy; model; earthquake

INTRODUCTION

Earthquake is a natural occurrence that origin in the deep parts of the Earth with rapid release of energy in the focus area. Such released energy spread through the ground and on the Earth's surface is manifested as earthquake. Related to the depth of the focus, earthquakes are divided in: depth earthquakes – their focus is deeper from the limit of the Earth's crust (depth to 800 km) and earthquakes whose focuses are in the Earth's crust, so called tectonic earthquakes. Depth earthquakes are formed in the subduction zones of the tectonic plates, mixing the subduction material leads to complex physical-hemical processes that generate large quantity of energy which is reason for this type of earthquakes (1).

Tectonic earthquakes are with focus in the Earth's crust and occurred because of the tectonic movements in it.

There are many parameters that define the earthquake, but the most important is determination of the energy. Tectonic earthquakes occurred as a result of movements and strain in the rock masses. Tectonic processes generate increasing of the potential energy in given area of the Earth's crust. Over time, in one moment, this energy exceeds the limits of load-bearing capacity of the rock masses and in that moment comes to breaking, movement of the rock blocks in relation to each other, or comes to the abrupt releasing of accumulated potential energy in the form of kinetic or seismic energy (3).

THEORETICAL MODEL

At the origin of earthquakes, the potential energy of strain is quickly released and transformed in part of the kinetic energy of seismic (elastic) waves, and some other types of energy. Energy characterizes the essence of occurrence of earthquakes and is one of the main physical characteristics of the focus. The seismic energy, spreading in space, excites particles and they begin to oscillate. The energy released in the focus is greater, and the oscillation of the ground is more intense.

To see the parameters from which depend seismic energy, considered one elastic wave that spread along the y axis (at the coordinate system x, y, z). That wave movement is defined with the formula:

$$x = a \cos w \left(t - y / v\right), \qquad (1)$$

where: a – wave amplitude, t – time, v – wave velocity, x, y – coordinates, and $w = 2\pi/T$ circular frequency, T is period. The total energy in the observed point in that coordinate system is consists of kinetic energy $E_к$ and potential energy E_p. Let this point be located in volume, which is a homogeneous elastic medium. It examines elementary volume q and with mass m, a speed of movement of its particles v, then the kinetic energy of the wave is:

$$E_к = \left(mv^2\right)/2 .$$

Because $m = gq$, where g is density of the medium, than according the formula (1) is obtained:

$$v = dx/dt = -aw \sin w\left(t - y/v\right). \quad (2)$$

The expression for the kinetic energy E_k, can be write as:

$$E_к = gqa^2w^2/2 \cdot \sin^2 w\left(t - y/v\right). \quad (3)$$

The expression for the potential energy of the analyzed volume that is exposed to relative deformation ($\Delta L/L$), is given in the form:

$$E_p = \left(E_j S/L\right) \cdot \left(\Delta L^2/2\right), \quad (4)$$

where E_j is Young's elasticity modulus, S and L are volume dimensions and ΔL is linear deformation.

Including the coefficient of elasticity $K = 1/E_j$, instead of the Young's modulus of elasticity and multiplying and dividing on the right side with L is obtained:

$$E_p = (1/K) \cdot \left(\Delta L^2/L\right)^2 LS/2 . \quad (5)$$

The product $L \cdot S$ represents the volume q on the deformed body; relative deformation $\Delta L/L$ can be written in form dx/dy, where dx is elementary movement of the studying point in given volume on the distance dy, so it is obtained:

$$E_p = (1/K) \cdot \left(dx/dy\right)^2 q/2 . \quad (6)$$

Differentiating the equation (1) after y is obtained:

$$dx/dy = \left(aw/v\right) \cdot \sin w\left(t - y/v\right) \quad (7)$$

and expression for potential energy, then can be written in form:

$$E_p = (1/K) \cdot (a^2w^2q)/\left(2v^2\right) \cdot \sin^2 w\left(t - y/v\right) \quad (8)$$

The total energy E in the studying point of the given volume (q) is sum of the energies E_p and $E_к$, and according the equations (3) and (8) is obtained:

$$E = E_p + E_к = (1/(Kv^2) + g) \cdot \\ \cdot (a^2w^2q/2) \cdot \sin^2 w\left(t - y/v\right) \quad (9)$$

Because the propagation velocity of the wave in the medium is $v = (1/(Kg))^{1/2}$, the expression for the total energy can be written in the form:

$$E = ga^2w^2q \sin^2 w\left(t - y/v\right). \quad (10)$$

Accordingly, the seismic energy is proportionnal to the square of the amplitude of oscillation, the square of the frequency and density of the middle.

If you introduce a term energy density Γ, which is defined as the ratio of the energy that contains elemental volume q, the size of this volume is:

$$\Gamma = E/q = ga^2w^2 \sin^2 w\left(t - y/v\right). \quad (11)$$

Energy density in a given point is variable quantity, as well as the energy. After the expiration of half the period of oscillation, the energy density gets the original value. Since the mean value of the square of a sine for a period is 1/2, then according to equation (11), the mean energy density is:

$$\overline{\Gamma} = (ga^2w^2)/2 \quad (12)$$

As energy not remains localized in the reviewed item from the volume, but spread across the medium, then you can introduce the notion of energy flux. Under the energy flux through the surface means size, numerically equal to the energy that is transferred through a given unit area, per unit time.

Assuming that the surface S is taken out, perpendicular to the direction of waves propagation with speed v, then through that area during the equal period T of oscillation, will pass amount of energy equal to the energy containing parallelepiped with cross section S and length v (Fig. 1). This amount of energy is equal to the mean density of energy over a period multiplied by the volume $v \cdot T \cdot S$, i.e.

$$E = \overline{\Gamma} vTS. \quad (13)$$

Mean energy flux Π is obtained when previous expression is divided with the time T, during which the energy E passes through the surface S:

$$\overline{\Pi} = \overline{\Gamma} vS. \quad (14)$$

If the value for $\overline{\Gamma}$ from the expression (12) is replaced in (14) is obtained:

$$\overline{\Pi} = (ga^2w^2vS)/2 . \quad (15)$$

From here it follows that the average energy flux through the surfaces perpendicular to the direction of propagation of the waves is equal to the product of the mean density of energy, the velocity of propagation of the wave and the size of the surface.

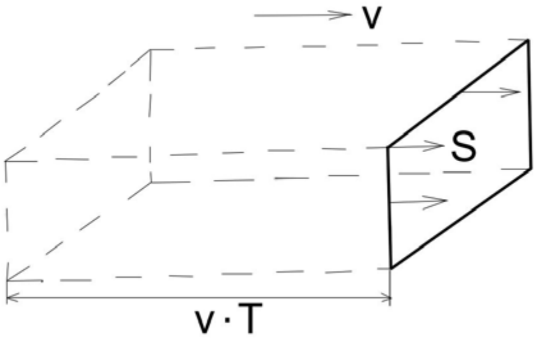

Fig. 1. In the time T, through the surface S passes energy that is contained in the volume of the parallelopiped vTS

The amount of energy per unit time that passes through unit area is called the flux density P. As by definition $P = \Pi/S$, then from the formula (14) is obtained:

$$\overline{P} = \overline{\Gamma} v, \tag{16}$$

i.e. the flux density is equal to the product of the mean energy density and velocity of wave propagation.

Since velocity is a vector, then the energy flux density can be represented by a vector, which is directed in the direction of propagation of the wave. This vector was first introduced by N. A. Umov and because of it is called Umov vector.

If you have a spherical wave that is propagated from point source will show that the mean density of the energy flux is inversely proportional to the square of the distance from the source. If is assumed point source of oscillation and describe a sphere with radius R centered at the source, then the wave and energy will be spread in the direction of the radius, i.e. perpendicular to the surface of the sphere. During the period T of the wave that across the surface of the sphere will pass energy equal to $\overline{\Pi} \cdot T$, where $\overline{\Pi}$ is the energy flux through the sphere. Flux density P is obtained if the energy is divided by the sphere surface and the time T:

$$\overline{P} = \left(\overline{\Pi} T\right) / \left(4\pi R^2 T\right) = \overline{\Pi} / \left(4\pi R^2\right). \tag{17}$$

Since the stationary wave process, when there is no absorption in the medium, the energy flux is constant and does not depend on the size of the sphere radius R, the last equation shows that the mean flux density is inversely proportional to the square of the distance from the point source.

Derived equation, as has been said, applies if there is no absorption in the medium; in other words, in the discussion is assumed that the energy of the wave process turns into another kind of energy. But, in reality, the energy of the oscillatory movement in any medium, part of it is changed in internal heat. That is the consequence because of the existing internal friction in every mechanical medium. Total amount of energy that is carried by the wave depends on its distance from the source: if the wave surface is far from the source, the wave has smaller energy. Since the energy is proportional to the square of the amplitude, due to the wave propagation, amplitude of oscillation will decrease.

From the above said, it is clear that released seismic energy (E) can be expressed with the Umov vector ($\overline{\mathrm{P}}$), taking in to account the time t_n (it is the time interval in which generated seismic oscillations) and closed spherical area S (sphere is taken to simplified the relations) in which is placed the source of the oscillations. Released oscillatory (seismic) energy (E) is given with the relation:

$$E = \int_0^{t_n} dt \int S\left(\vec{\mathrm{P}} \cdot \vec{n}\right) ds, \tag{18}$$

where $\vec{n}$ is unit vector normal to the elementary surface (Fig. 2).

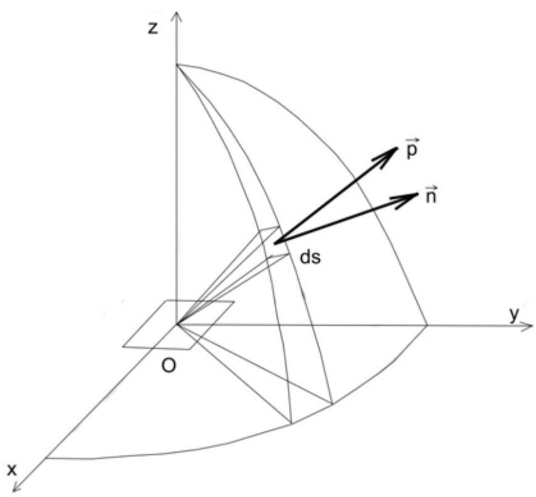

Fig. 2. Flux density of the energy

With increasing distance from the source of oscillation, the density of the energy flux P decreeases due to increasing surface waves front, but even greater changes in energy (E) cause anizotropy, inhomogenity, vertical and horizontal layering of the real environment through which propagated seismic waves.

Energy density Γ can be presented with the formula:

$$\overline{\Gamma} = \int_0^{t_n} \left(\overline{P} \cdot \overline{n} \right) dt . \qquad (19)$$

From the definition for the energy density follows that $\Gamma = dE/dS$.

If the total effect of attenuation of the seismic energy is presented as a function of the distance r and is marked with $\Phi(r)$, than the following relation can be written:

$$\Gamma\left(r_1\right) / \Gamma\left(r_2\right) = \Phi\left(r_1\right) / \Phi\left(r_2\right), \qquad (20)$$

where $\Gamma\left(r_1\right), \Gamma\left(r_2\right)$ are energy densities of certain earthquake on distances r_1 and r_2 from the focus, and $\Phi\left(r_1\right), \Phi\left(r_2\right)$ are functions of attenuation on that distances.

In the case of unlimited isotropic ideal elestic environments, the function of attenuation is defined by the formula:

$$\Phi\left(r\right) = 1 / r^2 . \qquad (21)$$

In real conditions of attenuation, significantly deviates from idealized sphericity and other idealized environmental features, and functions of the attenuation in general form are $\Phi\left(r\right) = 1 / r^n$ and n differ by 2. Or, the function of attenuation is change with the distance, so the size of the integral (18) will depend on the sphere (the radius) of integration. Different level of attenuation depends on the frequency, i.e. the waves with higher frequencies are less attenuated. The complex conditions of wave propagation in real medium made it more difficult and today there are no exact functions of attenuation that are used in practice. Therefore are derived an empirical formulas of attenuation which are used with variable success.

Because of the changeable function of attenuation of the seismic energy and lack of the spherical symmetry, integration by the area S is very difficult. That integral can be written in the following form:

$$\int S = \Gamma ds = 4\pi R^2 \Gamma\left(R\right), \qquad (22)$$

where $\Gamma(R)$ is mean value of the earthquake energy that would be obtain from several seismological station on average the same distance from the focus, but in practice is very rare case.

If in the formula (22) instead of $\Gamma(R)$, is replaced the formula (20), it is obtained:

$$E = 4\pi R^2 \Gamma(r) \Phi(r) / \Phi(r) . \qquad (23)$$

MODEL OF EARTHQUAKE ACCORDING THE TOTAL SEISMIC ENERGY

Possible are two fundamentally different approaches to classification of earthquakes according the energy. One of these classifications is by the size of the flux of energy of seismic waves through a sphere with a fixed radius (reference sphere). This classification is simple enough, because that correlate to the empirical setted calibration curves and appears to be quite good at solving multiple tasks (for example, in determining the seismic effect of the earthquake on the Earth's surface). The choice of the size of the radius R of the sphere depends, primarily, on the functions of attenuation of the seismic energy on that distance, the need the wave front can be roughly approximated to sphere and energy density on the surface and on the same distance in the Earth's crust not to differ. For these conditions and current practical researches reference sphere radius is taken $R = 10$ km.

The second type of classification is to reconcile earthquakes in the total seismic energy E_o released by the focus. Now the integration is performed over the surface S_o restricting the earthquake focus. The total seismic energy of the earthquake E_o is:

$$E_o = \int_{S_o} \Gamma ds. \qquad (24)$$

With approximation is possible to present S_o as a sphere with radius r_o that characterized the focus. Then, relation (24) has the form:

$$E_o = 4\pi r_o^2 \Gamma(r_o) = 4\pi r_o^2 \Gamma(r_o) \Phi(r_o) / \Phi(r) \quad (25)$$

To determine the value for the total seismic energy E_o it is necessary to know r_o and attenuation of the seismic waves near the focus. It is clear that here are included many approximation, that, more or less will affect the result.

Relation of the energy flux E/E_o, through the reference sphere with radius R and through the focus surface with radius r_o is equal to:

$$E / E_o = (R / r_o)^2 \cdot \Phi(R) / \Phi(r_o). \quad (26)$$

This formula will have a good line just in case if $r_o < R = 10$ km. In the opposite case, when $r_o > R = 10$ km there will be no physical sense.

Classification of the earthquakes according the value E_o is of great interest. From the work presented, the determination of released seismic energy of earthquakes is as follows: (I) Determination of energy density Γ in the observed point on the seismograms; (II) mentioned quantities $\Gamma(r)$ are of importance on distances R (or r_o) with selected radiuses of integration, knowing the empirical attenuation curves according the relations (20) and (23), is calculated earthquake energy E on the reference sphere or the total seismic energy on the focus surface E_o.

Described energetic classifications are base for determination the form of the empirical function of attenuation and determination of the integration sphere. Changing the size of the sphere radius will change the other characteristics of the attenuation function and lead to substantial differences in the size of energy.

There are several formulas that give energy as a function of various parameters. As the most famous are formulas of Golicin and Gutenberg. The energy in the Gutenberg formula is equal to the energy flux through a sphere with a radius equal to the focus depth h:

$$E = 3\pi^3 h^2 g V_s (A_o / T_o)^2 t_o . \quad (27)$$

When using this formula is taken average depth of Californian earthquakes $h = 16$ km; T_o (period of oscillation in epicentral zone) and t_o (duration of earthquake in epicentral zone) do not depend on the distance and for the A_o (amplitude in epicentral area) is obtained from the calibration curve $A = A(\Delta)$, attenuation of the amplitude A with the epicentral distance.

Because of the complicated settings for determination of the seismic energy released by earthquakes and due to numerous assumptions about the dimensions of the focus and characteristics of the medium, practically, in seismology, are used not theoretical, but empirical formulas for this purpose. empirical formulas are relation between the seismic energy (E) and earthquake magnitude (M).

CONCLUSION

From the physical aspect, released seismic energy is directly related to the volume of activated earthquake focus.

Important factor are the physical-mechanical characteristics of the rock masses located in the focus and as a medium for propagation of the seismic waves.

Presented theoretical assumptions in the paper and defined physical parameters are good base for modelling the specific earthquake.

To define the space model of an earthquake, necessary are detailed seismological, geophysical and geological explorations on the given seismic area.

REFERENCES

[1] Blažev, K., Arsovski, M.: *Basic Geology*, Faculty of Mining and Geology, Štip, 2003.

[2] Delipetrov, T.: *Energetic Characteristics of the Vardar zone*, Master thesis, Institute of Earthquake Engineering and Engineering Seismology, Skopje, 1982.

[3] Doneva, B.: *Defining correlation dependence between the seismic and magnetic field – model for the Republic of Macedonia*, Doctoral thesis, Faculty of Natural and Technical Sciences, Štip, 2014.

Р е з и м е

ТЕОРИСКИ МОДЕЛ ЗА ДЕФИНИРАЊЕ СЕИЗМИЧКА ЕНЕРГИЈА

Тодор Делипетров, Благица Донева, Марјан Делипетрев

Универзишеш „Гоце Делчев“, Факулшеш за йриродни и шехнички науки, Шший,
Гоце Делчев 89, 2000 Шший, Рейублика Македонија
todor.delipetrov@ugd.edu.mk

Клучни зборови: сеизмичност; енергија; модел; земјотрес

Во трудот е даден теориски приод на дефинирањето на енергијата на даден земјотрес. Енергијата како физичка величина е основен параметар со силно влијание на сите други карактеристики на земјотресот. Поради сложениот процес на земјотресот, процесите во жариштето, како и ширењето на сеизмичките бранови низ средината, мал број истражувачи се занимавале со просторно моделирање на земјотресот. Поголем е бројот на тие кои врз база на бројни проучувани земјотреси определувале емпириски формули за определување на сеизмичката енергија, обично како магнитуда на даден земјотрес.

Дадениот теориски модел може да претставува добра основа за дефинирање на модел на определено сеизмичко подрачје.

Дефинирањето на модел од ваков тип бара продлабочени познавања од поголем број научни дисциплини, со детални проучувања на сеизмичкиот режим и геологијата на теренот.