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SOLVING A TWO-PHASE TRANSPORTATION PROBLEM – TRANSSHIPMENT MODEL WITH THE APPLICATION OF LINGO SOFTWARE

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Abstract

Transportation, as a logistical subsystem, is the most cost-significant part of logistics. The costs arising from transportation are the highest incurred expenses within the logistics system. Considering the limited possibilities for reducing transportation costs, it is crucial for enterprises to find new ways to lower these costs, thereby increasing their profit. This paper addresses the solution to a two-phase transportation problem – a transshipment model with the application of computers, aiming to determine an optimal transportation plan for goods and minimize transportation costs both within a single enterprise and across the entire supply chain.

Key words: *transportation, two-phase problem, model, transshipment, computers.*

INTRODUCTION

The transshipment model is essentially a multi-phase transportation problem in which the flow of materials—raw materials and products—between the point of delivery and the point of reception is interrupted at least once. The product is not delivered directly from the supplier to the point of consumption/needs; instead, it first goes to a transshipment point, and from there to the point of consumption/needs, as shown in Figure 1.

Transshipment as a form of transportation has multiple advantages that should be utilized in today's increasingly competitive conditions. Many manufacturers/suppliers lack sufficient experience and competencies, as well as an established organizational structure, that would enable them to quickly and successfully deliver goods directly to the consumer. As a result, there may be significant losses in terms of breakage, damage, and loss of quality due to improper handling and transportation of products, leading to a loss of trust from customers. These indirect costs can reach high levels and, over the long term, can even exceed direct transportation costs. Therefore, it is preferable for products to be distributed with the help of an intermediary or distributor responsible for handling and transporting the goods [1].

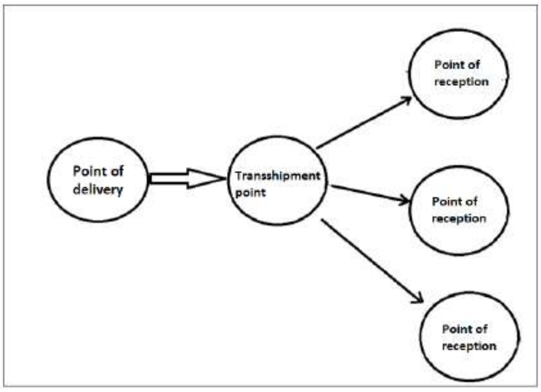

Figure 1. Indirect management of goods flow in the supply chain with multiple points of reception.

The simplest transshipment model is characterized by one point of delivery, one transshipment point, and multiple points of reception, known as a two-phase transportation problem, as shown in Figure 1. On the other hand, Figure 2 illustrates a two-phase transportation problem with multiple delivery points, multiple storage/transshipment points, and multiple reception points.

In addition to the previously mentioned model with one point of interruption in the flow of goods, there are also models with multiple interruption points during the delivery of goods, which are referred to as multi-phase transportation problems, and they are solved in a manner similar to the two-phase problem.

The production units delivered to the transshipment point and distributed from there can be heterogeneous or homogeneous. Heterogeneous units refer to a pre-assembled assortment for the needs of customers—which may pertain to a warehouse for supplying a regional market—and are distributed as such, while the production units delivered to the transshipment point are mainly homogeneous. Larger quantities of production units are typically transported from the production site to the regional warehouse, while smaller quantities are transported from the regional warehouse to the customers.

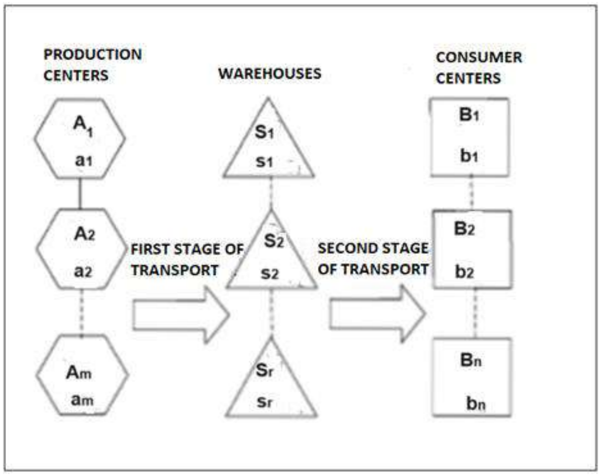

Figure 2 Two-phase transportation problem with multiple points of delivery, storage and reception

For the transshipment model, as a multi-phase logistical system, to be cost-effective, the sum of transportation costs, inventory costs, and storage costs must be less than the direct costs of delivering/transporting to the customer from the point of supply. It is a fact that additional logistical activities are carried out at transshipment points, which represent additional costs; however, from the previous discussion, it can be seen why the system of indirect delivery to customers is still more cost-effective in certain cases.

Transshipment points are often referred to as distribution centers because goods are distributed from them to the end consumers/customers. The reasons supporting the establishment of distribution centers include:

- Reduction of distribution costs (degressive effect of transportation costs from the manufacturer to distribution centers due to the quantities being transported),
- Reduction of delivery time (from distribution centers to customers),
- Reduction in the number of routes from manufacturers to distribution centers and from distribution centers to customers, compared to the number of routes directly from manufacturers to customers,

Opportunities for combining shipments from different manufacturers to one customer, with a potential for reducing transportation costs.

LITERATURE REVIEW

The study of transportation problems in general and the two-phase transportation problem, or transshipment problem, has been addressed by numerous researchers; however, we will mention only a few of them here. The most commonly used software used to solve the two-phase transportation problem are the following: LINGO, GAMS, Microsoft Excel Solver, WinQSB, Matlab, as well as codes in programming languages C/C++, Python, etc.

Pasagić, H. (2003) discussed the theoretical foundations of the two-phase transportation problem and presented an analytical solution to practical examples, [4]. Pupovac, D., Drašković, M. (2007) analyzed the two-phase transportation problem and applied calculation tables in Microsoft Excel software to optimize logistics networks [5]. Alić, B.M., Alić, A., Cobović, M. (2012) addressed the transshipment problem as a multi-tier metalogical system, where they solved a two-phase transportation problem with three factories, three distribution centers, and three consumers using the WinQSB software [1]. Božičković, R., Božičković, Z., and Popović, M. (2010) in paper [3] present models for solving a two-phase transportation problem by analyzing a case with five suppliers, three transshipment points, and four consumers, using the WinQSB and Microsoft Excel software. Anand Jaya Kumar, A., Raghunayagan, P. (2018) in their papers propose solving multi-phase transportation problems using the software LINGO. In paper [2], they present a solution to a transshipment problem with three factories, two warehouses, and three consumers.

METHODOLOGY OF SCIENTIFIC RESEARCH

Mathematical formulation of the two-phase transportation problem

When defining the two-phase transportation problem, the following questions arise:

- How to transport goods from the producers to the warehouses?
- How to transport goods from the warehouses to the consumers?

The mathematical model of the two-phase transportation problem [3] is formulated as follows:

Let the production locations be: A_1, A_2, ... , A_i, ……., A_m where the same goods are produced in the following quantities during the considered time period: a_1, a_2,........,a_i, …….., a_m. The demands from consumers are also known: B_1, B_2,……, B_j, ...,B_n for the same goods in quantities: b_1, b_2, ………, b_j, ……..,b_n. Each unit of goods is transported from the producers to one of the warehouses: S_1, S_2, ……., S_k, …..,S_r where the quantities that can be stored are: s_1, s_2, ………, s_k, ….., s_r. Let c_{ik} be the transportation cost of one unit of goods from producer A_i to warehouse S_k, c_{kj} be the transportation cost from warehouse S_k to consumer B_j, and c_k be the storage cost of one unit of goods in warehouse S_k.

The total capacity of the warehouses cannot be less than the total quantity of produced goods, i.e.:

$$\sum_{k=1}^{r} s_k \geq \sum_{i=1}^{m} a_i \quad (1)$$

and the total quantity of produced goods cannot be less than the total demand of consumers:

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j \quad (2)$$

Considering the aforementioned relationships, the optimal plan must ensure minimum total transportation and storage costs.

The following variables are introduced in the model:

x_{ik} – the value of the quantity of goods delivered from producer A_i to warehouse S_k

x_{kj} – the value of the quantity of goods delivered from warehouse S_k to consumer B_j.

These variables satisfy the following conditions:

1. Each consumer is supplied with goods from the warehouses to meet their needs:
$$\sum_{k=1}^{r} x_{kj} = b_j \;, \quad j = 1,2, \ldots, n \quad (3)$$
2. The quantity of goods produced by each producer that is transported to all warehouses cannot exceed the capacity of that producer:
$$\sum_{k=1}^{r} x_{ik} \leq a_i \;, \quad i = 1,2, \ldots m \quad (4)$$
3. The quantity of goods delivered to each warehouse does not exceed the capacity of that warehouse:
$$\sum_{i=1}^{m} x_{ik} \leq s_k \;, \quad k = 1,2, \ldots r \quad (5)$$
4. The quantity of goods delivered to each warehouse is equal to the quantity of goods shipped from that warehouse to the consumer:

$$\sum_{i=1}^{m} x_{ik} = \sum_{j=1}^{n} x_{kj}, \qquad (6)$$

The objective function encompasses the costs of transportation and storage, and it is:

$$F = \sum_{i=1}^{m} \sum_{k=1}^{r} c_{ik} x_{ik} + \sum_{k=1}^{r} \sum_{j=1}^{n} c_{kj} x_{kj} + \sum_{k=1}^{r} \sum_{j=1}^{n} x_{kj} \rightarrow min \qquad (7)$$

where the first sum represents the transportation costs of goods from the producers to the warehouses, the second sum represents the transportation costs from the warehouses to the consumers, and the third sum represents the storage costs. Formula (7) shows that the established linear problem of cost minimization is similar to the classical transportation problem.

If the sum of the capacities of the warehouses is equal to the total production from the producers and the total demand of the consumers, i.e., if the following condition is met:

$$\sum_{k=1}^{r} s_k = \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (8)$$

this means that the capacity of each warehouse will be fully utilized, and the transportation plan from the warehouses to the consumers does not depend on the transportation plan from the producers to the warehouses. In such a case, the two-phase transportation problem is divided into two independent transportation problems:

The optimal transportation plan from the producers to the warehouses is determined separately, and then from the warehouses to the consumers. However, in general case:

$\sum_{k=1}^{r} s_k > \sum_{i=1}^{m} a_i$ and

$\sum_{k=1}^{r} s_k > \sum_{j=1}^{n} b_j$

and the two-phase transportation problem cannot be divided into two independent transportation problems. In fact, it is shown that in the general case, the sum of the optimal solutions of the partial problems does not yield the optimal solution of the complete problem, which means that it is incorrect to divide the general problem into parts and solve it in segments. Only in some cases, one of which is mentioned, can the optimal solution be found in this manner.

The method for solving two-phase and multi-phase transportation problems was proposed by the Russian mathematician V.A. Mash, but a similar idea was introduced somewhat earlier by the American mathematician A. Orden.

Brief description of the LINGO software

LINGO is an optimization modeling software designed to help users formulate, solve, and analyze linear, nonlinear, integer, and stochastic optimization problems. It provides a user-friendly modeling environment that simplifies the process of defining complex optimization models. LINGO includes a powerful solver that handles large-scale problems efficiently, making it suitable for applications in logistics, supply chain management, finance, and engineering. Additionally, LINGO integrates various functions and tools to assist in model building, including sensitivity analysis and solution diagnostics, helping users to interpret and refine their models effectively.

Illustrative example for solving a two-phase transportation problem with the LINGO program

Example: Solve the two-phase transportation problem where products from two factories: F1 and F2 are transported to three warehouses: W1, W2, and W3. Then, the products from the warehouses are transported to four consumers: R1, R2, R3, and R4. The supplies from the factories are: 400 and 600, and the demands from the consumers are: 200, 300, 150, and 350. It is assumed that the warehouses can accept the supplies from the factories where the corresponding products are produced, i.e., they can receive the same products in quantities: w1=w2=w3=550. The unit transportation costs from the factories to the warehouses and from the warehouses to the consumers are shown in Table 1.

Table 1. Unit transportation costs

No.	From - To	Unit Cost
1	F1 - W1	1
2	F1 – W2	2
3	F1 – W3	3

4	F2 - W1	6
5	F2 – W2	4
6	F2 – W3	3
10	W1 – R1	5
11	W1 – R2	3
12	W1 – R3	1
13	W1 – R4	3
14	W2 – R1	1
15	W2 – R2	2
16	W2 – R3	3
17	W2 – R4	4
18	W3 – R1	8
19	W3 – R2	7
20	W3 – R3	6
21	W3 – R4	5

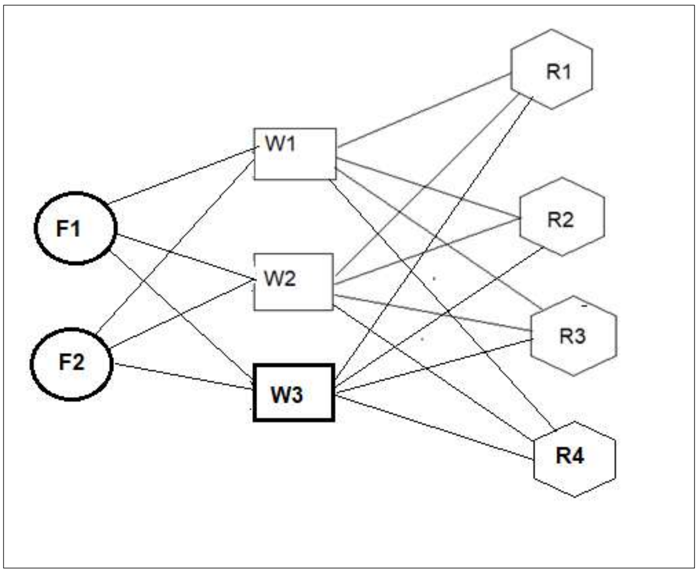

Figure 3. Schematic representation of the two-phase transportation problem from the example

First, it is necessary to create the initial table for solving the problem, in which the unknown quantities are presented, Table 2.

Table 2. Initial table for solving the two-phase transportation problem

		W1	W2	W3	R1	R2	R3	R4
		550	550	550	200	300	150	350
F1	400	1 X_{11}	2 X_{12}	3 X_{13}				
F2	600	6 X_{21}	4 X_{22}	3 X_{23}				
W1	550				5 Y_{11}	3 Y_{12}	1 Y_{13}	3 Y_{14}
W2	550				1 Y_{21}	2 Y_{22}	3 Y_{23}	4 Y_{24}
W3	550				8 Y_{31}	7 Y_{32}	6 Y_{33}	5 Y_{34}

Due to the complexity of manually solving the two-phase transportation problem, algorithms and computer programs have been developed to solve it in order to shorten the time needed to find the optimal solution. One of the software programs used for solving linear programming problems is the LINGO software [2], which has been used in this work.

OBTAINED RESULTS

The example was solved using the computer program LINGO, the listing of which is shown below:

```
Model:
Min =
1*X11+2*X12+3*X13+6*X21+4*X22+3*X23+5*Y11+3*Y12+1*Y13+3*Y14+1*Y21+2*Y22+3*
Y23+4*Y24+8*Y31+7*Y32+6*Y33+5*Y34;
X11+X12+X13<=400;
X21+X22+X23<=600;
Y11+Y21+Y31>=200;
Y12+Y22+Y32>=300;
Y13+Y23+Y33>=150;
Y14+Y24+Y34>=350;
X11+X21<=550;
X12+X22<=550;
X13+X23<=550;
Y11+Y12+Y13+Y14-X11-X21<=0;
Y21+Y22+Y23+Y24-X12-X22<=0;
Y31+Y32+Y33+Y34-X13-X23<=0;
@GIN(X11); @GIN(X12); @GIN(X13); @GIN(X21); @GIN(X22); @GIN(X23); @GIN(Y11);
@GIN(Y12); @GIN(Y13); @GIN(Y14); @GIN(Y21); @GIN(Y22); @GIN(Y23); @GIN(Y24);
@GIN(Y31); @GIN(Y32); @GIN(Y33); @GIN(Y34);
end
```

Table 3. Solution of the processed example

		W1	W2	W3	R1	R2	R3	R4
		550	550	550	200	300	150	350
F1	400	1 400	2	3				
F2	600	6	4 550	3 50				
W1	550				5	3	1 150	3 250
W2	550				1 200	2 300	3	4 50
W3	550				8	7	6	5 50

The value of the minimum transportation cost function is TC = 4,900 monetary units.

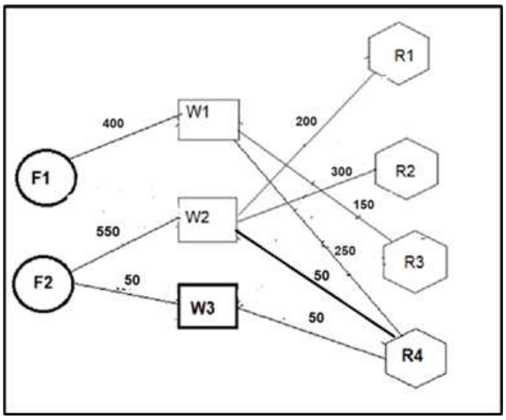

Figure 4. Graphical representation of the solution to the two-phase transportation problem

Verification of the solution to the problem using program code developed with the PuLP library in the Python programming language

To verify the results obtained with the LINGO software, we created a program code with the PuLP library in the Python programming language, based on the solution scheme for the two-phase problem shown in Fig. 5.

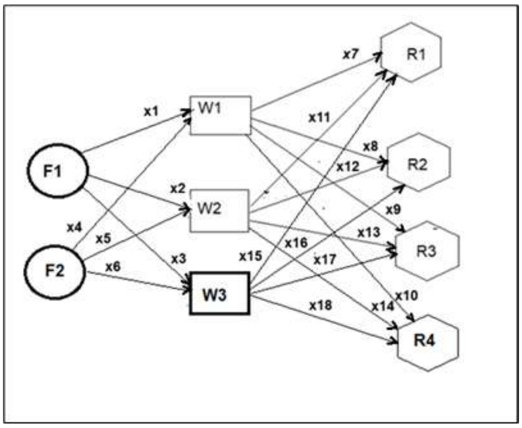

Figure 5. Schematic representation of the two-phase transportation problem adapted for solving with the PuLP library in the Python programming language

The program code for solving the two-phase problem from the example, developed in the Python programming language, is shown in Fig. 6.

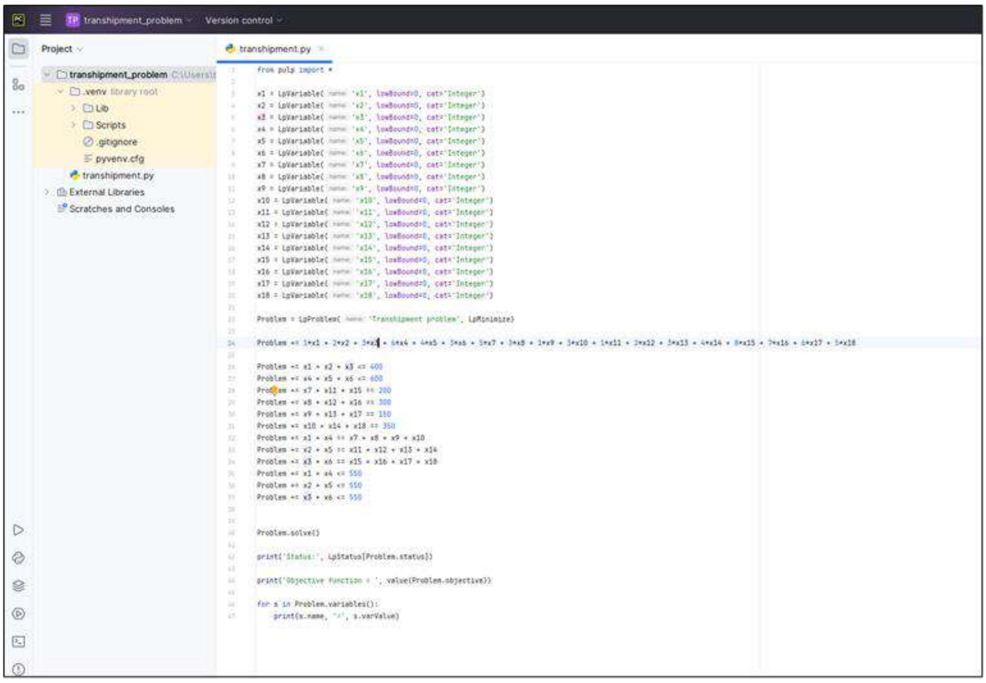

Figure 6. Program code for solving the problem in the Python programming language

By executing the program code in Python, completely identical results were obtained (Fig. 3.5.3), as with the LINGO software, confirming that the obtained results are accurate.

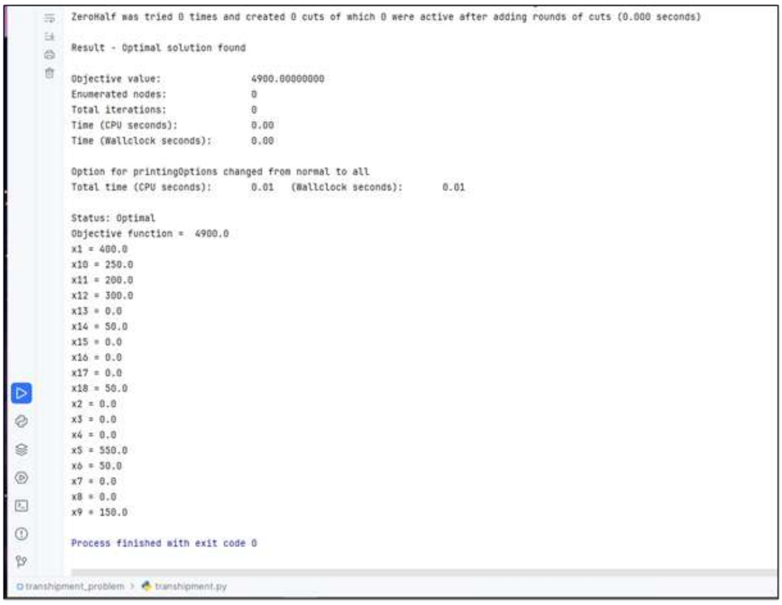

Figure 7. Results of the solution to the problem obtained in the Python programming language

CONCLUSION

In essence, the transportation problem refers to the movement of goods directly from the loading point to the unloading point with constant unit transportation costs, and in that case, it concerns one-phase logistical systems. However, in practice, the transportation of goods often occurs through an appropriate number of transshipment points, whose task is to regroup the goods into smaller quantities or to concentrate them into larger units for delivery, depending on demand needs. In this case, it refers to multi-phase logistical systems. The application of the transshipment model, or the two-phase transportation problem, contributes to the overall reduction of transportation costs. Solving these problems with the use of computers results in a reduction in the required processing time, a decrease in the number of experts needed, and the attainment of accurate and precise results for the optimal transportation plan.

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