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СОДРЖИНА
CONTENT

CALCULATION OF MULTI-STATE TWO TERMINAL RELIABILITY

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Abstract. Traditionally, reliability of the transportation system has been analyzed from a binary perspective**.** It is assumed that a system and its components can be in either a working or a failed state. But, many transportation systems as: telecommunication systems, water distribution, gas and oil production and hydropower generation systems are consisting of elements that may operate in more than two states. The problem that we consider in this paper is known as the multi-state two terminal reliability computation. The multi – state two terminal reliability can be computed with the formula of inclusion and exclusion, if the minimal path vector or minimal cut vector are known.

Keywords: multi-state systems, network reliability, minimal path vectors, minimal cut vectors.

1 Introduction

Two-terminal network reliability for binary transportation system has been studied in various ways. For the binary network it is assumed that a whole system and its components can be in two states: working or failed state. However, the binary approach does not completely describe some transportation systems. Such systems are telecommunication systems, water distribution, gas and oil production and hydropower generation systems. These networks and its components may operate in any of several intermediate states and better results may be obtained using a multi-state reliability approach.[1]
The authors developed a multi-state approach for exact computation of multi-state two-terminal reliability at demeaned level *d* ($M2TR_d$). The multi-state two terminal reliability is defined as the probability that a demand of *d* units can be transmitted from source to sink nodes through multi-state edges [2]. The multi – state two terminal reliability can be computed if the minimal path vector or minimal cut vectors are known. In the literature many algorithms for calculating on minimal path or cut vectors are known.

Some algorithms for obtaining minimal path or cut vectors are given in [1], [2], [3] and [4]. In [1] is developed a multi-state approach for exact computation of multi-state two-terminal reliability. In the paper is proposed algorithm for obtaining minimal path vector. Disadvantage of this algorithm is that it gives candidates minimal path vectors that are not minimal. In [2] is proposed algorithm for obtaining minimal cut vectors for the multi-state two-terminal transportation system. The disadvantage of this algorithm is that it works only for weak homogeneous components. The components can have different number of state, but the first state of the components has to be the same. In [3] is proposed algorithm for obtaining minimal path vectors. This algorithm has restriction for the values of capacities on the edges. The capacities on the edges can be only valued from the set of integer number. In [4], it is proposed an algorithm for obtaining the minimal path vector that does not require any restrictions for the values of the capacities of the links.

2 Basic definition

In this paragraph we will give some basic definitions for binary two terminal network.

Let $G(V,E)$ be a multi- state two terminal network. By V the set of nodes is denoted, and by $E=\{e_i \mid 1\le i\le |E|\}$ the set of edges is denoted. Multi-state edge is defined as an edge of the system, which has a set of states $\{r\le\cdot 100\% \mid r\in[0,1]\}$. The state 0 is appropriate in case when there is no stream over the edge. The state 100% is appropriate on the state, when the edge works with full capacity. Intermediate states are all states in which could be found edge between the state 0 and the state in which the system works with total capacity. State space set is the vector which presents the state of the components.

For example, let the edge of the transport system has three different states: 0%, 50%, 100%. Then the vector $(0,0.5,1)$ is the vector of the states of that edge.

For any multi-state edge, the vector of the capacity **(Capacity Vector - Capacity State Set)** is obtained as a product of the full capacity of the edge and the vector of the state of that link. Let suppose that the edge described before, in perfect conditions could transfer flow equal to 8 units. The vector of the capacities, obtained in this way is equal to $S_i=(8\cdot 0,8\cdot 0.5,8\cdot 1)=(0,4,8)$.

Capacity state set S, as the set of the all possible capacities from the source to the sink, for the total system is defined. For some simplifications, the states could be numerated in different ways. For example, it can be supposed that the perfect state at the level 2,50% corresponds to 1, and the state of the total breakdown corresponds to 0. In this way, the vector (0, 1, 2) is obtained.

Let x_i be the state of the link e_i. The vector $\vec{x}=(x_1,x_2,\ldots,x_n)$ which describes the states of all components of the system, is called **state vector.** The set of all state vectors is called state of the system $N=S_1\times S_2\times\ldots\times S_{|E|}$.

The function $\varphi: N\to S$ where $\varphi(\vec{x})$ is the potential capacity from the source to the sink. If the system is in the state $\vec{x}$, it is called **multi – state structural function.**

Definition 1. Multi-state two terminal reliability for the level d (M2TR$_d$) is the probability of the event, that the flow is larger or equal to *d* and could be successfully transfered from the source to the sink.

$$M2TR_d = P\left(\varphi\left(\vec{x}\right) \geq d\right) \tag{1}$$

Definition 2. A vector $\vec{y}$ is said to be less than $\vec{x}$, $\vec{y} < \vec{x}$**,** (or dominated by $\vec{x}$ **)** if $\forall i$, $y_i \leq x_i$ and for some k, $y_k < x_k$.

Definition 3. A vector $\vec{x}$ is said to be a **minimal path vector to level** d if $\varphi\left(\vec{x}\right) \geq d$ and if for every $\vec{y} < \vec{x}$**,** $\varphi\left(\vec{x}\right) < d$.

Definition 4. A vector $\vec{x}$ is said to be a **minimal cut vector to level** d if $\varphi\left(\vec{x}\right) < d$ and if for every other $\vec{y} > \vec{x}$, $\varphi\left(\vec{x}\right) \geq d$. [4,5]

In order to determine the structure of the transport system we define binary path vector. We are considering multi-state transport system with set of the nodes V and set of the edges $E = \{e_i \mid 1 \leq i \leq n\}$. We consider transportation system with the same nodes and edges. All the edges in this system are binaries, i.e. the set of the state of the links is {0,1}. Let $\vec{v}$ be the minimal path vector for this transportation system. We say that $\vec{v}$ is binary minimal path vector for the multi-state transport system. With BPV we denote the set of binary minimal path vectors.

With $TS = (V, E, BPV, S, VP)$, we denote the transportation system, where V is the set of the nodes, E is the set of the edges, BPV is the set of binary minimal path vectors, S is the set of capacity vectors of the components and VP is the set of the probabilities on the level of the components, where $\vec{p}_i$ is vector of the probabilities on the i-th edges i.e. $p_{id} = P(x_i = d)$.

3 Calculating on the multi – state two terminal reliability

We will show how the reliability of multi state system, when the minimal path vectors are known, can be calculated. For the binary system, reliability could be solved by the following formula:

$$R = P\left(\bigcup_{h=1}^{j} \mathcal{P}_h\right) = \sum_{h=1}^{j} P(\mathcal{P}_h) - \sum_{h<k}^{j} P(\mathcal{P}_h \cap \mathcal{P}_k) + \ldots (-1)^j P(\mathcal{P}_1 \cap \ldots \cap \mathcal{P}_j).$$

(2)

where j is a number of minimal paths, and P_h is a h - the minimal path [5].

This formula can be extended for the new structure of the vectors from the minimal sets. In the case of multi state system $M2TR_d$ can be obtained with the following modification of the formula of inclusion and exclusion

$$M2TR_d = \sum_{h=1}^{T} P\left(\vec{x} \geq \vec{y}_h\right) - \sum_{h<k}^{T} P\left(\vec{x} \geq \vec{y}_h \wedge \vec{x} \geq \vec{y}_k\right) + \ldots (-1)^T P\left(\vec{x} \geq \vec{y}_1 \wedge \ldots \wedge \vec{x} \geq \vec{y}_T\right)$$

(3)

where T is number of the MPV_d (minimal path vectors on level d) and $\vec{y}_h \in MPV_d$. By using of the notation

$$\max(\vec{z}_1, \ldots, \vec{z}_s) = (\max(z_1^{(1)}, \ldots, z_s^{(1)}), \ldots, \max(z_1^{(l)}, \ldots, z_s^{(l)}) \qquad (4)$$

where $z_u^{(v)}$ is v-th coordinate of the coordinate $\vec{z}_u$, and the equation (3) can be written in the form:

$$M2TR_d = \sum_{h=1}^{T} P(\vec{x} \geq \vec{y}_h) - \sum_{h<k}^{T} P(\vec{x} \geq \max(\vec{y}_h, \vec{y}_k)) + \ldots (-1)^T P(\vec{x} \geq \max(\vec{y}_1, \ldots, \vec{y}_T))$$

(5)

Algorithm for reliability calculating

Input: Binary minimal path vectors BPV, set of the capacity vectors of the components *S*, and the set of the probabilities of the components levels VP.

Output: Reliabilities $M2TR_d$, for $m<d<M$, m is the minimal level of the system work and M is the maximal level of the system work.

Step 1. Finding the minimal path vectors for level d , $D=\left\{\overrightarrow{z_1},\overrightarrow{z_2},\ldots,\overrightarrow{z_n}\right\}$ with the algorithm proposed in [1-4].

Step 2. Finding all possible non empty subsets of the set *D*.

Step 3. For every subset D_r $1\leq r\leq 2^T-1$ obtained in the step 2, it is find the supremum, $\vec{v}_r=\sup D_r$, according to the ordering relation f given in definition 2, in this way every coordinate is equal to the maximum of the appropriate vector coordinates in that subset.

Step 4. Finding of the probabilities $P\left(\varphi(\vec{x})\geq\vec{v}_r\right)$ and application of the formula 5 in order to obtain the reliability $M2TR_d$.

From the previous algorithm we can concluded that multi – state two terminal transportation system can be computed if minimal path vectors are given. In the Example 1, we will show minimal path vectors for level 1,2,3,4 for the system from Figure 1, and appropriate reliability.

Example 1. Let us consider the simple transportation system in Figure 1.

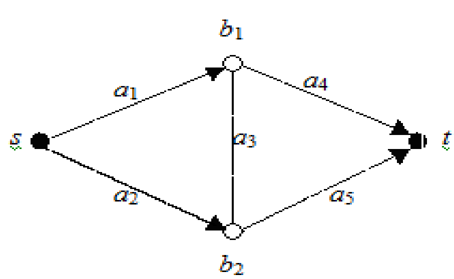

Figure 1. Two terminal transportation system

Capacities of the edges are S_1={0,1,2}, S_2={0,1,2}, S_3={0,1}, S_4={0,1} and S_5={0,1,2,3}. Probabilities that the edges can be in some state are: p_1=(0.1,0.1,0.8), p_2=(0.1,0.1,0.8), p_3=(0.1,0.9), p_4=(0.1,0.9) and p_5=(0.1,0.05,0.05,0.8).

The vector of maximal state will be *M*=(2,2,1,1,3).
We suppose that binary path vectors that are minimal path vectors to level 1 are known. With MP_i we will denote minimal path vector to level i.

MP_1={(1,0,1,0,1), (1,0,0,1,0), (0,1,1,1,0), (0,1,0,0,1)}
$M2TR_1$=0.97686.
MP_2 = {(2,0,1,1,1), (1,1,0,1,1), (1,1,1,0,2), (0,2,1,1,1), (0,2,0,0,2)}
$M2TR_2$=0.84614

MP_3 = {(2,1,1,1,2), (1,2,0,1,2), (1,2,1,0,3)}
$M2TR_3$=0.64584

MP_4 = {(2,2,1,1,3)}
$M2TR_4$=0.3686

4 Conclusion

This paper presents an algorithm for calculating a multi-state two terminal reliability. With this algorithm reliability to level d can be calculated when the minimal path vectors to level d are known.

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