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CONTENT

COMPARING OF THE BINOMIAL MODEL AND THE BLACK-SCHOLES MODEL FOR OPTIONS PRICING

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Abstract
In this paper will be considered the simple binomial model with one and more periods. It will be given the correspondence between binomial model and the Black-Scholes model for option pricing and also will be shown that the binomial model is more simple then the continuous Black-Scholes model from pedagogical point of view.
Kew words: *Binomial model, Black-Scholes model, Call option, Brownian motion, stochastic differential equation.*

1. Introduction

Call option is a financial instrument that gives its holder the right, not the obligation, to purchase from its seller one unit of the underlying security, at a predetermined price, at or before an expiry date. American and European versions differ in that the latter can be exercised only at the expiry date [2].

The Black-Scholes Model was first discovered in 1973 by Fischer Black and Myron Scholes who developed a formula for valuation of European contingent claims based on geometric Brownian motion model for the stock price process. Robert Merton developed another method to derive the formula with more applicability and generalized the formula in many directions. It was for the development of the Black-Scholes Model that Scholes and Merton received the Nobel Prize of Economics in 1997 (Black had passed away two years earlier). The idea of the Black-Scholes Model was first published in "The Pricing of Options and Corporate Liabilities", the Journal of Political Economy by Fischer Black and Myron Scholes [3], and then elaborated in "Theory of Rational Option Pricing" by Robert Merton in 1973. The three young Black-Scholes Model researchers, which were still in their twenties, set about trying to find an answer to derivatives pricing using mathematics, exactly the way a physicist or an engineer approaches a problem. They had shown that mathematics could be applied using a little known technique known as stochastic differential equations and that discovery led to the development of the Black-Scholes Model that we know today. The using of the Black-Scholes Model and the Black-Scholes formula require knowledge from partial differential equations and stochastic differential equations. Because of the advanced mathematical concepts students are faced with difficulties in understanding of this model and this formula. Cox, Ross, and Rubinstein (1979) have derived instead a binomial option model, which converges to the Black-Scholes model. The binomial approach shares the same statistical idea of finding the probability of some specific numbers of heads and tails from repeatedly tossing a biased coin. On the other hand binomial model is much easier for understanding by students as a simple algebraic form, and also has the same economic significance as a model of Black-Scholes. Good understanding of the binomial model facilitates understanding of the model of Black-Scholes. Also binomial model allows numerical approach for determining the price of derivatives for which it is impossible to determine a closed solution.

2. The Black-Scholes Model

The most famous model, which is used in finance, long time ago is the model in which the stock price can be described with Brownian motion. At the beginning this model, in which the stock price is described with many Brownian motions, was not accepted because of two reasons. One of the reasons is that the Brownian motion can receive a negative value, but the option price cannot be negative. Also, the stock prices have proportional increments, but the Brownian motion has stationary and independent increments. The different stocks have different volatility σ. It is expected that the rate of return μ is greater than risk free rate r, because every investor expects higher profit, with which would be recover the takeover risk.

For modeling stock price it will be used stochastic differential equation $\frac{dS_t}{S_t} = \sigma dB_t + \mu dt$, or equivalent integral form:

$$S_t = S_0 + \int_0^t S_s \sigma dB_s + \int_0^t S_s \mu ds, \qquad (1)$$

where B_t is Brownian motion [6], [5], [1].

The famous formula for option pricing is given with the equation

$$W_0 = x\Phi(g(x,T)) - Ke^{-rT}\Phi(h(x,T)) \qquad (2)$$

where

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{y^2}{2}} dy, \quad x = P_0 = S_0$$

S_0 is a initial option price.

$$g(x,T) = \frac{\log\left(\frac{x}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$h(x,T) = g(x,T) - \sigma\sqrt{T}$, K is strike price, T is expiry date and r is interest rate.

Without knowing the derivation of the Black-Scholes formula, the majority of students would see this formula as to complex and could not recognize the significance of the formula and its efficiency in the calculations. As a result, they cannot understand the value of securities. Therefore, in this paper will be considered the binomial model as introduction of the Black-Scholes Model and in that way the students will change the perception about the Black-Scholes formula.

3. Binomial model

The simplest model for the valuation of risky securities or shares is a binomial model. The binomial model with one period has initial time 0 and end time moment 1. The time unit can be chosen differentially: year, quarter, month, week etc.

Let suppose that there is one share, which has initial price $S(0) = S_0$ at the initial time 0 and thus can be obtained only two different values at the end of the period uS_0, $(u > 1)$ with probability p if there are price movements up (increase the price of the share) and dS_0, $(0 < d < 1)$ with probability q in the case of moving the price down (decreasing the price of the share). For the probabilities p and q hold $p + q = 1$ and $0 < p, q < 1$.

In this concept the European Call Option will be valued. The European Call Option is the opportunity to buy one unit of the share at the moment 1, with strike price K. If S_1 is the price of the share at time 1, then its value is given with $V_1 = (S_1 - K)^+$, where $x^+ = \max(x, 0)$.

The value V_1 of the European Call Option with binomial one period time model is given with:

$$V_1 = \frac{1}{1+r}\left[\frac{1+r-d}{u-d} V_1^u + \frac{u-(1+r)}{u-d} V_1^d\right]. \qquad (3)$$

The value of the European Call Option V will be obtain with the binomial one period time model and this value is also calculate with the programming packet Mathematica. If the initial value of the share is $S_0 = 50$, the strike price $K = 35$, the price of the share increases with factor $u = 1{,}5$, and decreases with $d = 0{,}5$ and the rate is $r = 0{,}03$, then the value of the option is

$$\begin{aligned} V_1 &= \frac{1}{1+r}\left[\bar{p}V_1^u + \bar{q}V_1^d\right] = \\ &= \frac{1}{1.03}\left[0.53 \cdot 40 + 0.46 \cdot 0\right] = 20.5825. \end{aligned}$$

where $\overline{p} = \frac{1+r-d}{u-d} = \frac{1.03-0.5}{1.5-0.5} = 0.53$.

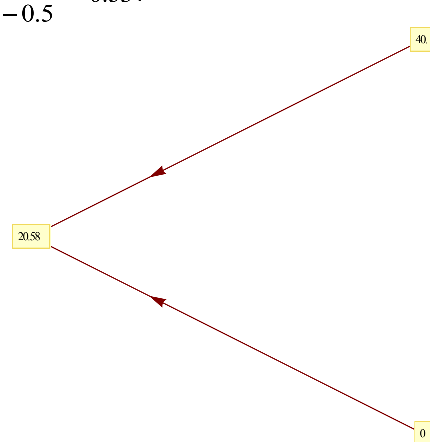

Figure 1: Binomial one period time model in Mathematica

First we consider the binomial model with 2 periods, i.e. with two time steps, and then make its generalizations to the n time steps.

$$\begin{aligned} V(S_0,0) = \frac{1}{(1+r)^2}\Big[& \overline{p}^2 \max\{0, u^2 S_0 - K\} + \\ & + 2\overline{p}\overline{q} \max\{0, udS_0 - K\} + \\ & + \overline{q}^2 \max\{0, d^2 S_0 - K\}\Big], \end{aligned} \tag{4}$$

the notations are the same as in the binomial one period time model.

If we consider the binomial model with two periods for European Call Option valuation, with strike price $K = 80$, initial stock price $S_0 = 80$, increase factor for the stock price $u = 1.5$, and decrease factor for the stock price $d = 0.5$ rate $r = 0.1$ in Mathematica we obtain the binomial tree, which is given at the figure 2:

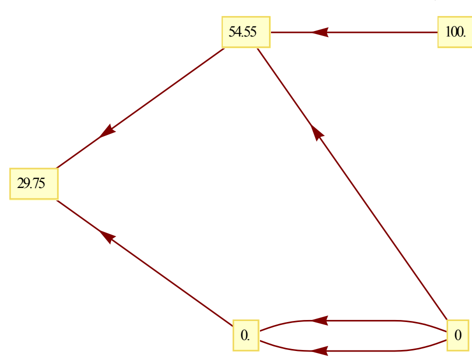

Figure 2: Binomial model with two time periods in Mathematica

About the generalizations of the binomial model with n time periods we consider the probability space Ω. We hold on the case when the probability space Ω can be presented as set of all sequences with length n, of increases of the stock price for factor u, denoted with G (gain) and of all decreases of the stock price for factor d, denoted with L (loss). Let the initial price of the stock S_0, is fixed. With $S_k(\omega) = u^j d^{n-j} S_0$ is denoted the stock price after k - periods (steps) if in the first n - outcomes $\omega \in \Omega$, j -outcomes are G, and the other

$n-j$ -outcomes are L. With simple extension of the binomial model with two periods, for European Call Option valuation, with strike price K in the case with n periods is obtained:

$$V(S_0,n) = \left(u^j d^{n-j} S_0 - K\right)^+ = \\ = \max\left\{0, u^j d^{n-j} S_0 - K\right\} \\ j = 0,1,2,\dots \qquad (5)$$

i.e. at the initial time

$$V(S_0,0) = \frac{1}{(1+r)^n} \sum_{i=1}^{n} \binom{n}{j} \bar{p}^{\,j} \bar{q}^{\,n-j} V(S_0,n). \qquad (6)$$

The formula (6) is obtained from the formula (4), with generalizations.

If we consider the binomial model for stock pricing with strike price $K = 10$, 3 periods $n = 3,\ u = 3,\ d = \frac{1}{2},\ r = 0{,}1,\ S_0 = 20$ and if V is European Call Option with strike price K and expiry date n, in Mathematica are calculated the values V_0, V_1 and V_2 in every moment. The binomial tree with three periods is given at the figure 3:

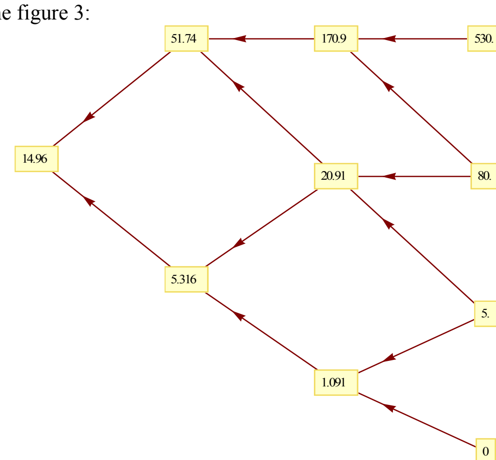

Figure 3: Binomial model with three time periods in Mathematica

With the figure obtained in Mathematica it is easy to notice that the binomial model is very simple for calculations, and that it converges to the Black-Scholes Model when the periods increase [8].

In his paper [4] illustrates, from a pedagogic perspective, how a simple binomial model, which converges to the Black-Scholes formula, can capture the economic insight in the original derivation. They have showed that Microsoft Excel TM plays an important pedagogic role in connecting the two models. With interactivity provided by scroll bars, in conjunction with Excel's graphical features, they have allowed students to visualize the impacts of individual input parameters on option pricing.

4. Conclusion

The binomial model has simplicity as its advantage and it is used often in financial institutions, in the discrete case, even in the case of very short time period. As advantages of the binomial model are the following:It shows the construction of the replicating portfolio.It clearly reveals that the probability distribution is not centrally involved, since expectations of outcomes are not used to value the derivatives. It reveals that we need more probability theory to get a complete understanding of path dependent probabilities of security prices. There are some disadvanatges of this model because the trading times are not really at discrete times, trading goes on continuously. Securities do not change value according to a Bernoulli (two-valued) distribution on a single time step, or a binomial distribution on multiple time periods, they change over a range of values with a continuous distribution. Also this model consists of tedious calculations. But the binomial model can be viewed as the discrete-time versions of the Black-Scholes model.The analytical difference between this two models does not affect the crucial idea that underlies the derivation of each model.In both cases risk free hedging is equally important. Being a continuous-time case, the Black-Scholes model relies on continuous risk-free hedging by revising the required hedge ratio instantaneously. The binomial model, in contrast, allows changes in the hedge ratio from one period to the next, thus allowing students to follow the analytical process involved.

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