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The Appendix of the first number of Balkan Journal of Applied Mathematics and Informatics, is devoted to the reports of the First Modelling Week in Macedonia, which was held in Stip, 12-16 February 2018.

The First Modelling Week in Macedonia was organized by Faculty of Computer Science -Department of Mathematics and Statistics, Faculty of Electrical Engineering and Faculty of Technology with the support of the TD 1409 MI-NET Cost Action. The aims of the Modelling Week were: widening, broadening and sharing knowledge relevant to the Action's objectives through working on modern and actual problems which can be solved with mathematics and mathematical modelling.

The Modelling Week was organized under auspices of Prof. Blazo Boev, Rector of the Goce Delcev University, Stip, Macedonia.

The Program Committee of the First Modelling Week were:

- 1. Vineta Srebrenkoska, PhD Macedonia
- 2. Tatjana Atanasova Pachemska, PhD Macedonia
- 3. Poul G. Hjorth, PhD Denmark
- 4. Wojciech Okrasinski, PhD Poland
- 5. Joerg Elzenbach, PhD Germany
- 6. Gregoris Makrides, PhD Cyprus
- 7. Biljana Jolevska Tuneska, PhD Macedonia
- 8. Limonka Koceva Lazarova, PhD Macedonia

In the First Modelling Week in Macedonia participated 34 participants from Macedonia, Bulgaria, Portugal and Denmark. The Modelling Week was aimed towards Masters, PhD students, Early Career Investigators (up to 8 years after their PhD). All the participants were split in three groups in order to solve the three problems which were set:

Problem 1 - Scheduling in kindergarten, proposed by Limonka Koceva Lazarova

Problem 2 - Determining the optimal number of cash boxes to increase the efficiency of the customer service and determining the way of storage of products in the warehouse. How to manage stocks in the warehouse, proposed by Tatjana Atanasova – Pachemska.

Problem 3 - Optimization of the industrial processes for production of advanced polymer composites by implementation of the full factorial experimental design, proposed by Vineta Srebrenkoska.

The third problem was split in three subproblems.

All of the solutions are presented in form of reports in this appendix.

Thanks for the editors of the Balkan Journal of Applied Mathematics and Informatics, about their support for publishing of the results from The First Modelling Week in Macedonia.

# 

# OPTIMIZATION MODELS FOR SCHEDULING IN KINDERGARTEN AND HEALTHCARE CENTERS

# Aleksandra Stojanova <sup>1</sup>, Mirjana Kocaleva <sup>1</sup>, Natasha Stojkovikj <sup>1</sup>, Dusan Bikov <sup>1</sup>, Marija Ljubenovska <sup>1</sup>, Savetka Zdravevska <sup>1</sup>, Biljana Zlatanovska <sup>1</sup>, Marija Miteva <sup>1</sup>, Limonka Koceva Lazarova <sup>1</sup>

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**Abstract.** Kindergartens and day care centers face up with a problem how the employees should be organize throughout the working day, or only partially included in a certain period of the day. It is necessary to make a schedule of employees in the kindergartens or in a day care center for children. Our aim was making an optimization model to deal with the large number of children and employees. This problem can be extend to problem for healthcare services for elderly people.

Keywords: kindergartens, healthcare services, job shop scheduling, genetic algorithm

# 1. Introduction

Considering the modern way of life, where both of the parents usually work more than 8 hours during the day and they usually spent much time outside the home, there is a necessity for caring of the children in kindergartens, or in a care centers for children. However, children as a sensitive category have specific requirements, which depend of their age. They require different approaches and care. Because of that, the kindergartens and day care centers face up with a problem that the employees have to be organize throughout the working day, or only partially included in a certain period of the day. Therefore, it is necessary to make a schedule of employees in the kindergartens or in a day care center for children.

An optimization model, in order to manage how to deal with the large number of children attending such centers and the employees, on the other hand, has to be create. That is our aim with this work. It is necessary to take into consideration that groups should be form depending on the children age. In addition, it is good to be consider the case when there are sick children in the group that requires particular care from a caregiver or a medical person. The children with special needs should be included in the regular groups, but in that case, a psychologist will be need. The basic needs of children should be consider, like feeding, sleeping, upbringing, and learning. In the day center, there are small children aged from one to six years who do not attend school, as well as children from six to ten years old who go to school.

This problem can be also extend into a problem for healthcare services, for elderly people, in rest home. It requires an optimization method and a model to handle with the increasing demand of supplying healthcare services to elderly. That means, by using the limited number of caregivers and making proper scheduling, as more as possible, elder patients to be service. The main problem here is scheduling patients (elderly people) to the available caregivers. Scheduling is the allocation of shared resources over time to competing activities.

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Figure 1 Daily activities of elderly and children

# 2. Literature review 2.1. General model proposed by Marta Ferreira

The general model was develop for the staff-scheduling problem of an organization that works continuously, 24 hours a day.

The day is divided in nS working shifts, where nS is number of working shifts. The model considers a set of nT teams of homogeneous (single skilled and full-time) employees, that must be assigned to either a work or a break shift, in each of the nD planning period days. nT and nD are number of teams and number of days, accordingly. Daily shift demand levels must be satisfied, meaning that the model must guarantee a required number of teams working in each shift on each day. Work rules include a minimum and a maximum number of consecutive working days for each team, as well as a predefined sequence of working shifts to be respect. Each shift change must have a break or non-working day in between. The objective is to minimize and to level the number of days each team works in each shift, in order to balance the workload. [1]

 $d \in \{1, \ldots, nD\}, day;$ 

```
t \in \{1, \ldots, nT\}, team;
```

 $s \in \{1, \ldots, nS\}$ , working shift;

s'  $\in \{1, \ldots, 2 \times nS\}$ , extended shift.

n(s') is the extended shift that follows the extended shift s' in a given sequence;

This model, also can be used for healthcare services to elderly people.

# 2.2. Mathematical models for scheduling problem

Mostly used mathematical models in the past for dealing with scheduling problem were:

- Base Model (FSMP)
- Single machine model
- Parallel machine model
- Flow shop model
- Job shop model
- Pure job shop model

Table 1 presents the number of variables and constraints necessary for a general MJP formulation of some of the scheduling problems. As we can conclude from the table, the number of variables and constraints grow rapidly, indicating serious limitations and solving larger problems using this method. [2]

		01							
Model	Variable	Number of	Number of						
	name	variables	constraints						
Base	$c_{ij}$	n * m	n * m						
Model	$Y_{ijk}$	m	n * m						
(FSMP)	x <sub>irj</sub>	$n * \sum M_j$	$\sum_{m=1}^{m}$						
		j=1	$n(n-1)*\sum_{j}M_{j}$						
0: 1	-	[n(n-1)m]/2	<i>j</i> =1						
Single	c <sub>ij</sub>	n	n						
machine	x <sub>ir</sub>	[n(n-1)]/2	n(n-1)						
model									
Parallel	c <sub>ij</sub>	n	n						
machine	$Y_{ik}$	$n * M_1$	n						
model	xir	[n(n-1)]/2	$n(n-1)M_1$						
Flow shop	c <sub>ij</sub>	n * m	n * m						
model	x <sub>irj</sub>	[n(n-1)m]/2	n(n-1)m						
Job shop	c <sub>ij</sub>	n * m	n * m						
model	$Y_{ijk}$	<i>m</i>	n * m						
	x <sub>iri</sub>	$n * \sum M_j$	m						
		$\overline{j=1}$	$n(n-1) * \sum_{j} M_j$						
		[n(n-1)m]/2	$\overline{j=1}$						
Pure job	c <sub>ij</sub>	n * m	n * m						
shop	$x_{irj}$	[n(n-1)m]/2	n(n-1)m						
model									

# Table 1 Models for scheduling problem

# 3. Proposed models

For problem solving, we proposed two models.

3.1. Job shop-scheduling model

- o Optimization of time needed for completing all tasks
- Agent based approach with AnyLogic [3]
- 3.2. Genetic algorithm
  - Used alone or in combination with JSS

### 3.1. Job shop scheduling model

This model can be described by a set of n jobs  $J_i$ , where  $1 \le i \le n$ , and each job has to be processed on a set of m machines  $M_r$ , where  $1 \le r \le m$ . Each job has a sequence of machines that must be processed. The processing of job  $J_i$  on machine  $M_r$  is called the operation  $O_{ir}$ . Operation  $O_{ir}$  requires the exclusive use of  $M_r$  for an uninterrupted duration  $p_{ir}$ , where  $p_{ir}$  is its processing time. A schedule is a set of completion times for each operation  $M_{sir}$ , where  $1 \le i \le n$  and  $1 \le r \le m$  that satisfies those constraints. (Fig 2)

The time required to complete all the jobs is called the makespan MS. The scheduling objective is makespan minimization, which means to minimize the completion time of the last operation of any job.



Figure 2 JSS model

In our problem, we have two sets

 $E = {E_1, ..., E_m}$  where E are employees (machine of the model)

 $K = \{K_1, ..., K_m\}$  where K are kids or elderlies (jobs of the model)

The aim is to minimize the time to perform all tasks (all needs of the kids or all needs of elderlies in a shortest possible time).

For the assignment problem of kids or elderlies to employs, we use MIP (mixed integer programming) model.

The MIP formulation is often used to model the classical deterministic JSSP. The MIP model yields optimum solutions for small problem instances, but it's not so good model for large problem size [4], [5], [6], [7].

# • MIP model for our problem is

#### 1) Parameters:

r<sub>ilk</sub> with value 1, if elderly (kid) i requires task l from employ k, and 0 otherwise.

p<sub>ik</sub> is servicing time in which an elderly (kid) i has to be serviced from employ k.

2) Decision variables:

sik is start time of servicing an elderly (kid) i by employ k.

y<sub>ijk</sub> has a value 1 when an elderly j precedes elderly i for caregiver k.

$$\sum_{k=1}^{m} r_{imk} (s_{ik} + p_{ik}) \le MS \quad i = 1, 2, ..., n$$
(1)

Constraint 1 gives the lower bound for the function MS.

$$\sum_{k=1}^{m} r_{lik} \left( s_{ik} + p_{ik} \right) - \sum_{k=1}^{m} r_{i,l+1,k^{2}ik} \le 0 \quad i = 1, 2, \dots, m; \ l = 1, 2, \dots, m-1; \quad (2)$$

Constraint 2 ensures that the starting time of servicing an elderly i with task l + 1 is not earlier than its finish time in its predecessor, task l.

$$K(1 - y_{ijk}) + s_{ik} - s_{ik} \ge p_{ik}, k = 1, 2, ..., m; 1 \le i < j \le n$$
(3)

$$Ky_{iik} + s_{ik} - s_{ik} \ge p_{ik}, k = 1, 2, ..., m; 1 \le i < j \le n$$
 (4)

Constraints 3 and 4 ensure that only one elderly is served from employer at any given time. The parameter K is a large number, sometimes taken as the sum of all processing times [8], [9].

### • Agent based approach of the problem

Agent-based models (ABMs) consist of a set of elements (agents) characterized by some attributes, which interact each other through the definition of appropriate rules in a given environment. ABMs are useful in reproduce of economics and social science systems.

Agents have to communicate among each other with messages. They are adaptive and autonomous entities who are able to assess their situation, make decisions, compete or cooperate with one another on the basis of a set of rules, and adapt future behaviors on the basis of past interactions.

- Our agent-based approach of the problem consists two types of agents.
  - kids (elderly people or patients)
  - employees (caregivers).

They communicate among each other in a manner that employees can provide different services to elderlies or kids. This approach is applicable in combination with discrete event simulations



Figure 3 AnyLogic Simulation for job shop scheduling problem

In Fig 3 a screenshot from simulation of our problem made in AnyLogic is given. The red color shows that certain employee (caregiver) is busy. That means there is a kid or elderly he/she is serving in that moment. In addition, if the employee is free his/her cube is marked green. Each serviced kid or elderly is presented with different color in front of employee (caregiver). In the moment, each caregiver can give service to one kid or elderly. When all kid or elderly are completely served, simulation ends. At the end of simulation, all employees (caregivers) are colored green.

There are also indicators presented in the table of simulation, which changes every moment according to a current situation. In that table, one can see how many kids or elderlies are in service in the moment, how many of them have been served, and how many are waiting to be served. Total makespan during the whole simulation is 0 and is changed at the end of simulation, representing total time spent for all kids or elderlies to be serviced by employees (caregivers).

In Fig 4 a) and b) Gantt chart and time plot chart are presented. They show how much time each caregiver is busy or free.

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#### Figure 4 a) Gantt chart presentation of Anylogic job shop scheduling

	JO	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J11	J12	J13	J14	J15	J16	J17
MO	19.098	16.558	12.847	7.771	8.264	14.787	19.985	19.436	8.07	14.562	19.807	14.257	17.745	9.069	16.457	8.697	10.537	14.66
M1	19.208	14.898	16.16	5.16	11.48	19.765	14.456	8.142	8.824	5.131	7.299	5.422	10.307	15.947	11.416	8.727	11.958	6.657
M2	19.056	7.351	7.13	7.416	8.497	8.101	18.644	7.59	16.669	14.468	15.659	13.872	13.979	18.301	19.988	15.603	7.659	16.619
M3	10.958	10.673	12.226	7.671	18.349	10.62	12.615	13.234	8.375	8.12	17.36	6.095	9.381	5.656	5.929	10.066	15.142	9.597
M4	10.213	7.096	13.168	13.106	5.575	11.95	12.372	13.332	19.747	18.205	6.855	6.111	18.147	11.866	13.795	14.11	15.734	18.209

Figure 4 b) Time plot chart

## 3.2. Approach with Genetic algorithm

Genetic algorithm can be used in combination with JSS. (Fig 5). To apply a genetic algorithm to a scheduling problem we must first represent it as a chromosome. After that it is necessary to implements each schedule as a chromosome/individual in a population of schedules. Each schedule should be evaluated with a fitness function. Schedules with greater fitness function values are allowed to "mate" with other schedules via crossover. Mutation is used in order to provide diversity in the population. With the help of crossover and mutation operations new populations of schedules are generated. New generations are created until a schedule is formed that is deemed acceptable. Generally speaking, a schedule is deemed acceptable if its fitness function value is high enough. [10], [11]



Figure 5 Genetic algorithm presentation

# 4. Future work

Our plans for future are to find application of General model and to try to adapt it for our problem. In addition, we want to optimize our proposed approach given in this paper and to adapt to other constraints needed. Our last aim is to extend proposed model with genetic algorithms and from this to conclude which algorithm or model gives the best results for our problem.

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<sup>[3]</sup> https://www.anylogic.com/