GOCE DELCEV UNIVERSITY - STIP
FACULTY OF COMPUTER SCIENCE

ISSN 2545-479X print
ISSN 2545-4803 on line

BALKAN JOURNAL OF APPLIED MATHEMATICS AND INFORMATICS (BJAMI)

YEAR 2018

VOLUME I, Number 1

GOCE DELCEV UNIVERSITY - STIP, REPUBLIC OF MACEDONIA
FACULTY OF COMPUTER SCIENCE

ISSN 2545-479X print
ISSN 2545-4803 on line

BALKAN JOURNAL OF APPLIED MATHEMATICS AND INFORMATICS

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OF APPLIED MATHEMATICS AND INFORMATICS

(BJAMI)

YEAR 2018 VOLUME I, Number 1

AIMS AND SCOPE:
BJAMI publishes original research articles in the areas of applied mathematics and informatics.

Topics:
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2. Computer and software engineering;
3. Information technology;
4. Computer security;
5. Electrical engineering;
6. Telecommunication;
7. Mathematics and its applications;
8. Articles of interdisciplinary of computer and information sciences with education, economics, environmental, health, and engineering.

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BALKAN JOURNAL
OF APPLIED MATHEMATICS AND INFORMATICS (BJAMI), Vol 1

ISSN 2545-479X print
ISSN 2545-4803 on line
Vol. 1, No. 1, Year 2018

EDITORIAL BOARD

CONTENT

The Appendix of the first number of Balkan Journal of Applied Mathematics and Informatics, is devoted to the reports of the First Modelling Week in Macedonia, which was held in Stip, 12-16 February 2018.

The First Modelling Week in Macedonia was organized by Faculty of Computer Science - Department of Mathematics and Statistics, Faculty of Electrical Engineering and Faculty of Technology with the support of the TD 1409 MI-NET Cost Action. The aims of the Modelling Week were: widening, broadening and sharing knowledge relevant to the Action's objectives through working on modern and actual problems which can be solved with mathematics and mathematical modelling.

The Modelling Week was organized under auspices of Prof. Blazo Boev, Rector of the Goce Delcev University, Stip, Macedonia.

The Program Committee of the First Modelling Week were:

1. Vineta Srebrenkoska, PhD – Macedonia
2. Tatjana Atanasova – Pachemska, PhD – Macedonia
3. Poul G. Hjorth, PhD – Denmark
4. Wojciech Okrasinski, PhD – Poland
5. Joerg Elzenbach, PhD – Germany
6. Gregoris Makrides, PhD – Cyprus
7. Biljana Jolevska – Tuneska, PhD – Macedonia
8. Limonka Koceva Lazarova, PhD - Macedonia

In the First Modelling Week in Macedonia participated 34 participants from Macedonia, Bulgaria, Portugal and Denmark. The Modelling Week was aimed towards Masters, PhD students, Early Career Investigators (up to 8 years after their PhD). All the participants were split in three groups in order to solve the three problems which were set:

Problem 1 - Scheduling in kindergarten, proposed by Limonka Koceva Lazarova

Problem 2 - Determining the optimal number of cash boxes to increase the efficiency of the customer service and determining the way of storage of products in the warehouse. How to manage stocks in the warehouse, proposed by Tatjana Atanasova – Pachemska.

Problem 3 - Optimization of the industrial processes for production of advanced polymer composites by implementation of the full factorial experimental design, proposed by Vineta Srebrenkoska.

The third problem was split in three subproblems.

All of the solutions are presented in form of reports in this appendix.

Thanks for the editors of the Balkan Journal of Applied Mathematics and Informatics, about their support for publishing of the results from The First Modelling Week in Macedonia.

UDK: 519.872:339.372.5

PROPOSED QUEUING MODEL M/M/3 WITH INFINITE WAITING LINE IN A SUPERMARKET

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Abstract: In this paper M/M/c queuing model is considered and applied to solve a problem in a supermarket when are generated crowds by buyers on the cash boxes. The model of the supermarket has three cash-settling boxes and one queue created from buyers. The performance of the supermarket is analyzed using three cash boxes with arriving rate of 50 customers/ hour and service rate 18 customers/ hour. Also, it is considered whether if is necessary to include additional cash boxes in order to reduce the waiting queue and time.

Keywords: waiting lines, queuing models, M/M/c queuing model, supermarket

1. Introduction

Waiting in lines is part of everyday life. In waiting line systems, the terms server and channel are used interchangeably. Waiting line systems can have single or multiple lines. System serving capacity is a function of the number of service facilities and server proficiency. Single-server examples include small retail stores with a single checkout counter. Multiserver systems have parallel service providers offering the same service. The study of waiting lines, called queuing theory, is one of the oldest and most widely used quantitative analysis techniques. Queuing theory had its beginning in the research work of a Danish engineer named A. K. Erlang. In 1909, Erlang experimented with fluctuating demand in telephone traffic. Eight years later, he published a report addressing the delays in automatic dialing equipment. At the end of World War II, Erlang's early work was extended to more general problems and to business applications of waiting lines [7]. In real life, many queueing situations arise in which there may be tendency of customers to be discouraged by a long queue. As a result, the customers either decide not join the queue (i.e. balk) or depart after joining the queue without getting served due to impatience (i.e. renege. Queueing theory deals with one of the most unpleasant experiences of life, waiting. Queueing is quite common in many fields, for example, in telephone exchange, in a supermarket, at a petrol station, at computer systems, etc. Queueing theory became a field of applied probability and many of its results have been used in operations research, computer science, telecommunication, traffic engineering, reliability theory, just to mention some.

G. Kendall developed a notation that has been widely accepted for specifying the pattern of arrivals, the service time distribution, and the number of channels in a queuing model. This notation is often seen in software for queuing models. It is a three-part code a/b/c where the first letter specifies the interarrival time distribution, the second one the service time distribution and the third and last letter specifies the number of servers. The letters that are used in the Kendal notation are G, M and D, where G stands for general distribution, M for Poisson distribution for number of occurrences (or exponential times) and D for constant (deterministic) rate. If there are c distinct service channels in the queuing system with Poisson arrivals and exponential service times, the Kendall notation would be M/M/c. M/D/c is a queuing model for c-channel system with Poisson arrivals and constant service and c-channel system with Poisson arrivals and service times that are normally distributed would be identified as M/G/c. The notation can be extended with an extra letter to cover other queuing models. For more details about the queuing models see [2, 4, 5, 8]. Many authors work with the M/M/c queuing model, for example the authors in [1] presents an analysis for the M/M/c queue with balking and reneging. They assumed that arriving customers balk with a fixed probability and renege according to a negative exponential distribution. Also, in [6] M/M/c queuing model is used to model and analyze the characteristic of queuing system at the pharmacy unit of hospital in Malaysia. Instead of exponentially distributed times some service systems have constant service times. When customers or equipment are processed according to a fixed cycle, as in the case of an automatic car wash or an amusement park ride, constant service rates are appropriate. Because constant rates are certain, the values for average queue (system) length and average waiting time in the queue (system) are always less than they would be in the model M/M/m which have variable service times. As a matter of fact, both the average queue length and the average waiting time in the queue are halved with the constant service rate model.

The length of a line can be either limited or unlimited. Queuing models are treated in this module under an assumption of unlimited queue length. A queue is unlimited when its size is unrestricted. Service systems are usually classified in terms of their number of channels (for example, number of servers) and number of phases (for example, number of service stops that must be made). A single- channel queuing system has only one server with one waiting line while the multiple- channel queuing system has several servers with one waiting line. In a single phase system, the customers is served from only one server and then exits the system, while in multiphase system the customer is served from several servers before exiting. In Figure 1 are shown this channel configurations.

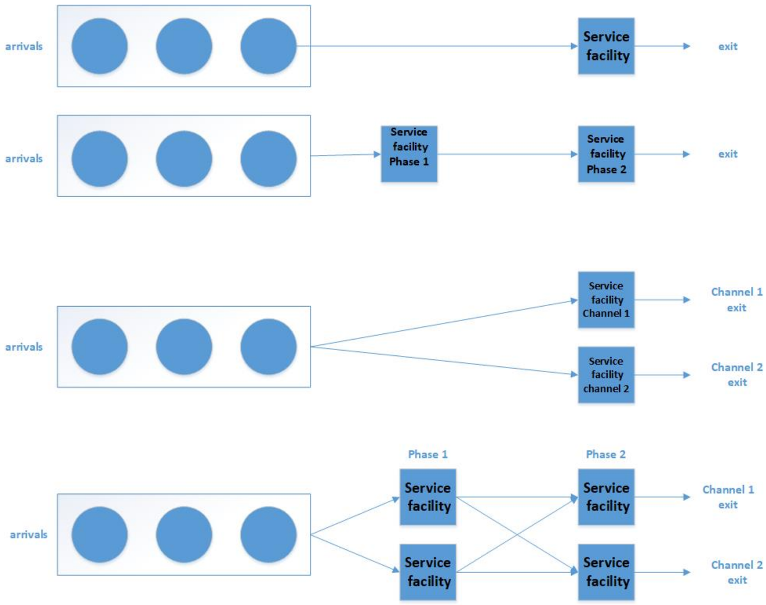

Figure 1. Queuing system design (a) single channel, single phase system (b) single channel, multiphase system, (c) multichannel, single- phase system, (d) multichannel, multiphase system

This paper is organized as follows. In Section 2 is given an analytical approach to determine important measures of performance in a typical M/M/c service system. The performance of the supermarket is analyzed in Section 3 using three cash boxes with arriving rate of 50 customers/ hour and service rate 18 customers/ hour. And in the same section is considered whether it is necessary to include additional cash boxes in order to reduce the waiting queue. Section 4 concludes this paper with comparison of the supermarket performances for 3 and 4 available cash boxes.

2. Multichannel queuing model – M/M/c

In this section an analytical approach to determine important measures of performance in a typical M/M/c service system is represented. It is assumed that customers awaiting service form one single line and then proceed to the first available cash box. An example of such a multichannel, single-phase waiting line is found in many supermarkets and banks today. The multiple-channel system presented here again assumes that arrivals follow a Poisson probability distribution and that service times are distributed exponentially. Service is first come, first served, and all servers are assumed to perform at the same rate.

The following formulas can we used in a waiting line analysis:

1. The probability that there are zero customers in a supermarket:

$$P_0 = \frac{1}{\sum_{n=0}^{c-1}\left(\frac{1}{n!}\right)\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{(c-1)!}\left(\frac{\lambda}{\mu}\right)^{c-1}}$$

where c is number of an open cash boxes, λ is an average arrival rate and μ is average service rate at each cash box.

2. The average number of customers the supermarket:

$$L = \frac{\lambda\mu(\lambda/\mu)^c}{(c-1)!(c\mu-\lambda)^2}P_0 + \frac{\lambda}{\mu}$$

3. The average time a customer spends in the waiting line or being serviced:

$$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

4. The average number of customers in line waiting for service:

$$L_q = L - \frac{\lambda}{\mu}$$

5. The average time a customer spends in the queue waiting for service:

$$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

6. Utilization rate of the cash boxes:

$$\rho = \frac{\lambda}{c\mu}$$

7. Probability of n customers in store at given time:

$$P_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!}P_0, & n \le c \\ \frac{(\lambda/\mu)^n}{c!c^{n-s}}P_0, & n > c \end{cases}$$

3. Model simulation and results

In this section, a single-phase queuing model with a single queue and multiple parallel cash boxes (services) is represented. Arrivals are random and are independent of one another and their occurrence cannot be predicted exactly. The queuing model deployed in this paper assumes that the arriving customer is patient and waits in the queue until is served. Unfortunately, in real life customers can balk or renege [11]. Customer who bulks refuse to join the queue because the queue is too long and exits the supermarket. Reneging customer enters the queue but after some time leaves the queue because become impatient and exit without being served. The customers after finding all cash boxes busy join the queue (Fig.2) and are served with FIFO discipline. The first customer in the queue will be proceed to any of the three cash boxes as soon as one is available and there is no priority classification for any arrival. Arrivals of the customers follow a Poisson probability distribution with arrival rate λ customers per hour. The service times vary from one customer to next but their average is known. The service rate is distributed exponentially with serving rate μ customers per hour. All the cash boxes are assumed to perform with same rate. There is no limit to the number of customers of the queue and they come from infinite or very large population.

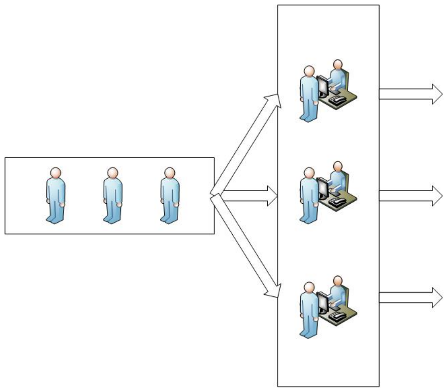

Figure 2. Multiple server single queue model

First, the performance of the supermarket is analyzed using three cash boxes with arriving rate of 50 customers/ hour and service rate 18 customers/ hour. The performance measurements are represented in Table 1. The probability of n customers in the supermarket is shown in Fig. 3.

Table 1: Performance measurements for 3 cash boxes

arrival rate (lamda)	50
service rate (mu)	18
numer of services (s)	3
average time between arrivals	0,02000
average service time	0,05556
Average service utilization	0,92593
Probability the system is empty	0,01790
Average number in the system	13,56912 customers
Average number waiting in the queue	10,79134 customers
Average time in the system	0,27138 hors
Average time in the queue	0,21583 hours
Probability all servers are busy	0.8633

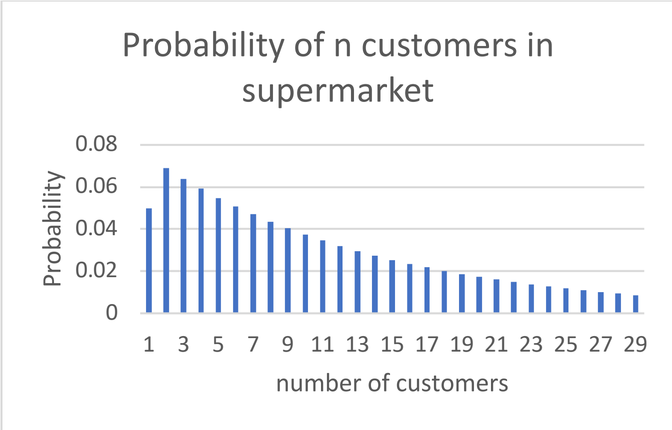

Figure 3. Probability of n customers in the supermarket (c=3, λ=50, μ=18)

For multiple server queue, the average service utilization must be strictly less that one (i.e. $\lambda<c\mu$). That is, the total service rate ($c\mu$) of c servers must be strictly greater that the arriving rate (λ) otherwise the queue will grow without bond. Thus, the minimum number of cash boxes is calculated from:

$$\frac{50}{18c}<1$$

$$c>3$$

At least four cash boxes are required in order the queue not to grow without bond. This means that in order to avoid the crowds and long waiting queue another cash box is opened. The arrival rate and service rate are not changed, and all four cash boxes are assumed to work at same rate. In Table 2 are represented the performance measurements of the supermarket with 4 cash boxes. Figure 4 shows the probability of n customers in the supermarket with 4 cash boxes.

Table 1: Performance measurements for 4 cash boxes

arrival rate (lamda)	50
service rate (mu)	18
numer of services (s)	4
average time between arrivals	0,02000
average service time	0,05556
Average service utilization	0,69444
Probability the system is empty	0,05174
Average number in the system	3,73250 customers
Average number waiting in the queue	0,95472 customers
Average time in the system	0,07465 hours
Average time in the queue	0,01909 hours
Probability all servers are busy	0.4200

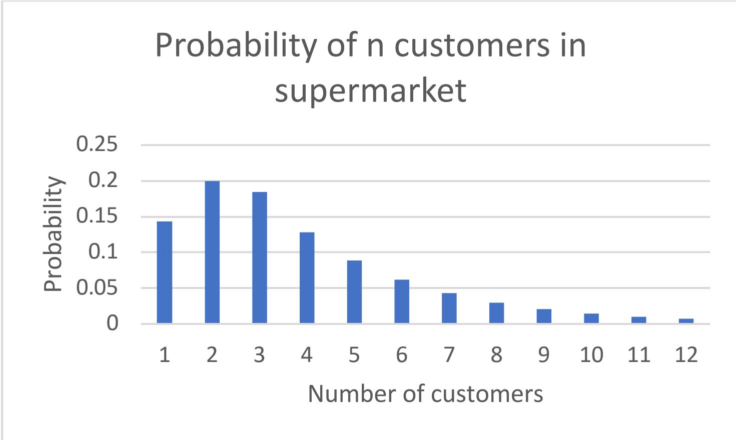

Figure 4. Probability of n customers in the supermarket (c=4, λ=50, μ=18)

When the arriving rate is λ=50 customers/ hour, service rate μ=18 customers/hour and three cash boxes opened, the cash boxes are busy with average waiting time of the customers around 13 minutes. When four cash boxes are opened with same arrival and service rate as with three cash boxes, the customers almost wait no time, and the busy rate of the cash boxes decreases from 86.33% to 42.0%.

4. Conclusion

In this paper is given analytical approach for supermarket application with a single phase and three parallel cash boxes. The customers arriving rate is with Poisson distribution with λ arriving customers per hour and the service rate is distributed exponentially with serving rate μ customers per hour. There is no limit to the number of customers of the queue and they come from infinite or very large population. When three cash boxes opened the average number of customers in the supermarket is 13.56, while with four cash boxes the average number of customers in the supermarket decrease to 3.7 customers. The average waiting time in the queue with three cash boxes is 12.6 minutes and with four cash boxes the average waiting time in the queue is 1.15minutes. Also, the average number of customers waiting in the queue decreases from 10.71 customers to 1 customer when the forth cash box is opened.

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