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Address of the editorial office
Goce Delcev University – Stip
Faculty of philology
Krste Misirkov 10-A
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CONTENT

The Appendix

In honor of the first Doctor of Mathematical Sciences Acad. Blagoj Popov, a mathematician dedicated to differential equations, the idea of holding the "Day of Differential Equations" was born, prompted by Prof. Ph.D. Boro Piperevski, Prof. Ph.D. Borko Ilievski, and Prof. Ph.D. Lazo Dimov. Acad. Blagoj Popov presented his doctoral dissertation on 05.05.1952 in the field of differential equations. This is the main reason for holding the " Day of Differential Equations" at the beginning of May.

This year on May 10th, the "Day of Differential Equations" was held for the fifth time at the Faculty of Computer Sciences at "Goce Delcev" University in Stip under the auspices of Dean Prof. Ph.D. Cveta Martinovska - Bande, organized by Prof. Ph.D. Biljana Zlatanovska.

Acknowledgments to Prof. Ph.D. Boro Piperevski, Prof. Ph.D. Borko Ilievski and Prof. Ph.D. Lazo Dimov for the wonderful idea and the successful realization of the event this year and in previous years.

Acknowledgments to the Dean of the Faculty of Computer Sciences, Prof. Ph.D. Cveta Martinovska - Bande for her overall support of the organization and implementation of the "Day of Differential Equations".

The papers that emerged from the "Day of Differential Equations" are in the appendix to this issue of BJAMI.

UDC 517.93:519.6

NUMERICAL ANALYSIS OF THE BEHAVIOR OF THE DUAL LORENZ SYSTEM BY USING MATHEMATICA

Biljana Zlatanovska

Abstract. The Dual Lorenz system as a system of differential equations was obtained [1] and its dynamical analysis was done in the paper [2]. In this paper, by using Runge-Kuta method via mathematical software Mathematica a numerical analysis of the behavior of the Dual Lorenz system for a finite time t (as in the paper [3]) will be made. From the papers [1] and [2], the main role in the behavior of the Dual Lorenz system has the value of the parameter r, therefore the numerical analysis of the Dual Lorenz system will be made via changing the parameter r so that: we will change the parameter r and the parameter σ and b will remain fixed, we will change the parameters r and σ (or b) and the third parameter b (or σ) will remain fixed, or we will change all three parameters in parametric space (σ,r,b).
Mathematical Subject Classification: 65L06, 65L07.

1. Introduction

Dynamical systems as systems of differential equations are studied extensively in numerous mathematical literature (for example [1] – [15]). The behavior is often analyzed via graphical visualizations with using numerical methods (see [3], [4], [5], [6], [7], [8] and [9]).

The Dual Lorenz system is a nonlinear autonomous dynamic system and it is obtained in the paper [1] by the following form

$$\begin{aligned} \dot{x} &= \sigma(y-x) \\ \dot{y} &= x(r-z)-y \qquad (1.1) \\ \dot{z} &= -xy-bz \\ \sigma, r, b &> 0 \end{aligned}$$

where x, y, z are real functions from the real argument *t*. It has three fixed points $O(0,0,0)$, $O_1(\sqrt{b(1-r)}, \sqrt{b(1-r)}, r-1), O_2(-\sqrt{b(1-r)}, -\sqrt{b(1-r)}, r-1)$ for $0<r<1$ and one fixed point $O(0,0,0)$ for $r>1$.

From [1], for the Dual Lorenz system (1.1), appropriate characteristic equation in the fixed point $O(0,0,0)$ is given with

$$(\lambda+b)[\lambda^2+(\sigma+1)\lambda+\sigma(1-r)]=0 \qquad (1.2)$$

for $r>1$. For $0<r<1$ all roots of this equation (1.2) are negative. The characteristic equation in other fixed points $O_1(\sqrt{b(1-r)}, \sqrt{b(1-r)}, r-1), O_2(-\sqrt{b(1-r)}, -\sqrt{b(1-r)}, r-1)$ is given with

$$\lambda^3+(\sigma+b+1)\lambda^2+b(\sigma+r)\lambda+2\sigma b(r-1)=0 \quad (1.3)$$

for $0<r<1$. The characteristic equation (1.3) does not have purely imaginary roots.

In the paper [2] a dynamical analysis of the Dual Lorenz system (1.1) was done where the following conclusions are obtained:

- For $0<r<1$, the Dual Lorenz system (1.1) has unstable fixed points O_1 and O_2, but for $t\to\infty$, the trajectories of the system (1.1) are most likely to weigh to the fixed point *O*;
- The Dual Lorenz system (1.1) does not have chaos, when $0<r<1$. For $0<r<1$, the Dual Lorenz system (1.1) has a stable fixed point *O*. But, for $r>1$, the fixed point *O* is the unstable fixed point;
- For $r=1$, the fixed point *O* appears as a subcritical pitchfork bifurcation point for the Dual Lorenz system (1.1).

In this paper, the numerical analysis of the Dual Lorenz system (1.1) done in the same way as the numerical analysis of the Lorenz system in the paper [3] for a finite time t. The numerical analysis of the Dual Lorenz system (1.1) is made with the mathematical package Mathematica, which numerically approximates the solutions of the system by the method of Runge – Kuta. The three fixed points for the Dual Lorenz system (1.1) will exist when the parameter $0<r<1$, therefore its numerical analysis will be made via the changing of the parameter $0<r<1$.

We will consider the following cases:

1. The parameter $0<r<1$ is changing, the other two parameters σ and b are fixed. This is the case when the parameters are changing along a line in the parametric space (σ, r, b);
2. The parameters $0<r<1$ and σ (or b) are changing, the third is fixed. This is the case when the parameters are moving in a plane in the parametric space (σ, r, b);
3. All three parameters $0<r<1$, σ and b are changing. This is the case when all three parameters are moving in the parametric space (σ, r, b).

For the solutions $x = x(t)$, $y = y(t)$, $z = z(t)$ of the Dual Lorenz system (1.1) only *2D* graphs will be given because the software Mathematica does not give a good *3D* graph $(x(t), y(t), z(t))$. On *2D* graphs of the Dual Lorenz system (1.1), the red color correspondents with $x = x(t)$, the green color correspondents with $y = y(t)$ and the blue color correspondents with $z = z(t)$. Regardless of the above cases, for $t=0$ the curves $x = x(t)$, $y = y(t)$, $z = z(t)$ always start from initial values $x_0=x(0)$, $y_0=y(0)$, $z_0=z(0)$ in *2D* coordinate systems *Oxt*, *Oyt*, *Ozt* respectively.

In the paper [2], we proved that the Dual Lorenz system (1.1) does not have chaos for $0<r<1$. For improving the conclusions of this paper the graphical visualization of the solutions of the characteristic equations (1.2) and (1.3) when $0<r<1$ will be given.

2. Numerical analysis of the behavior for the Dual Lorenz system (1.1) when the parameter $0<r<1$ changes

When the parameter $0<r<1$ changes and the parameters σ and b are fixed, it moves in a straight line in the parameter space (σ,r,b). For small the time t, the instability of the Dual Lorenz system (1.1) close to the fixed points O_1 and O_2 can be seen when the parameters are changing in the following way:

- $0<\sigma\leq 1$, $0<b\leq 1$, $0.9\leq r<1$;
- $0<\sigma\leq 1$, $b>1$, $0.65\leq r<1$.

Example 1: The parameter r is changing from 0.9 to 0.95 by step 0.01. Let $\sigma=0.5$, $b=0.6$ and the initial values $x_0=0.1$, $y_0=0.5$, $z_0=-0.5$.

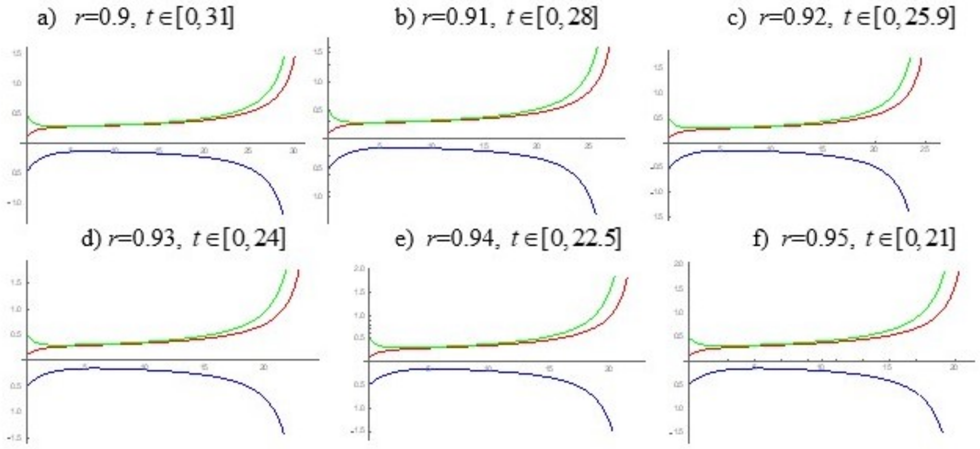
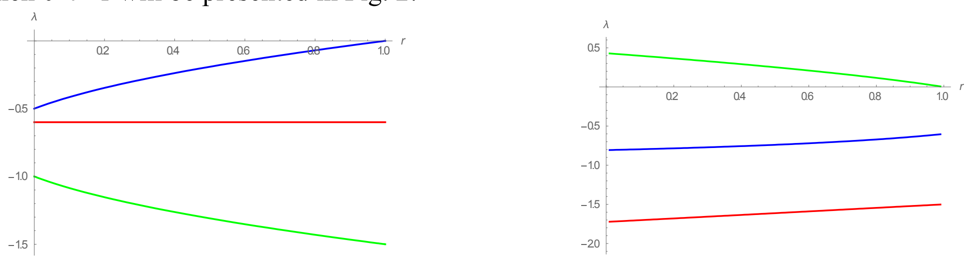


Figure 1: The solutions $x(t)$, $y(t)$, $z(t)$ of the Dual Lorenz system (1.1) in *2D* coordinate system

Figure 1 shows the graphical visualization of the solutions $x(t)$, $y(t)$, $z(t)$ of the Dual Lorenz system (1.1) and it is obtained the picture that the solutions $x(t)$, $y(t)$, $z(t)$ are away far from 0.

Next, the graphical visualization of the solutions of the characteristic equations (1.2) and (1.3) when $0<r<1$ will be presented in Fig. 2:



a) the solutions of the characteristic equation (1.2)   b) the solutions of the characteristic (1.3)

Figure 2: The graphical visualization of the solutions for the characteristic equations (1.2) and (1.3)

Figure 2, a) shows that all solutions of the characteristic equation (1.2) are negative numbers. Figure 2, b) shows that the two solutions of the characteristic equation (1.3) are negative numbers and one solution is a positive number.

3. Numerical analysis of the behavior for the Dual Lorenz system (1.1) when the parameter *r* and σ (or *b*) change

When the parameters $0<r<1$ and σ (or b) are changing, the third b (or σ) remains fixed; then they are moving in a plane in the parametric space (σ,r,b). For short time t, the instability of the Dual Lorenz system (1.1) close to the fixed points O_1 and O_2 can be seen when the parameters are changing in the following way:

- $0<\sigma<1$, $b>0$, $0.8\leq r<1$ (when the parametric σ and $0<r<1$ are changing and the parameter b remains fixed);
- $\sigma\geq 1$, $b>1$, $0.65\leq r<1$ (when the parametric σ and $0<r<1$ are changing and the parameter b remains fixed);
- but, small subintervals exist for $0<r<1$, $\sigma>0$, large b (when the parametric $0<r<1$ and b are changing and the parameter σ remains fixed).

Example 2: The parameter r is changing from 0.9 to 0.95 by step 0.05 and the parameter σ is changing from 0.7 to 0.8 by step 0.05. Let b=0.6 and the initial values x_0=0.1, y_0=0.5, z_0=-0.5.

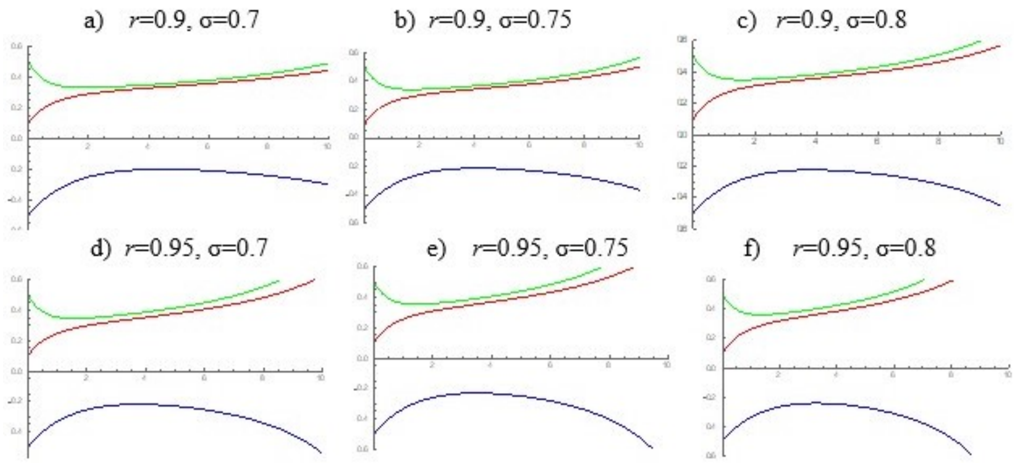
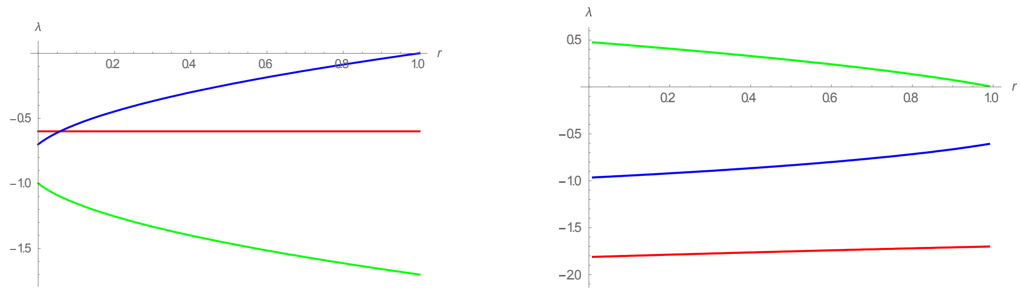


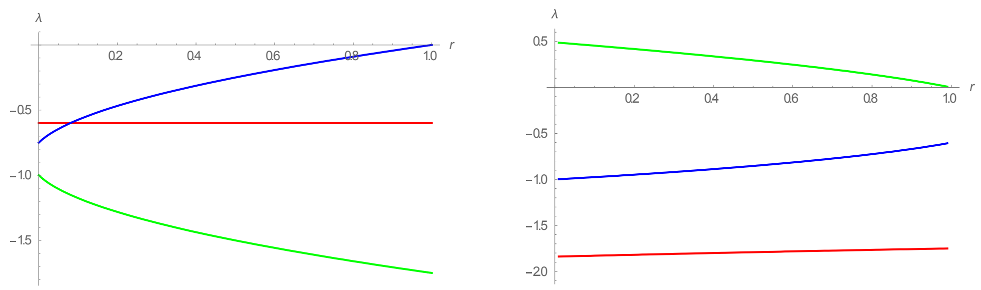
Figure 3: The solutions $x(t)$, $y(t)$, $z(t)$ of the Dual Lorenz system (1.1) in *2D* coordinate system for $t\in[0,10]$

In Figure 3 the graphical visualization of the solutions $x(t)$, $y(t)$, $z(t)$ of the Dual Lorenz system (1.1) is shown and the picture that the solutions $x(t)$, $y(t)$, $z(t)$ are away far from 0 is obtained.

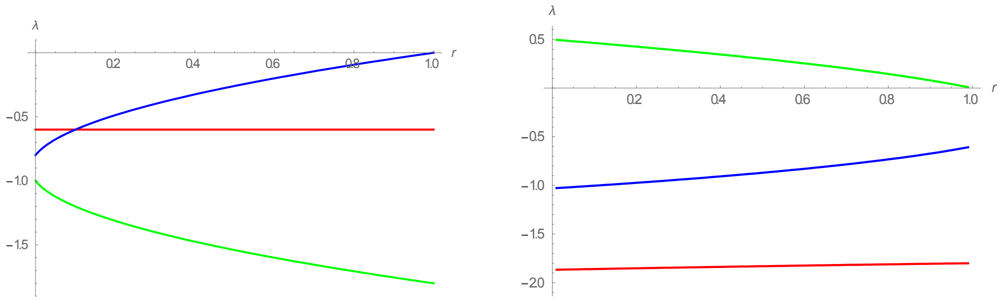
Next, the graphical visualization of the solutions of the characteristic equations (1.2) and (1.3) when $0<r<1$ in Fig. 4 will be presented, for the different value of σ:



a) the solutions of the characteristic equations (1.2) and (1.3) when σ=0.7



b) the solutions of the characteristic equations (1.2) and (1.3) when σ=0.75



c) the solutions of the characteristic equations (1.2) and (1.3) for σ=0.8

Figure 4: The graphical visualization of the solutions for the characteristic equations (2) and (3)

Figure 4 a) shows that all solutions of the characteristic equation (1.2) are negative numbers (the picture on the left), but the two solutions of the characteristic equation (1.3) are negative numbers and one solution is a positive number (the picture on the right) for σ=0.7. In Figure 4, b) and c) identical results are shown.

4. Numerical analysis of the behavior of the Dual Lorenz system (1.1) when all three parameters are changed

When all three parameters are changing, then the three parameters are moving in the parametric space (σ,r,b). For short time t, the instability of the Dual Lorenz system (1.1) close to the fixed points O_1 and O_2 can be seen in some small subintervals for $0<r<1$ close to 1, $\sigma>0$, about $b=0.05$.

Example 3: The parameter r is changing from 0.9 to 1 by step 0.05, the parameter σ is changing from 0.5 to 0.6 by step 0.05 and the parameter b is changing from 0.04 to 0.06 by step 0.01. Let $t\in[0,8]$ and the initial values t $x_0=0.1$, $y_0=0.5$, $z_0=-0.5$.

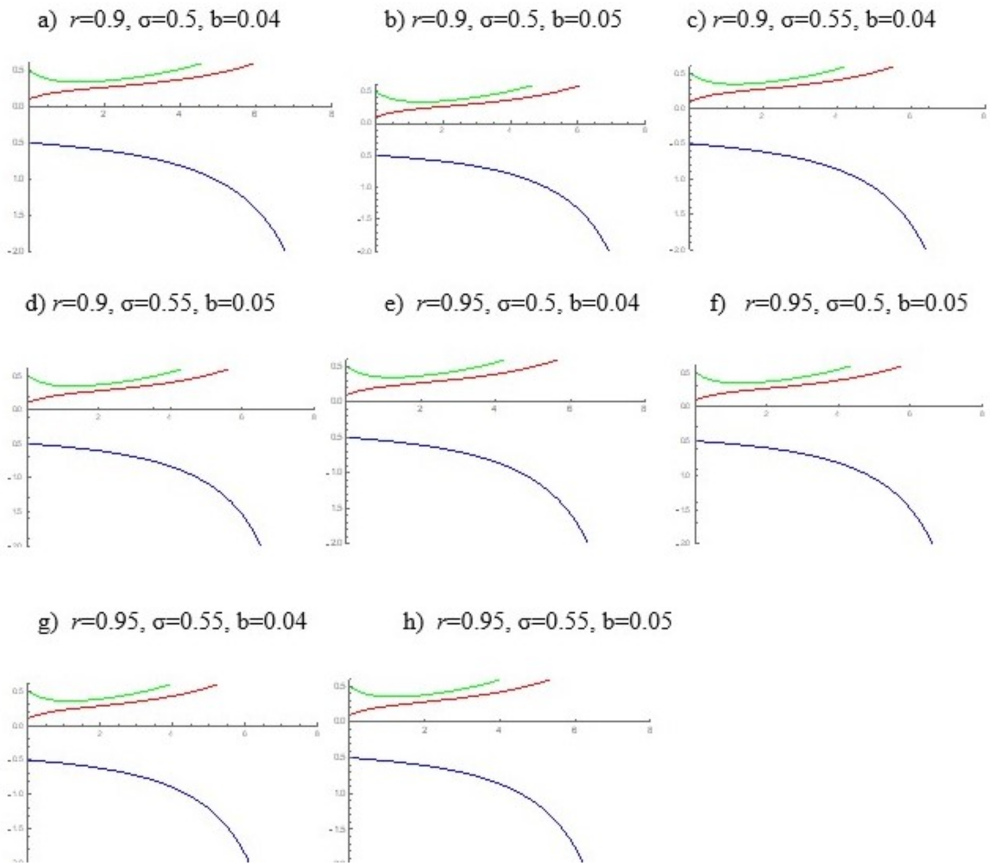
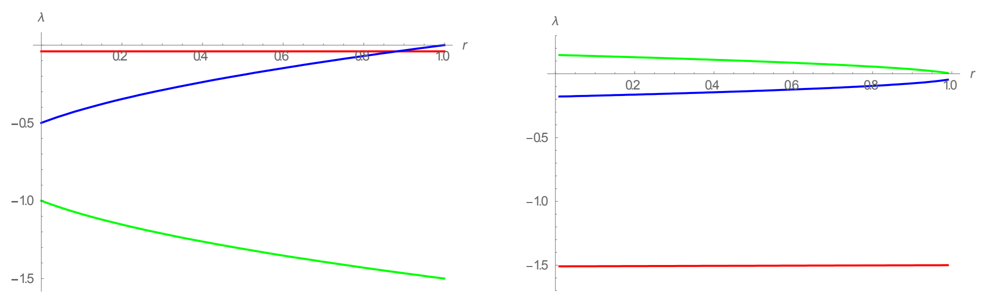


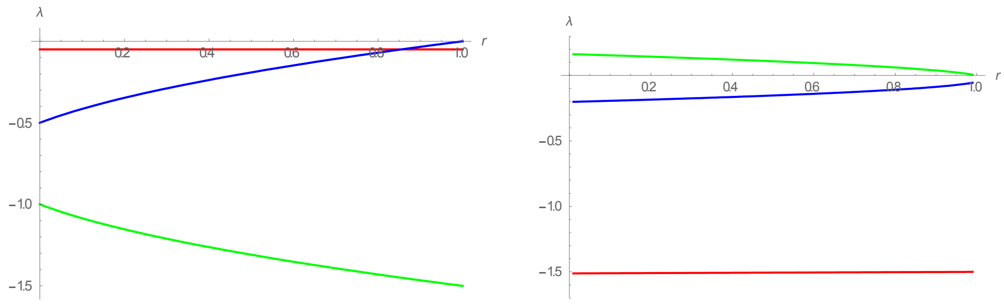
Figure 5: The graphs in *2D* coordinate system for $t\in[0,8]$

In Figure 5 the graphical visualization of the solutions $x(t)$, $y(t)$, $z(t)$ of the Dual Lorenz system (1.1) is shown and the picture that the solutions $x(t)$, $y(t)$, $z(t)$ are away far from 0 is obtained.

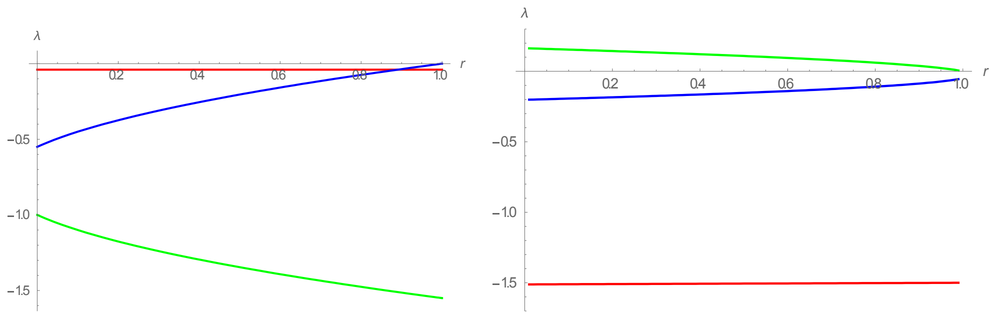
Next, the graphical visualization of the solutions of the characteristic equations (1.2) and (1.3) when $0<r<1$ will be presented in Fig. 6, for the different values of σ and b:



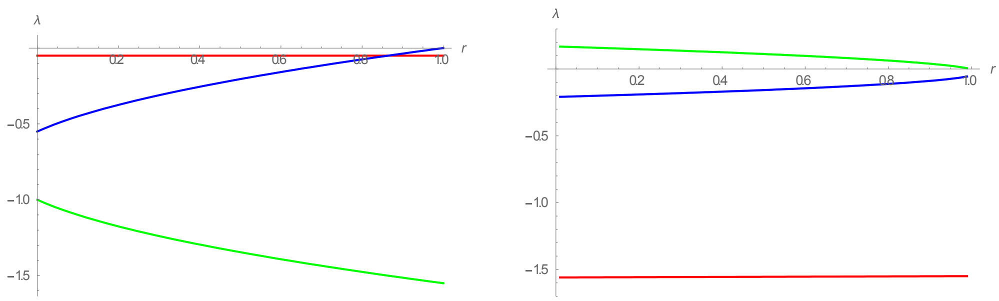
a) the solutions of the characteristic equations (1.2) and (1.3) when σ=0.5 and b=0.04



b) the solutions of the characteristic equations (1.2) and (1.3) when σ=0.5 and *b*=0.05



c) the solutions of the characteristic equations (1.2) and (1.3) when σ=0.55 and *b*=0.04



d) the solutions of the characteristic equations (1.2) and (1.3) when σ=0.55 and *b*=0.05

Figure 6: The graphical visualization of the solutions for the characteristic equations (1.2) and (1.3)

Figure 6, a) shows that all solutions of the characteristic equation (1.2) are negative numbers (the picture on the left), but the two solutions of the characteristic equation (1.3) are negative numbers and one solution is a positive number (the picture on the right) for σ=0.5 and *b*=0.04. In Figure 6, b), c) and d) identical results are shown.

Remark: In Figure 2, 4 and 6 the graphical visualization of the solutions of the characteristic equations (1.2) and (1.3) are shown when $0<r<1$ which are in accordance with the proofs, which are presented in the paper [2].

5. Conclusion

For the Dual Lorenz system (1.1), the explicit solutions are not known, therefore in this paper its behavior is analyzed with the Runge-Kuta method. The Runge-Kuta method gives us a geometrical visualization of the Dual Lorenz system. The Dual Lorenz system (1.1) does not have chaos and its behavior is the opposite of the behavior of the Lorenz system. The behavior of the Dual Lorenz system (1.1) was described in the papers [1] and [2], where appropriate proofs were given. The result from the numerical analysis of the Dual Lorenz system (1.1), which is made in this paper, is in accordance with the presenting of the proofs in the papers [1] and [2].

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Biljana Zlatanovska
University “Goce Delcev”
Faculty of Computer Science,
“Krste Misirkov” No.10-A, Stip
Republic of North Macedonia
E-mail address: biljana.zlatanovska@ugd.edu.mk