GOCE DELCEV UNIVERSITY - STIP
FACULTY OF COMPUTER SCIENCE

ISSN 2545-4803 on line

BALKAN JOURNAL OF APPLIED MATHEMATICS AND INFORMATICS (BJAMI)

YEAR 2020

VOLUME III, Number 1

GOCE DELCEV UNIVERSITY - STIP, REPUBLIC OF NORTH MACEDONIA
FACULTY OF COMPUTER SCIENCE

ISSN 2545-4803 on line

BALKAN JOURNAL OF APPLIED MATHEMATICS AND INFORMATICS

(BJAMI)

YEAR 2020 VOLUME III, Number 1

AIMS AND SCOPE:
BJAMI publishes original research articles in the areas of applied mathematics and informatics.

Topics:
1. Computer science;
2. Computer and software engineering;
3. Information technology;
4. Computer security;
5. Electrical engineering;
6. Telecommunication;
7. Mathematics and its applications;
8. Articles of interdisciplinary of computer and information sciences with education, economics, environmental, health, and engineering.

Address of the editorial office
Goce Delcev University – Stip
Faculty of philology
Krste Misirkov 10-A
PO box 201, 2000 Štip,
Republic of North Macedonia

BALKAN JOURNAL
OF APPLIED MATHEMATICS AND INFORMATICS (BJAMI), Vol 3

ISSN 2545-4803 on line
Vol. 3, No. 1, Year 2020

EDITORIAL BOARD

CONTENT

UDC: 621.31.011.7:519.876.5

CALCULATION FOR PHASE ANGLE AT *RL* CIRCUIT SUPPLIED WITH SQUARE VOLTAGE PULSE

GOCE STEFANOV, VASILIJA SARAC, MAJA KUKUSEVA PANEVA

Abstract. The serial connection of resistor *R* and inductance *L* is a basic electronic circuit. This circuit is the main circuit in industrial power consumers such as electric motors, induction furnaces, welding devices and other electrical consumers. In the paper, the analysis of serial *RL*- circuit supplied by the square voltage pulse is made. The equations for maximum current, time equivalent and phase angle in the serial *RL*- circuit are derived. For verification of theoretical calculations, a prototype of the bridge converter based on AMega 328P microcontroller was designed. In addition, the voltages waveforms in the circuit are represented.

1. Introduction

The ratio of the values on resistor and inductance in the serial circuit determines the amount of active energy that the power source delivers to the consumer. From the power point of view, the amount of this energy should be as high as possible, i.e. the circuit should work with a good power factor and good efficiency.

When the *RL*-circuit is supplied by sinusoidal-prostoperiodic voltage, the amount of active energy is determined by the values of effective voltage and current in the circuit, and the power factor determined by the phase difference between voltage and current. In the literature, there is detailed information about the work of the serial *RL*-circuit supplied by sinusoidal voltage [1, 2, 3].

The situation is quite different when the *RL*-circuit is supplied by voltage with square waveform. The amount of delivered energy then depends not only on the phase difference between the voltage and the current, but also on the harmonic distortion of the magnitudes of the voltage and current. Such *RL*-circuits, supplied by voltage with square waveform are found in all power converters such as speed regulators in directional and alternating motor, induction heating devices, and voltage regulators for different *RL*-consumers. In the literature [1, 2, 3] which deals with this matter, there is room for clarification of the nature of the behavior on *RL*-circuit supplied by voltage with square waveform.

Keywords: first- order differential equation, maximal current, phase angle, simulation, experiment

In this paper, an analysis of a serial *RL*- circuit supplied by square voltage pulse is made. The equations for maximum current, time equivalent, and phase angle in the serial *RL*-circuit are derived.

2. Response of *RL* Circuit Supplied by Step Voltage

A serial connection on resistor *R* and inductance *L* is a base circuit in power consumers such as electrical drive, induction-heating device. Figure 1 shows a serial *RL*-circuit.

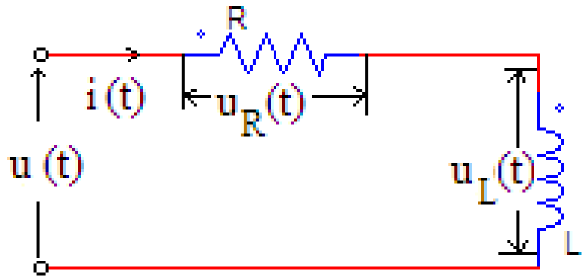


Figure 1. *Serial RL-circuit*

In Figure 1, u(t) is input voltage, i(t) is the current in the circuit, $u_R(t)$ is the resistor voltage, and $u_L(t)$ is the inductance voltage. This circuit is described by a linear nonhomogeneous differential equation of first order [1], [2], [3]:

$$\frac{\mathrm{d}i(t)}{\mathrm{d}t}+\frac{R}{L}i(t)=\frac{1}{L}\frac{\mathrm{d}u(t)}{\mathrm{d}t} \tag{1}$$

In case when the circuit is supplied by voltage with a sinus waveform, phase angle between the voltage and the current is given as in [3]:

$$\varphi=\mathrm{arctg}(\frac{\omega_s L}{R}) \tag{2}$$

In (2) ω_s is circular frequency and it is specified by switching frequency f_s as, $\omega_s = 2\pi f_s$. Figure 2 shows a circuit for simulations in the PowerSim program on RL-circuit supplied by step voltage, turned on at t = 0 s with conditions i(0-) = i(0+) = 0 A. The values on RL are: R = 1.9 Ω and L = 4,39 mH.
Figure 3 shows the waveform of the input step voltage u, the inductor voltage u_L and the current i of the circuit from Figure 2.

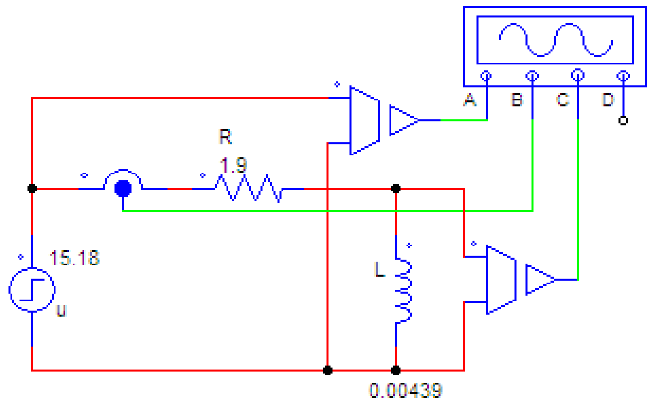


Figure 2. *Circuit for simulations in the PowerSim program on RL-circuit supplied by step voltage, turned on at t = 0 s with conditions i(0-) = i(0+) = 0 A.*

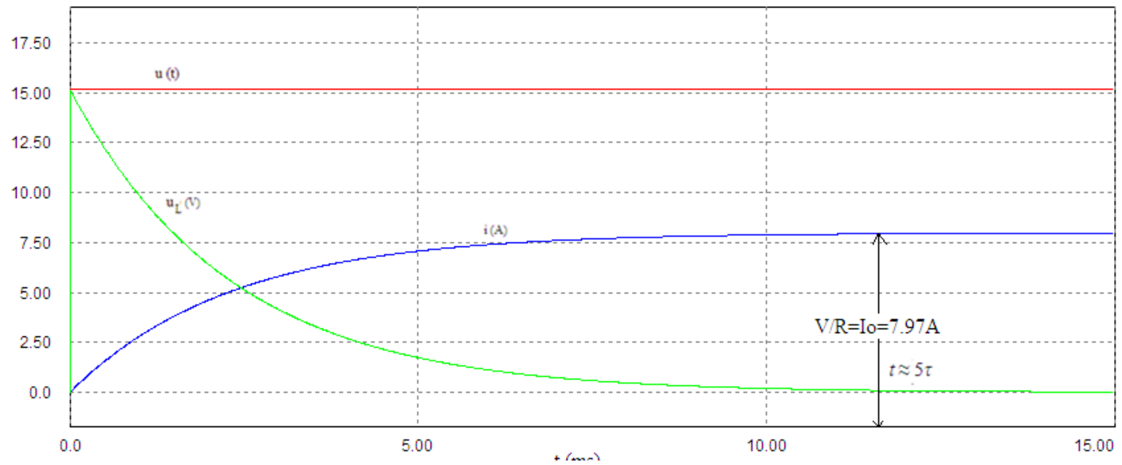


Figure 3. *Waveform of the input step voltage u, the inductor voltage u_L and the current i of the circuit from Figure 2 turned at t = 0 s with conditions i(0-) = i(0+) = 0 A.*

The amplitude on step voltage is V = 15.18 V. For this condition in [1], the solution of (1) for the current gives the expression:

$$i(t) = \frac{V}{R}(1 - e^{-\frac{t}{\tau}}) \tag{3}$$

and the inductor voltage is:

$$u_L(t) = Ve^{-\frac{t}{\tau}} \tag{4}$$

In (3), $V/R = I_{\max} = 7.99$ A is the maximum value of the current in circuit, and $\tau = L/R = 2.31$ ms is the time constant on RL-circuit. From Figure 3 it can be seen that for $t \approx 5\tau$, the current in circuit reaches its maximum values.

In Table 1 the values of the current and the inductor voltage for values on the time multiple on time constant τ are given.

Table 1 *Values on the current and the inductor voltage for values on the time multiple on time constant τ*

V (V)	t (ms)	i(t) (A)	U_L(V)	U_L/V (%)	i/I_{max} (%)
15.18	τ/2 = 1.16	3.13	9.23	60.80	39.17
15.18	τ = 2.31	5.05	5.58	36.76	63.20
15.18	2τ = 4.62	6.91	2.05	13.50	86.48
15.18	3τ = 6.93	7.59	0.75	4.94	95.23
15.18	4τ = 9.24	7.84	0.28	1.84	98.12
15.18	5τ = 11.55	7.94	0.10	0.66	99.25

From Figure 3 and Table 1 it can be seen that for t ≈ 5τ, the current in circuit reaches its maximum values and the inductor voltage is close to zero. It can also be seen that for t = τ, the current is close to 63 % of its maximum value and the inductor voltage is close to 37 % of step amplitude V.

3. *RL* Circuit Supplied by Square Voltage Pulse

For determining the phase angle between the voltage on *RL*-circuit and the current, in case of voltage supply with square waveform, the circuit shown on Figure 4 is used. This circuit is designed in the PowerSim simulation program.

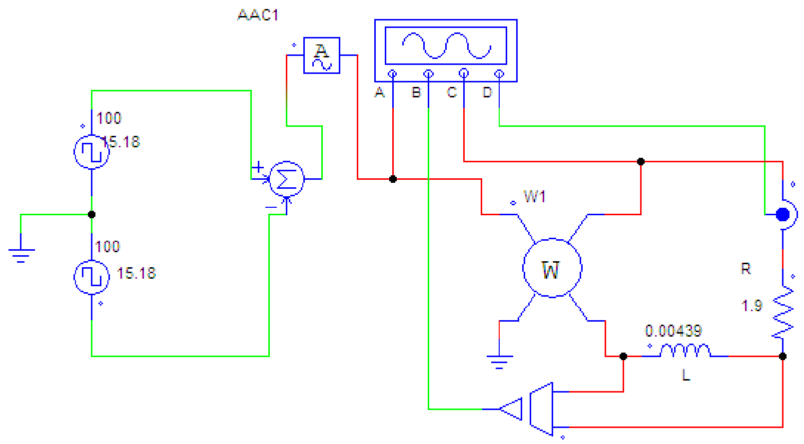


Figure 4. *RL-circuit in the PowerSim program used for determining the phase angle between the voltage and the current, in case of voltage supply with square waveform*

Figure 5 shows the waveform on the voltage supply $u(t)$, the inductor voltage $u_L(t)$ and the current $i(t)$ in the circuit from Figure 4 is in steady state. Simulations are performed using the square voltage pulse with switching frequency f_s = 50 Hz, amplitude V = 15.18 V and duty cycle D = 0.5.

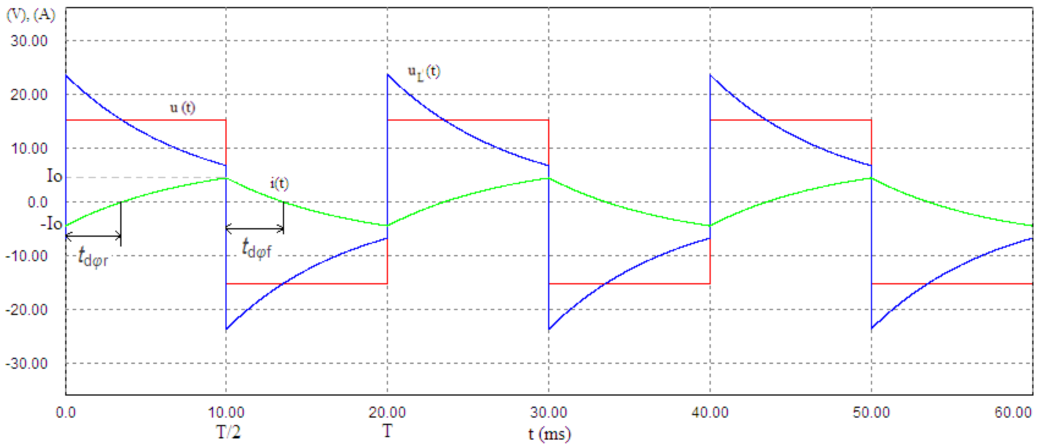


Figure 5. *Waveform of the suppling voltage u(t), the inductor voltage $u_L(t)$ and current i(t) in the circuit from Figure 4, in steady state for the square voltage pulse with switching frequency f_s = 50 Hz, amplitude V = 15.18 V and duty cycle D = 0.5*

In Figure 5, $t_{d\varphi r}$ and $t_{d\varphi f}$ are time equivalent of the phase angle of increasing and decreasing current, respectively. From Figure 5 it can be seen that the value of the current at t = 0 is i(0) = $-I_o$, and the current at t = T/2 is i(T/2) = I_o , i.e. the values of the current at these moments are the same in absolute value.

3.1 Calculation for the current at the beginning and end of the half cycle

In impulse electronics the following equation is used for the determination of the current values I_o and $-I_o$ at the moments t = 0 and t = T/2:

$$i(t) = i(\infty) - [i(\infty) - i(0)]e^{-\frac{t}{\tau}} \tag{5}$$

For defined conditions and values of the parameters above, $i(0) = -I_o$, $i(\infty) = V/R$. With this, from (5) is obtained:

$$i(t) = \frac{V}{R} - \left[\frac{V}{R} + I_o\right]e^{-\frac{t}{\tau}} \tag{6}$$

For $t = T/2$, $i(T/2) = I_o$ and (6) gets the form:

$$i(\frac{T}{2}) = \frac{V}{R} - \left[\frac{V}{R} + I_o\right]e^{-\frac{T}{2\tau}} = I_o \tag{7}$$

After several mathematical transformations from (7) the expression for the current I_o, i.e. is obtained:

$$I_o = \frac{V}{R}\frac{1-e^{-\frac{T}{2\tau}}}{1+e^{-\frac{T}{2\tau}}} \tag{8}$$

3.2 Calculation for time equivalent and phase angle

The inductor voltage is determined with (4). From Figure 5 it can be seen that in a steady state at the moment $t = 0$, the inductor voltage is:

$$u_L(t) = (V + |-I_0|R)e^{-\frac{0}{\tau}} = (V + I_0 R) \tag{9}$$

Also, from Figure 5 it can be seen that for $t = t_{d\varphi r}$, the current passes through zero and the inductor voltage is equal to the voltage V. With this and (9), the time equivalent t_φ on the phase angle φ_r when the current increases can be calculated. From (9) by $t = t_\varphi$ is obtained:

$$u_L(t) = (V + I_0 R)e^{-\frac{t_{d\varphi r}}{\tau}} = V \tag{10}$$

From (10) and (8) for the current I_0, as well as after some mathematical operations for the time equivalent, t_φ is obtained:

$$t_\varphi = \tau \ln\left[1 + \frac{1 - e^{-\frac{T}{2\tau}}}{1 + e^{-\frac{T}{2\tau}}}\right] \tag{11}$$

From (11) and according to [4, 5], for the relation between the time equivalent and the phase angle $t_\varphi/T = \varphi_\varphi/360°$ for phase angle we obtain:

$$\varphi_r = \frac{360}{T}\tau \ln\left[1 + \frac{1 - e^{-\frac{T}{2\tau}}}{1 + e^{-\frac{T}{2\tau}}}\right] \tag{12}$$

4. Results from Calculations, Simulations and Experiments

In this section, results from the calculations, the simulations and experimental measurements are represented. The calculations are made using the equations given above. The simulations are made in the PowerSim program [7] using the same values for RL and parameters defined above. The circuit used in the simulations is shown in Figure 4. In the experimental measurements for generating a square voltage, a prototype of a converter with 4 MOS transistors is developed. In addition, basic BST7960 bridge converter is used. The control electronics are realized with the AMega 328P microcontroller. The prototype is shown in Figure 6.

Figures 7 and 8 show the experimentally obtained waveforms of the voltage and the current in *RL*-circuit. Figure 7a shows the experimentally obtained waveforms of the voltage and the current in *RL*-circuit in case of a square voltage pulse supply and switching frequency 30 Hz. From this figure it can be seen that the time equivalent is $t_d = 2.2$ ms and the phase angle is $\varphi_{square} = t_d 360 \cdot f_s = 2.2 \cdot 10^{-3} \cdot 360 \cdot 30 = 23.76$ °. This value is the same as the value in Table 2. Figure 7b shows the waveforms of the voltage and current for the frequency of 50 Hz. Here the time equivalent is 1.8 ms and the phase angle is 32.40 °. Figure 8a shows the waveform of the voltage and current for the frequency of 60 Hz. The time equivalent is 1.7 ms and the phase angle is 36.72 °.

Figure 6. *Amega 328P microcontroller based circuit prototype and BST7960 bridge converter implemented for generating square voltage*

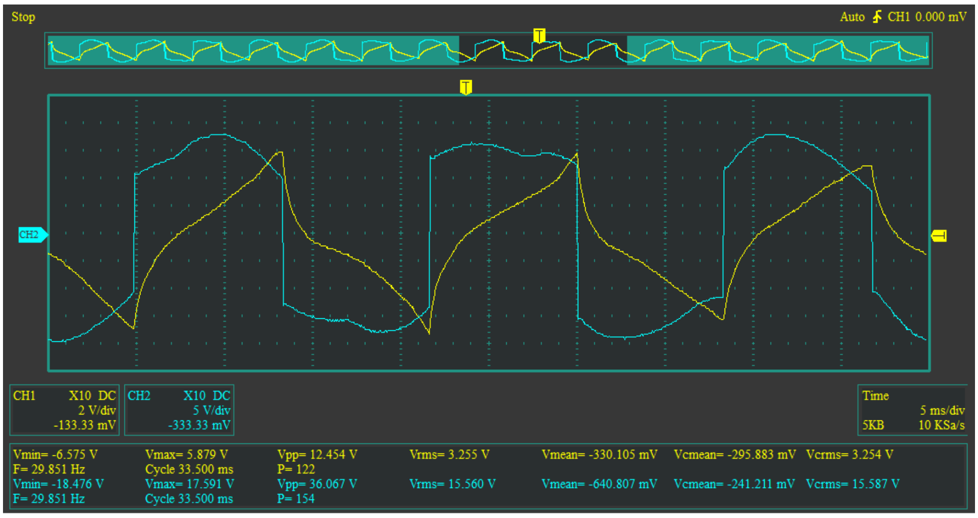
Figure 8b shows the waveforms of the voltage and current for the frequency of 90 Hz. The time equivalent is 1.5 ms and the phase angle is 48.60 °.

In Table 2a, the data for the phase angle for two cases is given, for sinus voltage supply calculated with (2), and square voltage pulse supply calculated with (12). Here the parameter is the ratio of the half-period and the time constant, i.e. $T/2\tau$.

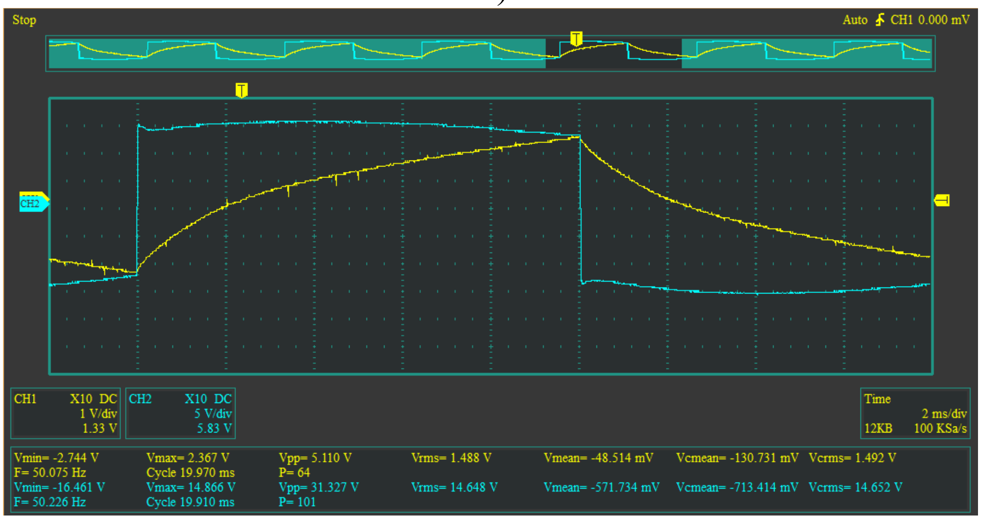
Table 2 *Data for phase angle for two cases, for sinus voltage supply and square voltage supply*

		calculations			simulations			experiment		
$T/2\tau$	f (Hz)	φ_{sinus} (°)	φ_{square} (°)	Imaxsq (A)	φ_{sinus} (°)	φ_{square} (°)	Imaxsq (A)	φ_{sinus} (°)	φ_{square} (°)	Imaxsq (A)
10.82	20	16.19	11.53	7.99	17.28	11.88	7.99		13.68	8.07
7.21	30	23.54	17.28	7.98	23.76	17.82	7.98		23.76	5.88
5.41	40	30.15	22.91	7.92	30.24	23.76	7.92		28.80	5.56
4.33	50	35.98	28.28	7.78	35.46	28.13	7.78	36	32.40	5.11
3.61	60	41.06	33.26	7.57	40.95	33.61	7.56		36.72	1.99
3.09	70	45.47	37.77	7.29	43.97	38.08	7.29		40.32	1.56
2.71	80	49.28	41.82	6.99	48.07	41.76	6.97		46.08	1.46
2.40	90	52.58	45.42	6.67	52.78	45.88	6.63		48.60	1.22
2.16	100	55.45	48.61	6.34	54.61	48.78	6.34		54.00	1.16

Due to the lack of sine voltage source with different frequency, in Table 2 the phase angle only for the frequency of 50 Hz is given.

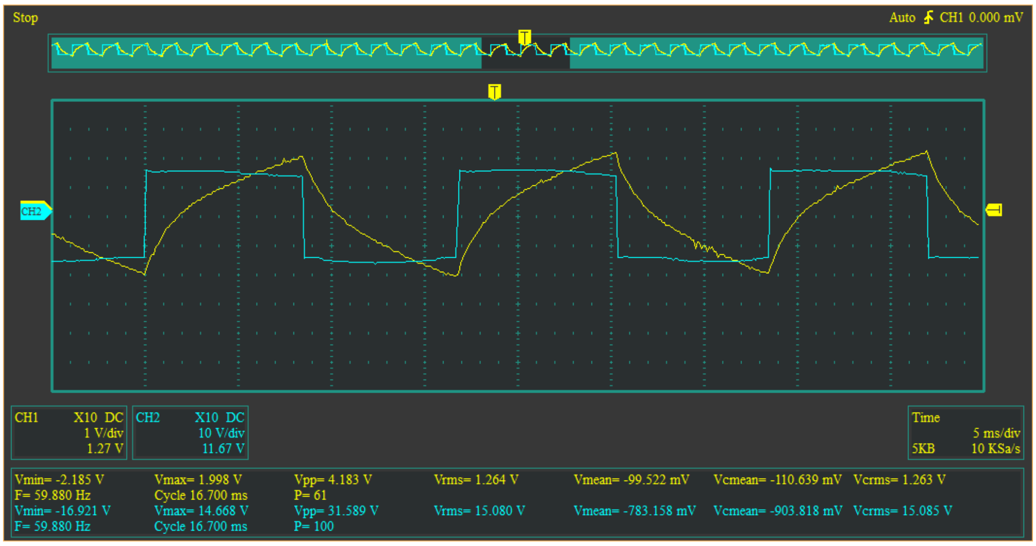


a.)

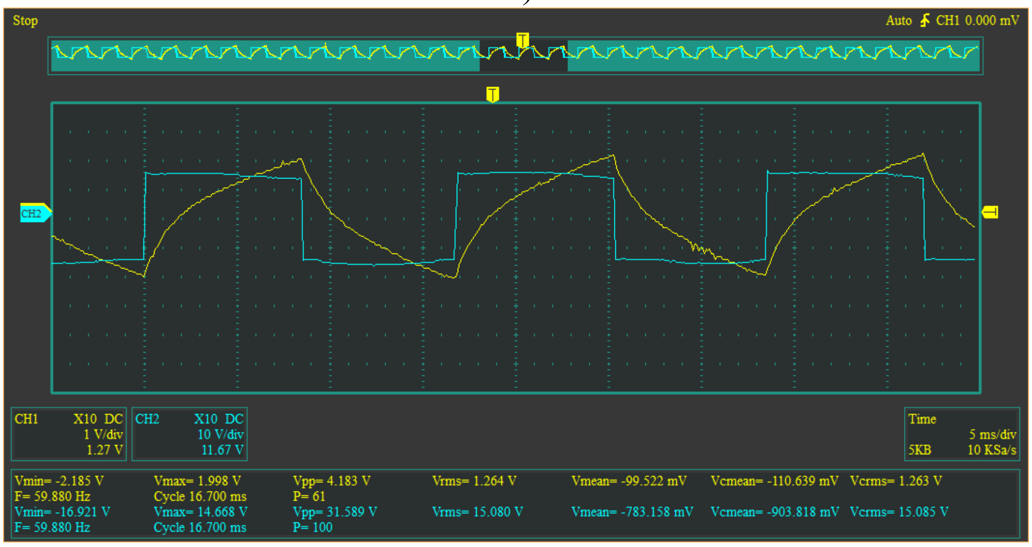


b.)

Figure 7 *Experimentally obtained waveforms on the voltage and the current in RL-circuit in case of square voltage pulse supply: a.) switching frequency 30 Hz, time equivalent* t_d *= 2.2 ms and phase angle* φ_{square} *= 23.76 °, b.) switching frequency 50 Hz, time equivalent 1.8 ms and phase angle 32.40 °*



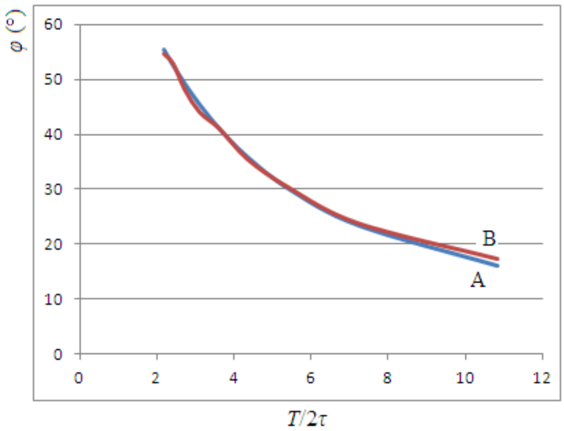
a.)

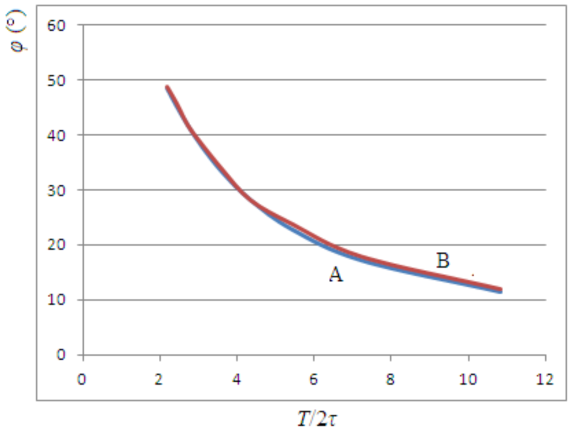


b)

Figure 8. *Experimentally obtained waveforms on the voltage and the current in RL-circuit in case the supply by the square voltage pulse: a.) switching frequency is 60 Hz, time equivalent is 1.7 ms and phase angle is 36.72 ° and b.) switching frequency is 90 Hz, time equivalent is 1.5 ms and phase angle is 48.60 °.*

Figure 9a shows the graph of the phase angle for the sinus supply, obtained with the calculation (curve A) by (2) and simulations (curve B) in the PowerSim program. Figure 9b shows the graph of the phase angle for the square supply, obtained with the calculation (curve A) by (12) and simulations (curve B) in the PowerSim program. From Figure 9a and 9b it can be concluded that the graphs of the phase angle obtained with the calculations and simulations in case of sinus and square supply are the same.





a.) b.)

Figure 9. *Graph of the phase angle: a.) the sinus supply obtained with the calculation (curve A) by (2) and simulations (curve B) in the PowerSim program and b.) the square supply obtained with the calculation (curve A) by (12) and simulations (curve B) in the PowerSim program*

Figure 10 shows the graph of the phase angle for the square supply obtained with the calculation (curve A) by (12), for the sinus supply obtained with the calculation (curve B) by (2) and the experimentally obtained graph (curve C) for the square supply.

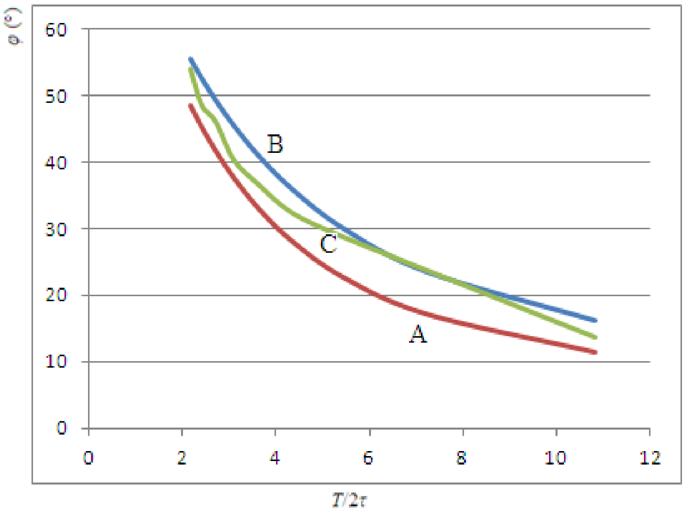


Figure 10. *The graphs on the phase angle in RL-circuit for the square supply obtained with the calculation (curve A) by (12), for the sinus supply obtained with the calculation (curve B) by (2) and the experimentally obtained graph (curve C) for the square supply*

From Figure 10 it can be concluded that the experimentally obtained graph (curve C) has values between the graphs obtained by calculating for sine (curve B) and squared voltages (curve A).

5. Conclusion

In this paper, an analysis of the serial *RL*- circuit supplied by a square voltage pulse is made. The equations for maximum current, time equivalent and phase angle in the serial RL- circuit are derived. These equations are experimentally verified and compared to the case when the circuit is supplied by sinus voltage. The results show that for the operation of the circuit on maximum current the half pulse period should be greater than 5τ.

References

[1] Williams W. B., (2006). Principles and Elements of Power Electronics, University of Strathclyde, Glasgow.
[2] Ericson R.W., & Maksimovic D., (2002). Fundamentals of Power Electronics, Kluwer.
[3] Mohan N., Undeland T. M., & Robbins W. P., (2003). Power Electronics: Converters, Applications, and Design. John Wiley & Sons.
[4] Kumar A.A., (2014). Pulse and Digital Circuits, PHI.
[5] Rao V.K., Rama S.K., Rao M.G., (2010). Pulse and Digital Circuits, Pearson.
[6] Stefanov G., Karadzinov Lj., Zlatanovska B., (2011). Mathematical Calculation of H–Bridge IGBT Power Converter, Comptes rendus de l´Academie bulgare des Sciences, Volume 64, Issue No6, pp.897–904.
[7] PowerSim Software, http://www.powersim.com/

Goce Stefanov
University of Goce Delcev Stip
Electrical Engineering Faculty
R. N. Macedonia
goce.stefanov@ugd.edu.mk

Vasilija Sarac
University of Goce Delcev Stip
Electrical Engineering Faculty
R. N. Macedonia
vasilija.sarac@ugd.edu.mk

Maja Kukuseva Paneva
University of Goce Delcev Stip
Electrical Engineering Faculty
R. N. Macedonia
maja.kukuseva@ugd.edu.mk