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CONTENT

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APPLICATION OF THE FOUR-COLOR THEOREM FOR COLORING A CITY MAP

NATASHA STOJKOVIKJ , MIRJANA KOCALEVA, CVETA MARTINOVSKA BANDE ,
ALEKSANDRA STOJANOVA AND BILJANA ZLATANOVSKA

Abstract. A graph can be defined as a mathematical representation of a network, or as a set of points connected by lines. All kinds of transportation networks (air, rail), a telecommunication system, the internet … can be defined as graphs. Graph theory is a branch of mathematics and it had its beginnings in math problems. However, today it has grown into a significant area of mathematical research, with applications almost everywhere (in chemistry, operations research, social sciences, and computer science). In this paper a basic definition of graph theory some definitions of a chromatic number of a graph and certain theorems related to graph coloring are given. Then the problem of map coloring with four colors is considered. One way to consider this problem is with the theory of graphs, because graphs are closely related to maps. We considered the map of the metropolitan area of Shtip and Skopje. For coloring, we use the software "Four color theorem – map solver".

Keywords – graph, graph theory, maps, map coloring, algorithms.

1. Introduction

Graphs are a very practical mathematical model and they are used in various areas of science and everyday life. With graphs, networks of places and paths that connect them, structural formulas of chemical compounds, people and relations between them and other problems can be represented.

We are considering the problem of map coloring with four colors. Francis Garth set this problem in 1852. The problem consisted of the possibility of coloring the world map with only four colors, but two neighboring countries must not be colored with the same color. The problem intrigued many scientists and in 1976, Aphel and Hacken, with computer help, proved that coloring the world map required four colors [1].

This problem can be considered with the theory of graphs, more precisely as a problem of coloring a graph. Graphs are very closely related to maps, as every map can be treated as a graph. A graph consists of curves called links, and of their intersections called nodes. The problem of coloring the graph is reduced to coloring the nodes of the graph, so each node is accompanied by one color, and the adjacent nodes are not of the same color. Such a graph is said to be properly painted. When converting a map to a graph, the regions are converted to nodes and set of links to represent neighboring relations between regions [2].

In this paper, we regarded the map of the metropolitan area of Shtip and Skopje. For coloring, we used the software "Four color theorem – map solver".

In the second section, a basic definition of graph theory is given. In the third section, the definition of a chromatic number of a graph and certain theorems that are related to graph coloring are given. In addition, the greedy algorithm for graph coloring is given. We will work our examples based on this algorithm. In the last sections, the software "Four color theorem – map solver" will be applied on the maps of the metropolitan area of Shtip and Skopje. Also, for approximatively obtaining chromatic numbers, we are going to use the theorems from the third section [14].

2. Basic definition of the graph theory

Definition 1 A graph *G (V, E)* consists of a finite non-empty set *V* whose elements are called nodes, vertices, or points or nodes and a finite set *E* of unordered pairs of distinct nodes, (i.e.) links of *G* [7], [8], [9].

A directed graph is a graph, i.e., a set of objects (called vertices or nodes) that are connected together, where all the edges are directed from one vertex to another. A directed graph is sometimes called a digraph or a directed network. In contrast, a graph where the edges are bidirectional is called an undirected graph.

Definition 2. The degree of a node of a graph is the number of links incident to the vertex. The degree of a vertex u is denoted deg (u) or degu.

Theorem 1. The sum of the degrees of the nodes in the graph *G (V, E)* is twice bigger than the number of links.

$$\sum_{\vartheta \in V} \deg(\vartheta) = 2n \tag{2.1}$$

where $n = |E|$.

Definition 3. A regular graph with a degree m is a graph where each node has the same degree m.

Definition 4. A complete graph with p nodes (denoted *Kp*) is a graph with n nodes in which each node is connected to each of the others (with one link between each pair of nodes). In a complete graph with p nodes, each node has a degree p. Here are the first five complete graphs (Fig.1.):

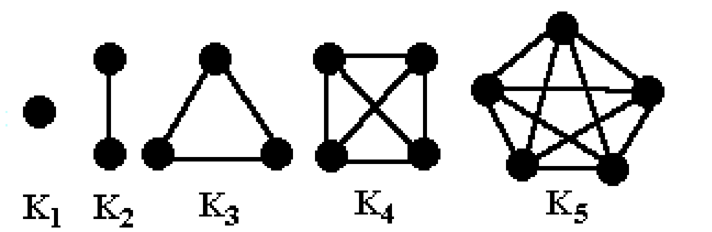

Figure 1. *Complete graphs*

According to Theorem 1, it follows that the number of links in the complete graph with p nodes is

$$q = \frac{p(p-1)}{2} \tag{2.2}$$

Definition 5. The graphs G_1 (V_1, E_1) and G_2 (V_2, E_2) are isomorphic if there is a bijective function *f* from V_1 to V_2 with the property that *u* and *v* are adjacent in G_1 if and only if *f (u)* and *f (V)* are adjacent in G_2.

Definition 6. A graph $G' = (V', E')$ is a subgraph of another graph $G = (V, E)$ if

1. $V' \subseteq V$, and
2. $E' \subseteq E \wedge ((u, v) \in E' \rightarrow u, v \in V')$.

- If u is a node from G, then with $(G - u)$ is denoted subgraph from G that is obtained with the removal of the node u and each link that is incident with u.
- If e is a link from G, then with $(G - e)$ is denoted subgraph from G that is obtained with the removal of the link e from G.
- If u and v are two nonadjacent different nodes from G, with $(G + (u, v))$ is denoted the graph that is obtained by adding on links between nodes u and v.

Definition 7. A graph $G = (V, E)$ is connected, if for any two vertices $u, v \in V$, there is a path whose endpoints are u and v.

A subgraph G' of G is called a connected component if G' is a maximal (non-extensible) connected subgraph of G. The number of connected components of a graph G is (often) denoted with k (G).

Definition 8. A walk from a node v to a node u is a sequence W of alternating nodes and links

$v_1, e_1, v_2, e_2, \ldots, e_{k-1}, v_k,$

such that $v_1 = v$, $v_k = u$ and $\forall i = 1, \ldots, k - 1$, e_i is incident with v_i and v_{i+1}; $k - 1$ is called the length of W. A walk without repeated edges is called a trail. A walk (trail) with $v_1 = v_k$ is called closed. If all nodes in W are distinct, W is called a path. If all nodes in a closed walk W are distinct and $k \geq 3$, then W is called a cycle.

Definition 9. A bipartite graph (or bigraph) is a graph whose nodes can be divided into two disjoint and independent sets U and U' such that for every link $(u,v) \in G$ is valid $u \in U$ and $v \in U'$.

Definition 10. Subset of nodes S from graph $G (V, E)$ for which is valid $E(S) \cap S = \varnothing$, where $E (u) = \{v | v \in V, (u, v)\}$ and $E(S) = \cup\{E(u)|u \in V\}$.

In other words, internally stable sets are sets of nodes in a graph, no two of which are adjacent. On the other hand, it is a set S of nodes such that for every two vertices in S, there is no link connecting the two. Equivalently, each link in the graph has at most one endpoint in S. The size of internally stable sets is the number of nodes they contain. A maximal independent set is either an independent set such that adding any other vertex to the set forces the set to contain an edge or the set of all vertices of the empty graph.

A maximum internally stable set is an internally stable set of largest possible size for a given graph G. This size is called the independence number of G is called stability number and is denoted with $\alpha (G)$.

Formally, let the family of all internally stable subset of odes on the graph $G (V, E)$ be denoted with Φ, then $\alpha (G) = max |S|, S \in \Phi$.

Definition 11. A planar graph is a graph that can be embedded in the plane, i.e. it can be drawn on the plane in such a way that its links intersect only at their endpoints. In other words, it can be drawn in such a way that no edges cross each other [7], [8], [9].

3. Chromatic number on graph

A nodes coloring (or simply coloring) of a graph G, is a labelling $f: V (G) \rightarrow \{1, 2 \ldots\}$. The labels are called colors, such that no two adjacent nodes get the same color and each node gets one color. A k - coloring of a graph G consists of k different colors and G is then called k - colorable. It follows from the definition that the k - coloring of a graph $G (V, E)$ partitions the set of nodes V into k independent sets $V_1, V_2, ..., Vk$ such that $V = V_1 \cup V_2 \cup ... \cup V_k$. The independent sets $V_1, V_2, ..., Vk$ are called the color classes and the function $f: V(G) \to \{1, 2, ..., k\}$ such that $f(v) = i$ where $v \in V_i$, $1 \leq i \leq k$, is called the color function [3], [4].

Definition 12. The smallest number of colors with which one graph can be colored is called the chromatic number of the graph, χ.

If the graph contains only isolated nodes, then $\chi = 1$, and if it is bipartisan, $\chi = 2$.

For a defining chromatic number, we are using the following theorems (proved in [2], [4]).

Theorem 2. If $\alpha (G)$ is stability number and $\chi(G)$ is chromatic number of graph G with n nodes, then $\alpha(G)\ \chi(G) \geq$ n.

Theorem 3. (König theorem) A nonempty graph G is bi -colorable (has chromatic number 2) if and only if G is bipartite.

Theorem 4. If G is a (simple) graph, then $\chi (G) \leq \Delta (G) +1$, where $\Delta (G)$ is the most degree of the nodes on the graph G.

Theorem 5. (Brooks' Theorem): If G is a connected (simple) graph and is not a complete graph or a cycle on an odd number of nodes, then it is valid $\chi (G) \leq \Delta (G)$, where $\Delta (G)$ is the highest degree of the nodes on the graph G.

By analyzing the properties of the different types of graphs, there are certain conclusions about chromatic numbers:

1. A graph is 1-chromatic if and only if it is totally disconnected (every node is isolated).
2. A graph that has at least one link is at least 2-chromatic (bi-chromatic).
3. Let G be a graph with n nodes, then $\chi (G) \leq n$.
4. If H is a subgraph of a graph G, then $\chi (H) \leq \chi (G)$.
5. A complete graph with n nodes is n-chromatic.
6. A cycle with length $n \geq 3$ is 2-chromatic if n is even and 3-chromatic if n is odd.

For map coloring, we will use the following reedy algorithm. The algorithm is as follows:

Algorithm 1

Input: graph
Output: colored graph

define Graph G;
sort the nodes in descending order by degrees.
assign color B_1 to the first node, and to all nodes that are not neighbors to the previous node, but are not neighbors to each other.
then repeat step 2, with color B_2 for the next uncolored node.
repeat step 3 **while** there are nodes without colors.
end

4. “Four color theorem – map solver” short software explanation

“Four color theorem – map solver” is a software which paints a map with 4-coloring algorithm. This software is accessed online and inside it, within an empty rectangular window, the map is painted with clicking on the button “Solve”. By clicking on the button “Reset”, the deletion is deleted and the window is ready for a new drawing on a map. By clicking on the button “Generate Image”, the map is stored below and it is ready to return to the window and color it. The Button “Toggle log” is not of interest to the particular problem (Fig.2.) [10], [11], [12], [13].

Figure 2. *The working window in which the maps are drawn and painted, as well as a display of the buttons that are in the function of the work window*

5. Coloring a map of Shtip with “four color theorem – map solver” software

Shtip contains seven cadastral municipalities: Shtip 1, Shtip 2, Shtip 3, Shtip 4, Shtip 5, Shtip 6 and Shtip Out City Area. To establish a relationship with the original definition of a 4-color graph coloring algorithm, the nodes of the graph will be the cadastral municipalities themselves, and the links between the nodes, in fact, will be the borders by which they are separated (Shtip 2 is a neighbor with Shtip 1, Shtip 3 and Shtip Out City Area; they are separated with the borders on the left, bottom border and right border sequentially). Here, the points (pixels) that are based on the algorithm itself are not taken as links between two cadastral municipalities (Fig.3. and Fig.4.).

Figure 3. *Displaying the metropolitan area of Shtip according to the official site of the Real Estate Cadaster Agency*

In Fig. 4., the areas of Shtip are shown by graph.

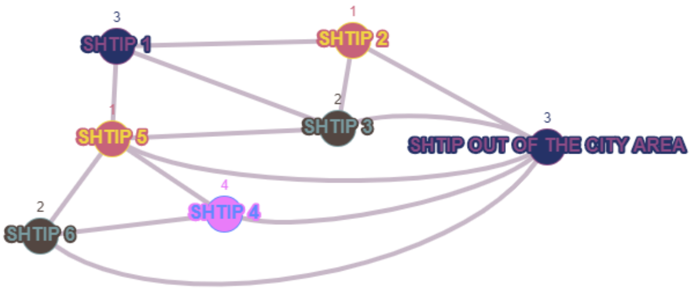

Figure 4. *Map of Shtip shown by graph*

Figure 5. *View of the metropolitan area of Shtip, painted according to the software "Four color theorem – map solver"*

Before, the used of algorithm for founding a chromatic number, we will use the theorem given in the previous section, with which the lower and upper bounds of chromatics number will be found. From Fig. 3, we can see that the graph is neither complete nor regular, from the corollary of König's theorem, we have that $3 \leq \chi(G)$. On the other side, we have that: deg(Shtip 1) = 3, deg(Shtip 2) = 3, deg(Shtip 3) = 4, deg(Shtip 4) = 3, deg(Shtip 4) = 3 , deg(Shtip 5) = 4, deg(Shtip 6) = 3 , deg (Shtip Out City Area) = 5. From the Brooks theorem it is valid that $\chi(G) \leq \Delta(G) = 5$, so $\chi(G) \in \{3, 4, 5\}$. For obtaining the chromatic number of graphs, we will use Algorithm 1 (this algorithm is given in the previous section) [5], [6].

Regarding the Color grading algorithm in 4 colors, to paint the metropolitan area of Shtip, 4 colors are needed. The steps of the algorithm in the concrete case are the following:

1. Determine the number of neighbors and the neighbors themselves in all Cadaster Municipalities, which are part of the metropolitan area of Shtip.
2. They are arranged in a descending order from the municipality with the most neighbors to the municipality with the least neighbors.
3. The first article of the subordinate list and all subsequent on the list that are not adjacent to it, shall be taken and painted with one color. Also, the algorithm takes into account that non-neighboring cadaster municipalities in relation to the first cadastral municipality are not neighborly with each other.
4. Step 3 is repeated for each next non-color member (the colored ones are colored and the same is completed), until the graph has been painted.

Now, the same four steps will be implemented in the concrete example (Fig.5.):

Step 1:

Shtip 1 has 3 neighbors (Shtip 2, Shtip 3 and Shtip 5);
Shtip 2 has 3 neighbors (Shtip 1, Shtip 3 and Shtip Out City Area);
Shtip 3 has 4 neighbors (Shtip 1, Shtip 2, Shtip 5 and Shtip Out City Area);
Shtip 4 has 3 neighbors (Shtip 5, Shtip 6 and Shtip Out City Area);
Shtip 5 has 4 neighbors (Shtip 1, Shtip 3, Shtip 4 and Shtip Out City Area);
Shtip 6 has 3 neighbors (Shtip 4, Shtip 5 and Shtip Out City Area);
Shtip Out of the City Area has 5 neighbors (Shtip 2, Shtip 3, Shtip 4, Shtip 5 and Shtip 6);

Step2:

Shtip Out of the City Area has 5 neighbors (Shtip 2, Shtip 3, Shtip 4, Shtip 5 and Shtip 6);
Shtip 3 has 4 neighbors (Shtip 1, Shtip 2, Shtip 5 and Shtip Out City Area);
Shtip 5 has 4 neighbors (Shtip 1, Shtip 3, Shtip 4 and Shtip Out City Area);
Shtip 1 has 3 neighbors (Shtip 2, Shtip 3 and Shtip 5);
Shtip 2 has 3 neighbors (Shtip 1, Shtip 3 and Shtip Out City Area);
Shtip 4 has 3 neighbors (Shtip 5, Shtip 6 and Shtip Out City Area);
Shtip 6 has 3 neighbors (Shtip 4, Shtip 5 and Shtip Out City Area);

Step3 and Step 4:

Shtip Out of the City Area is painted with one color (red) and with the same color all non-neighboring municipalities of it are painted. Because this municipality does not border with Shtip 1, Shtip 1 will be painted with the same color as a Shtip Out of the City Area (red). Next on the list is Shtip 3 or Shtip 5, because they have the same number of neighbors. We will take Shtip 3. We will paint Shtip 3 with a different color (green) and with this color we paint its non-neighbors. Non-neighbors of Shtip 3 are Shtip 4 and Shtip 6. Because these two municipalities are neighbors with each other, one of these two will be painted with the same color (green - that is Shtip 6). Now, Shtip 1 and Shtip Out of the City Area colored with red and Shtip 3 and Shtip 6 with green. Shtip 5 will be painted with another color (blue) and also every non-neighborly municipality. Because only Shtip 2 is a non-neighborly and unpainted cadastral municipality, Shtip 2 will be painted with the same color as Shtip 5 (blue). Only Shtip 6 is not painted and it will be painted with another color (yellow).

6. Coloring a map of Skopje with “four color theorem – map solver” software

It Skopje these Municipalities will be taken: Novo Selo, Lepenec, Bardovci, Vizbegovo, Butel, Staro Skopje, Gazi Baba, Chair, Gorce Petrov, Centar 1, Karposh, Zlocucani, Vlae, Gorce Petrov 6, Mirce Acev, Sisevo, Saraj, Gorce Petrov 3. To establish a relationship with the original definition of a 4-color graph-coloring algorithm, here the nodes of the graph will be the cadastral municipalities themselves, and the links between the nodes, in fact, will be the borders by which they are separated (Fig.6. and Fig.7.).

Figure. 6. *Displaying the metropolitan area of Skopje according to the official site of the Real Estate Cadastre Agency.*

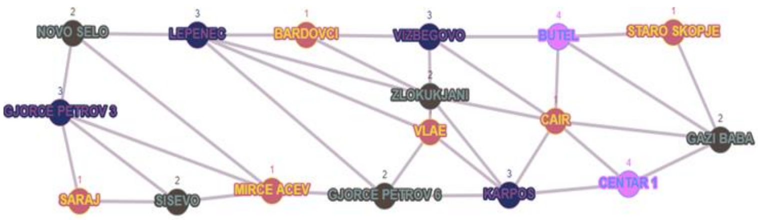

Figure 7. *Map of Skopje shown by graph*

Figure 8. *Map of the municipalities of Skopje, properly painted with 4 colors.*

Regarding the color-grading algorithm in 4 colors, to paint the metropolitan area of Skopje, 4 colors are needed [5], [6]. The steps of the algorithm in this concrete case are the following (Fig.8.):

Step1:

Novo Selo has 3 neighbors (Gorce p.3, Mirce A., Lepenec);
Lepenec has 5 neighbors (New V., Gorce p.6, Vlae, Zlocucani, Bardovci);
Bardovci has 3 neighbors (Lepenec, Zlocucani, Vizbegovo);
Vizbegovo has 4 neighbors (Bardovci, Zlocucani, Chair, Butel);
Butel has 4 neighbors (Vizbegovo, Chair, Gazi B., Staro S.);
Staro Skopje has 2 neighbors (Butel, Gazi B.);
Gazi Baba has 4 neighbors (Staro S., Butel, Chair, Centar 1);
Centar 1 has 3 neighbors (Karpos, Cair, Gazi B.,);
Chair has 6 neighbors (Vizbegovo, Butel, Gazi B., Centar 1, Karposh, Zlocucani);
Karposh has 5 neighbors (Centar 1, Chair, Zlocucani, Vlae, Gorce p.6);
Zlocucani has 6 neighbors (Karposh, Chair, Vizbegovo, Bardovci, Lepenec, Vlae);
Vlae has 4 neighbors (Gorce p.6, Karposh, Zlocucani, Lepenec);
Gorce Petrov 6 has 3 neighbors (Vlae, Lepenec, Mirce A.);
Mirce Acev has 4 neighbors (Gorce p.6, New V., Gorcep.3, Sisevo);
Sisevo has 3 neighbors (Mirce A., Gorce p.3, Saraj);
Saraj has 2 neighbors (Sisevo, Gorce p.3);
Gorce Petrov 3 has 4 neighbors (Saraj, Sisevo, Mirce A., New V.);

Step 2:

Chair has 6 neighbors (Vizbegovo, Butel, Gazi B., Centar 1, Karposh, Zlocucani);
Zlocucani has 6 neighbors (Karposh, Chair, Vizbegovo, Bardovci, Lepenec, Vlae);
Lepenec has 5 neighbors (New V., Gorce p.6, Vlae, Zlicucani, Bardovci);
Karposh has 5 neighbors (Centar 1, Chair, Zlocucani, Vlae, Gorce p.6);
Vizbegovo has 4 neighbors (Bardovci, Zlocucani, Chair, Butel);
Butel has 4 neighbors (Vizbegovo, Cair, Gazi B., Staro S.);
Gazi Baba has 4 neighbors (Staro S., Butel, Cair, Centar 1);
Vlae has 4 neighbors (Gorce p.6, Karposh, Zlocucani, Lepenec);
Mirce Acev has 4 neighbors (Gorce p.6, New V., Gorcep.3, Sisevo);

Gorce Petrov 3 has 4 neighbors (Saraj, Sisevo, Mirce A., New V.);
Sisevo has 3 neighbors (Mirce A., Gorce p.3, Saraj);
Bardovci has 3 neighbors (Lepenec, Zlocucani, Vizbegovo);
Centar 1 has 3 neighbors (Karposh, Chair, Gazi B.,);
Gorce Petrov 6 has 3 neighbors (Vlae, Lepenec, Mirce A.);
New Village has 3 neighbors (Gorce p.3, Mirce A., Lepenec);
Staro Skopje has 2 neighbors (Butel, Gazi B.);
Saraj has 2 neighbors (Sisevo, Gorce p.3);

Step 3 and Step 4:

Chair and Zlocucani have the same number of neighbors, so we choose one of them. Chair is painted with one color (red) and all of its non-neighboring municipalities (Staro S., Lepenec, Gorce P.3) will be painted with the same color. After that, we paint Zlocucani with another color (blue) and all of its non-neighboring municipalities (Centar 1, Butel, New V., Sisevo, Gorce p.6) will be painted with the same color. Because Lepenec is already painted, we take the next municipality Karposh and we paint it with a third color (green) and all of its non-neighboring municipalities (Gazi B., Vizbegovo, Saraj, Mirce A.) will be painted with the same color. Because Vizbegovo, Butel and Gazi Baba are already colored, we take the next municipality from the list and it is Vlae, Vlae is painted with a fourth color (yellow) and there is only one non-neighboring, no painted municipality and it is Bardovci, so Bardovci will be painted with the same color as Vlae.

In the end, four colors are needed for coloring the map of Skopje.

Like the example of the municipality of Shtip, for this case we can also find lower and upper bounds of chromatic number of the graph (Fig. 7). This graph is neither complete nor regular, so from corollary 1, we have that $3 \leq \chi(G)$. On the other hand, we have that $\Delta(G) = \max\{2, 3, 4, 5, 6\}=6$, $\chi(G) \leq \Delta(G) = 6$, and $\chi(G) \in \{3, 4, 5, 6\}$. Using Algorithm 1, we will find that the chromatic number of this graph is 4.

7. Conclusion

The real-life applications of the four color theorem are activity scheduling, identifying spatial regional poles and turnpikes of economic growth, security camera placement optimization in a large building with many corners such that you minimize overlap, construction of a wildlife reserve … In the paper we present the "Four color theorem – map solver" software. We considered a greedy algorithm for graph coloring, because our graph coloring software is based on it. What we did was color the maps of Shtip and Skopje with only four colors. Our next goal is to apply map coloring to the whole Republic of North Macedonia and the Republic of North Macedonia with neighbors.

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