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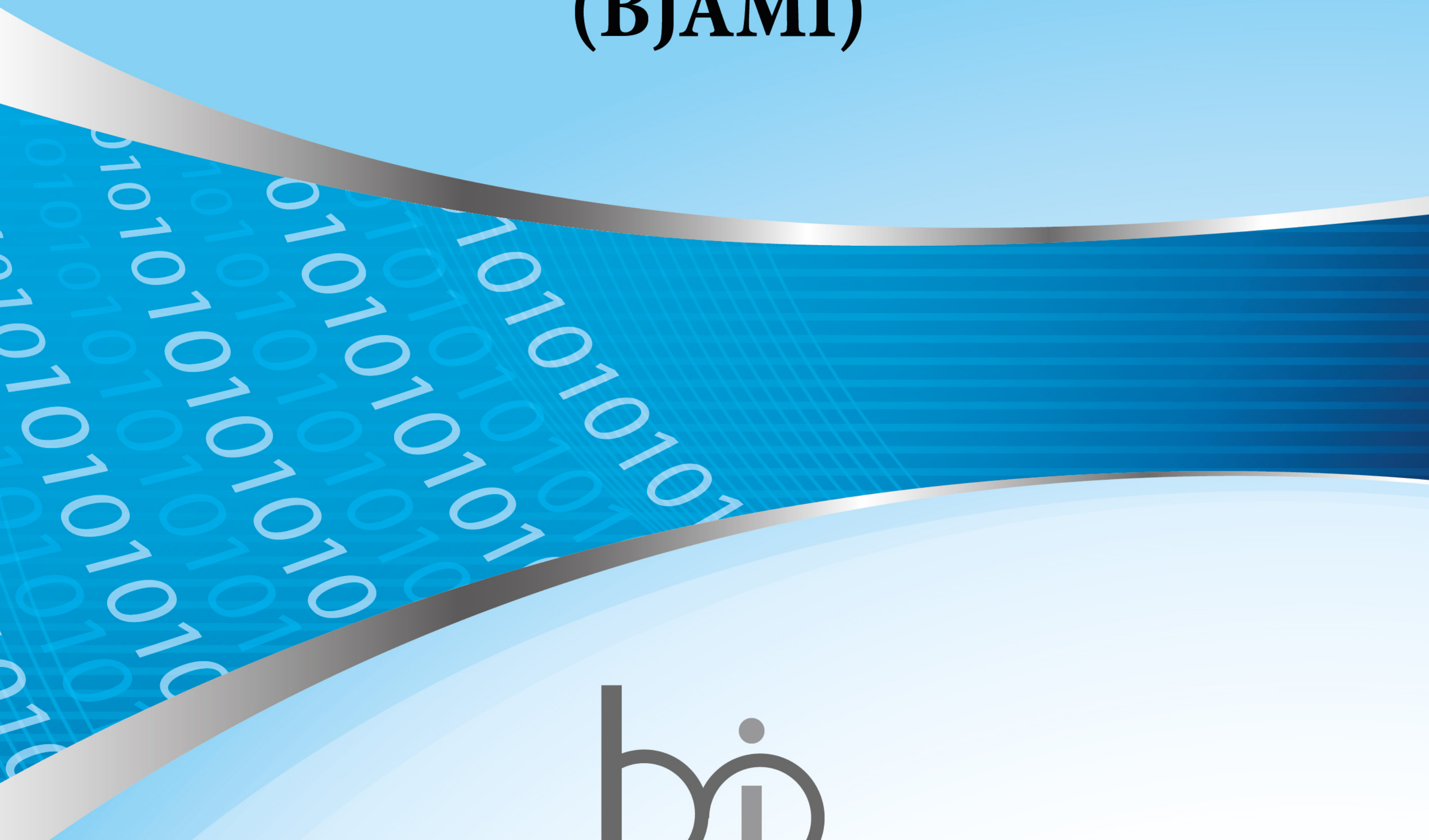

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# CONTENT

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# DECISION MAKING FOR THE OPTIMUM PROFIT BY USING THE PRINCIPLE OF GAME THEORY

**Shakoor Muhammad[1], Nekmat Ullah[1], Muhammad Tahir[2], [3]Noor Zeb Khan**

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**Abstract:** Game theory is a mathematical study of planning and strategy and interaction among the competing objects. The procreation of game techniques are the best methods that have been used to obtain various feasible problems. For example, politicians want to nominate proper candidates in order to win, and businesspersons organize their businesses in proper locations for maximum income. This paper applies the principle of game theory to produce rules for most favorable settings of three different varieties launching in three different localities in order to maximize profit.

**Keywords**: Pay-off, matrix games, decision making, continuous and discrete, maximizer, minimizer.

## Introduction

The mathematical game theory was basically presented by John von Neumann along with Oskar Morgenstern in 1944. The participants in a game are called players. These players are trying to exploit their pay-off, and formulate their plans that are known as "*Strategies*". Each player has his/her own strategies regardless of the strategies of the other player. For the result of the game, the net outcome of all the strategies selected by the participants in a game may result in a win or loss or a draw to a participant.

Game theory is related to the distinct optimization box connecting two or more contestants to dashing passions. Game theory problems may be discrete or continuous. Discrete game problems are generally represented in matrix forms. These matrices may have order (n x m) or (m x n) see [6], [1].

Table 1: Typical Game Matrix

| | | Player Q Chooses | | | | |
|---|---|---|---|---|---|---|
| | | $Q_1$ | $Q_2$ | $Q_3$ | … | $Q_n$ |
| Player P Chooses | $P_1$ | t11 | t12 | t13 | … | t1n |
| | $P_2$ | t21 | t22 | t23 | … | t2n |
| | $P_3$ | t31 | t32 | t33 | … | t3n |
| | … | … | … | … | … | … |
| | $P_m$ | tm1 | tm2 | tm3 | … | tmn |

In a continuous game, the choices of P and Q are continuous instead of discrete [2]. Therefore, there must be a continuous pay-off function H (P, Q) instead of a pay-off matrix $H_{ij}$ as illustrated in discrete games.

We look for a pair of choices

$$H(P^o, Q) \leq G(P^o, Q^o) \leq H(P, Q^o) \text{ for all } P, Q \qquad (1)$$

The necessary and sufficient conditions for P°,Q° are

$$\partial H/\partial P = 0, \ \partial H/\partial Q = 0 \qquad (2)$$

If condition (2) does not satisfy, then we apply the following condition (3)

$$\partial^2 H/\partial P^2 \geq 0, \ \partial^2 H/\partial Q^2 \leq 0 \qquad (3)$$

When any P°, Q° fulfill the sufficient conditions, it is said to be the game-theoretic saddle point [7], [5].

## MINMAX (MAXMIN) Principle

In game theory**, minmax** is a decision making rule used to minimize the worst-case potential loss. In each competition, players are interested to optimize their self-interest. As each game has its own conflicts, and moreover the lack of information regarding the specific strategies selected by the opponent player(s), optimality for the outcome of the game has to be based on conservative principles [1], [7].
Due to the huge importance of maxmin (minmax) rule which is used for the optimal strategies of the opponents in this paper, we define this rule as follows.
Consider a two-player game as illustrated in Table 2:

Table 2: (3 × 3) Discrete Game Matrix

| | | Player Z | | |
|---|---|---|---|---|
| | | $Z_1$ | $Z_2$ | $Z_3$ |
| Player X | $X_1$ | $E_{12}=6$ | $E_{12}=1$ | $E_{13}=7$ |
| | $X_2$ | $E_{21}=4$ | $E_{22}=3$ | $E_{23}=5$ |
| | $X_3$ | $E_{31}=5$ | $E_{32}=1$ | $E_{33}= -2$ |

If Player X (the maximizer), selects his first plan ($X_1$) he could get 6, 1, or 7 depending on the strategy selected by player Z.
Thus, player X is guaranteed to gain at least 1 = min (6, 1, 7) if he selects strategy $X_1$ regardless of the strategy preferred by player Z.

In the same way, X is sure to gain as a minimum

3 = min (4, 3, 5) for $X_2$ strategy selection

-2 = min (5, 1, -2) for $X_3$ strategy selection

Consequently, for player X to maximize his gain regardless the strategies of Z, he has to maximize his minimum gain i.e.

3 = max (1, 3, -2)

Similarly, if player Z chooses strategy $Z_1$ he loses 6, 4 or 5 depending on the strategy selected by player X.

As a result, player Z loses no more than

6 = max (6, 4, 5) for $Z_1$ strategy

3 = max (1, 3, 1) for $Z_2$ strategy

7 = max (7,5,-2) for $Z_3$ strategy

Thus for player Z to reduce his loss, regardless of player X, he has to minimize his utmost losses by selecting min (6, 3, 7) = 3

It is the minmax value of the game for player Z.

Hence:

$$\underset{Z\,X}{\text{maxmin}}\; H_{ij} = 3 = \underset{X\,Z}{\text{minmax}}\; H_{ij}$$

(X plays first) (Z plays first)

## Methodology

Game theory is to be used for solving problems in a condition of variance and contention involving two or more challengers. The mode at this time is the thought of opposes in terms of varieties in a particular feasibility situation.

We present three different varieties for sale $V_1$, $V_2$ and $V_3$ having different quantities in three different localities: locality 1, locality 2 and locality 3 of a city, respectively [4, 6].

If we agree to a feasibility investigation regarding the situation that 45% of the people of the city close to locality 1, 35% of the population of the city lives near locality 2, and the remaining 20% of the population of the city lives near locality 3.

In locality 1, approximately 30% of the people like variety 1, 50% of the people like variety 2 and 20% like variety 3.

In locality 2, approximately 80% of the people like variety 1, 15% of the people like variety 2 and 5% of the people like variety 3.

In locality 3, approximately 20% of the people like variety 1, 20% of the people like variety 2 and 60% of the people like variety 3.

Out of the three different localities $L_1$, $L_2$, and $L_3$, we will compare two of them for the three varieties $V_1$, $V_2$, and $V_3$ by rules of matrices. Firstly, we compare $L_1$ and $L_2$, then $L_2$ and $L_3$, and then $L_3$ and $L_1$.

Here we use the principle of the game theory in order to find the best possible outcomes for the three localities by assuming that the varieties contain no other competitors in the metropolitan. The pay-off matrix to the game is given in the following table as:

Table 3 (3 × 3 matrix) Game illustration

| | L2 | | | |
|---|---|---|---|---|
| | | $b_{1=V1}$ | $b_{2=V2}$ | $b_{3=V3}$ |
| L1 | $a_{1=V1}$ | $F_{11}$ | $F_{12}$ | $F_{13}$ |
| | $a_{2=V2}$ | $F_{21}$ | $F_{22}$ | $F_{23}$ |
| | $a_{3=V3}$ | $F_{31}$ | $F_{32}$ | $F_{33}$ |

Here we use the notations $L_1$, $L_2$ and $L_3$ for locality 1, locality 2 and locality 3 respectively. Similarly, we use $V_1$ for variety 1, $V_2$ for variety 2 and $V_3$ for variety 3 respectively.

The function $F_{ij}$ represents the percentage business in $L_1$ if it is located for locality i and $L_2$ for locality j. Similar reasoning applies for $L_2$ and $L_3$, and for $L_3$ and $L_1$ respectively.

The elements $F_{11}$, $F_{22}$ and $F_{33}$ correspond to the cases where $V_1$, $V_2$ and $V_3$ are located in the same locality. In the following decision making competition, we will have to check which variety has more profit in a particular locality.

### I. Competition for profit between $L_1$ & $L_2$

If the same variety $V_1$ is located in $L_1$ and $L_2$, then $V_1$ gets 30% of the business of $L_1$ (45% of the population) and 80% of $L_2$ (35% of the population) which gives a total of:

$$G_{11}=30(0.45) + 80(0.35) = 41.5\%$$

Now $V_1$ gets 30% of $L_1$ (45% population) and $V_2$ gets 15% of $L_2$ (35% population)

$$G_{12}=30(0.45) + 15(0.35) = 18.75\%$$

Now $V_1$ gets 30% of $L_1$(45% population) and $V_3$ gets 5% of $L_2$ (35% population)

$$G_{13}=30(0.45) + 5(0.35) = 15.25\%$$

Now $V_2$ gets 50% of $L_1$ (45% population) and $V_1$ gets 80% of $L_2$ (35% population)

$$G_{21}=50(0.45) + 80(0.35) = 50.5\%$$

Now $V_2$ gets 50% of $L_1$ (45% population) and $V_2$ gets 15% of $L_2$ (35% population)

$$G_{22}=50(0.45) + 15(0.35) = 27.75\%$$

Now $V_2$ gets 50% of $L_1$ (45% population) and $V_3$ gets 5% of $L_2$(35% population)

$$G_{23}=50(0.45) + 5(0.35) = 24.25\%$$

Now $V_3$ gets 20% of $L_1$ (45% population) and $V_1$ gets 80% of $L_2$(35% population)

$$G_{31}=20(0.45) + 80(0.35) = 37\%$$

Now $V_3$ gets 20% of $L_1$ (45% population) and $V_2$ gets 15% of $L_2$(35% population)

$$G_{32}=20(0.45) + 15(0.35) = 14.25\%$$

Now $V_3$ gets 20% of $L_1$ (45% population) and $V_3$ gets 5% of $L_2$(35% population)

$$G_{33}=20(0.45) + 5(0.35) = 10.75\%$$

Now $G_{ij}$ can be written in matrix form and we will use the minmax and maxmin rule in order to get the desired results.

Table 4 (3 × 3 Matrices) Game representation

| | | | | |
|---|---|---|---|---|
| | | $L_2$ | | |
| | | $b_1=V_1$ | $b_2=V_2$ | $b_3=V_3$ |
| $L_1$ | $a_1=V_1$ | $G_{11}=41.5$ | $G_{12}=18.75$ | $G_{13}=15.25$ |
| | $a_2=V_2$ | $G_{21}=50.5$ | $G_{22}=27.75$ | $G_{23}=24.25$ |
| | $a_3=V_3$ | $G_{31}=37$ | $G_{32}=14.25$ | $G_{33}=10.75$ |

min (41.5, 18.75, 15.25) =15.25
min (50.5, 27.75, 24.25) =24.25
min (37, 14.25, 10.75) = 10.75
max (15.25, 24.25 10.75) = 24.25
By the said rules, we get 24.25% pay-off for $V_2$ in $L_1$ and for $V_3$ in $L_2$, which gives a saddle point of 24.25%.

**II. Competition for profit between $L_2$ & $L_3$**

If $V_1$ is located in $L_2$ and $L_3$, where $V_1$ attains 80% of the business of $L_2$ (35% of the population) and $V_1$ gets 20% of $L_3$ (20% of the population) which gives a total pay-off:

$$H_{11}=80(0.35) + 20(0.20) = 32\%$$

If $V_1$ gets 80% of $L_2$ (35% population) and $V_2$ gets 20% of $L_3$ (20% population),

$$H_{12}=80(0.35) + 20(0.20) = 32\%$$

If $V_1$ gets 80% of $L_2$ (35% population) and $V_3$ gets 60% of $L_3$ (20% population),

$$H_{13}=80(0.35) + 60(0.20) = 40\%$$

If $V_2$ gets 15% of $L_2$ (35% population) and $V_1$ gets 20% of $L_3$ (20% population),

$$H_{21}=15(0.35) + 20(0.20) = 9.25\%$$

If $V_2$ gets 15% of $L_2$ (35% population) and $V_2$ gets 20% of $L_3$ (20% population),

$$H_{22}=15(0.35) + 20(0.20) = 9.25\%$$

If $V_2$ gets 15% of $L_2$ (35% population) and $V_3$ gets 60% of $L_3$ (20% population),

$$H_{23}=15(0.35) + 60(0.20) = 17.25\%$$

If $V_3$ gets 5% of $L_2$ (35% population) and $V_1$ gets 20% of $L_3$ (20% population),

$$H_{31}=5(0.35) + 20(0.20) = 5.75\%$$

If $V_3$ gets 5% of $L_2$ (35% population) and $V_2$ gets 20% of $L_3$ (20% population),

$$H_{32}=5(0.35) + 20(0.20) = 5.75\%$$

If $V_3$ gets 5% of $L_2$ (35% population) and $V_3$ gets 60% of $L_2$ (20% population),

$$H_{33}=5(0.35) + 60(0.20) = 13.75\%$$

Now $H_{ij}$ can be written in matrix form and use the minmax and maxmin rule in order to get the desired results.

Table 5 (3 × 3 Matrices) Game representation

| | $L_3$ | | | |
|---|---|---|---|---|
| | | $c_1=V_1$ | $c_2=V_2$ | $c_3=V_3$ |
| $L_2$ | $b_1=V_1$ | $H_{11}=32$ | $H_{12}=32$ | $H_{13}=40$ |
| | $b_2=V_2$ | $H_{21}=9.25$ | $H_{22}=9.25$ | $H_{23}=17.25$ |
| | $b_3=V_3$ | $H_{31}=5.75$ | $H_{32}=5.75$ | $H_{33}=13.75$ |

min (32, 32, 40) = 32
min (9.25, 9.25, 17.25) =9.25
min (5.75, 5.75, 13.75) =5.75
max (32, 9.25, 5.75) =32
Saddle point of $L_2$ and $L_3$ is 32
By minmax and maxmin rules, we get 32% pay-off for $V_1$ in $L_2$ and for $V_2$ in $L_3$.

**III. Competition for profit between $L_3$ & $L_1$**

If $V_1$ is located in $L_3$ and $L_1$, where $V_1$ gets 20% of the business of $L_3$ (20% of the population) and $V_1$ gets 50% of $L_1$ (45% of the population) which gives a total pay-off:

$I_{11}=20(0.20) + 30(0.45) = 17.5\%$

If $V_1$ gets 20% of $L_3$ (20% population) and $V_2$ gets 50% of $L_1$ (45% population), then

$I_{12}=20(0.20) + 50(0.45) = 26.5\%$

If $V_1$ gets 20% of $L_3$ (20% population) and $V_3$ gets 20% of $L_1$(45% population), then

$I_{13}=20(0.20) + 20(0.45) = 13\%$

If $V_2$ gets 20% of $L_3$ (20% population) and $V_1$ gets 30% of $L_1$ (45% population), then

$I_{21}=20(0.20) + 30(0.45) = 17.5\%$

If $V_2$ gets 20% of $L_3$ (20% population) and $V_2$ gets 50% of $L_1$ (45% population), then

$I_{22}=20(0.20) + 50(0.45) = 26.5\%$

If $V_2$ gets 20% of $L_3$ (20% population) and $V_3$ gets 20% of $L_1$ (45% population), then

$I_{23}=20(0.20) + 20(0.45) = 13\%$

If $V_3$ gets 60% of $L_3$ (20% population) and $V_1$ gets 30% of $L_1$ (45% population), then

$I_{31}=60(0.20) + 30(0.45) = 25.5\%$

If $V_3$ gets 60% of $L_3$ (20% population) and $V_2$ gets 50% of $L_1$ (45% population), then

$I_{32}=60(0.20) + 50(0.45) = 37.5\%$

If V3 gets 60% of $L_3$ (20% population) and $V_3$ gets 20% of $L_1$ (45% population), then

$I_{33}=60(0.20) + 20(0.45) = 21\%$

Now $I_{ij}$ can be written in matrix form and use the minmax and maxmin rule in order to get the desired results.

Table 6 (3 × 3 Matrices) Game representation

| | $L_1$ | | | |
|---|---|---|---|---|
| | | $a_1=V_1$ | $a_2=V_2$ | $a_3=V_3$ |
| $L_3$ | $c_1=V_1$ | $I_{11}=17.5$ | $I_{12}=26.5$ | $I_{13}=13$ |
| | $c_2=V_2$ | $I_{21}=17.5$ | $I_{22}=26.5$ | $I_{23}=13$ |
| | $c_3=V_3$ | $I_{31}=25.5$ | $I_{32}=37.5$ | $I_{33}=21$ |

min (17.5, 26.5, 13) =13
min (17.5, 26.5, 13) =13
min (25.5, 37.5, 21) =21
max (13, 13, 21) = 21
Saddle Point of $L_3$ & $L_1$ is 21.
By minmax and maxmin rules, we get 21% payoff for V3 in $L_3$ and for $V_3$ in $L_1$

## Conclusion:

From the above analysis, we conclude that the better optimal strategy for variety $V_3$ is to locate its branch in locality $L_3$ (24.5%) than in $L_1$ (21%) where it gains 3.5% more profit. The same strategy for $V_2$ is to locate its branch in $L_2$ (32%) rather than in $L_3$ (24.5%), where it gets 8.5% more business.

If we launch both varieties $V_2$ and $V_3$ in $L_3$, then variety $V_2$ (32%) will get 8.5% more business than variety $V_3$ (24.5%). Similarly, if we launch both varieties ($V_1$ & $V_3$) in $L_{1,}$ then variety $V_2$ (32%) will get 11% more than variety $V_1$ (21%).

From the last paragraph, we conclude that variety $V_1$ takes place of the business of variety $V_3$. Moreover, it will get 11% additional business as well (variety $V_3$ will have less business).

Thus, the optimal strategy is to launch variety $V_3$ in localities $L_1$ and $L_3$, and variety $V_2$ in locality $L_2$.

## References:

1. Rao S. S., *Optimization Theory and Application*, Second Edition, Willey Eastern Limited, 1984.
2. Shehu D. M, *Optimal Analysis and Application of Discrete Games in Decision Making Environment*, In proceedings of SSCE Conference, Minna, Nigeria, 2006
3. Jack M., *Introduction to Optimal Control*, MIR Publishers, Moscow, 1977.
4. Raifla L. *Games and Decisions*, Princeton University Press, N.J., 1957.
5. Straffin Philips D., *Game Theory and Strategy*, MIT Press Cambridge, 1993.
6. Emilio O. R, *Modern Optimal Control*, Books/Cole Publishing Company, California,1989
7. Hanson Y., *Applied Optimal Control*, Wiley-Inter Science, New York, 1969.