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CONTENT

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EIGENVALUES AND EIGENVECTORS OF A BUILDING MODEL AS A ONE-DIMENSIONAL ELEMENT

MIRJANA KOCALEVA AND VLADO GICEV

Abstract. To estimate vulnerability of existing and newly designed buildings, we need to know their natural periods. Knowing the location of the building and its fundamental (longest) natural period, based on spectral seismic maps, we can estimate the level of its damage due to a seismic event. Moreover, using the impulse response method, we can separate the fixed-base frequency, f_1 from the system frequency, fs. With this research, we will try to define which parameters are needed for creating empirical equations for the estimation of fundamental periods of the existing and the newly designed buildings in North Macedonia. We expect that the newly proposed equations, compared with the existing ones, will give a better estimate of fundamental system periods Ts and fundamental fixed-base periods T_1 for buildings in our country. In correlation with future seismic microzoning of big cities with the Uniform Hazard Spectrum (UHS) method, this research will be an excellent base for estimation of vulnerability and economic loss due to earthquakes.

1. Introduction

Fundamental periods are a major factor in all standards for building seismic resistant structures and it is therefore important to determine them as accurately as possible. The main purpose of our research is to propose an empirical equation (formula) for obtaining the natural frequencies, i.e. the fundamental periods of the objects. In our country, the old Yugoslavian standards of 1981 are still used, where the fundamental periods are determined by empirical formulae that consider only the height, respectively the floor of the buildings.

Because of the importance of the problem, plenty of research has been done in the past. The authors propose empirical equations or recommendations for obtaining natural frequencies of structures. In the paper [1] the vulnerability of the existing buildings in India is presented. Earthquake vulnerability assessment was carried out in two steps – (i) a preliminary evaluation and (ii) a detailed evaluation. For preliminary evaluation, they used the method Rapid Visual Screening (RVS), which visually examines the building to identify features that affect the seismic performance of the building, such as the building type, seismicity, soil conditions, and irregularities. This method can be performed for 30 minutes without going inside the building and is based on a survey. They also calculate the performance score (PS) for each building based on numerical values on the RVS form using the formula $PS = (BS) - \sum[(VSM) \cdot (VS)]$ where VSM is vulnerability score modifiers and VS is vulnerability score that is multiplied with VSM to obtain the actual modifier to be applied to the BS (basic score). The study [2] is focused on evaluation of the reliability of the existing code formula. The authors made measurement on around 50 buildings and proposed a new improved empirical formula. In order to identify the natural frequencies, they calculate the averaged normalized power spectrum. They start with the existing formula $T = 0.09H/\sqrt{B}$ where the frequency depends only on the height of the building in meters (H) and the full plan dimension of the building, in meters, in the direction parallel to the applied forces without regarding shear wall dimensions (B). The new proposed formula is $T = C_1 \frac{1}{\sqrt{L_w}} H^{\beta} + C_2 = b_1 \frac{1}{\sqrt{L_w}} H^{\beta_1} + b_2$ where H is the height of the building in meters, L_w is the wall length per unit plan area, C_i is the constant determined by regression analysis and b_1, b_2, β_1 are the unbiased estimates of C_1, C_2, β. Goel and Chopra [3] first evaluate the empirical US formulae given in that period, using the data for the fundamental periods of the building obtained in the period of eight earthquakes in California. They show that the current code formula is inadequate for new buildings, and for that reason they propose a new improved formula similar to the one in paper [2], $T = C \frac{1}{\sqrt{A_e}} H$ where A_e is the equivalent shear area expressed as a percentage, C is the seismic coefficient and H is the height of the building in meters. Pulkit Velani and Pradeep Kumar Ramancharla [4] talk about the code formula in India. The aim of this paper is to study the reliability of the empirical expression of the fundamental period for tall buildings in India. For this purpose, ambient vibration tests have been carried out for 21 reinforced concrete (RC) buildings, located in Mumbai and Hyderabad cities. The measured periods have been compared with the code provisions. It was found that, as the height of the building increases, the natural period is not linearly proportional to the height. Hence, there is an urgent need for revision of the empirical expression. For estimating the fundamental natural periods for RC SW buildings with infill panels they proposed the formula $T = 0.01\, H^{1.1}$ and for RC buildings above 20 floors they proposed another formula $T = 0.009\, H^{1.1}$.

In this paper we again derive and recall the analytical solution of a shear beam excited by dynamic loadings causing initial displacement and (or) initial velocity of the beam. Because roughly, in one dimension (1D) the building can be modeled as a shear beam, we can get a clue what parameters of the beam or 1D model of the building are important in determining natural periods. The basic (fundamental) natural period corresponds to the first transverse mode of motion of the object.

2. Eigenvalues and eigenvectors

The equation that describes our problem is the one - dimensional (1-D) wave equation

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial y^2} \tag{1}$$

which computes the horizontal displacement of the building u, at an arbitrary height - distance y, at any instant of time t (Figure 1). In equation (1), C is the propagation speed of the wave defined as $C = \sqrt{\frac{\mu}{\rho}}$ where μ is the stiffness of the material and ρ is the density of the material. The quantities μ and ρ are material properties and they are different for different materials. The velocity of propagation of shear wave, C, is constant and is always a positive number.

Boundary conditions for shear beam as a model of building (Figure 1) are

$$\begin{cases} u(y=0)=0 \\ \frac{\partial u}{\partial y}\Big|_{y=H} \quad =0 \end{cases} \tag{1a}$$

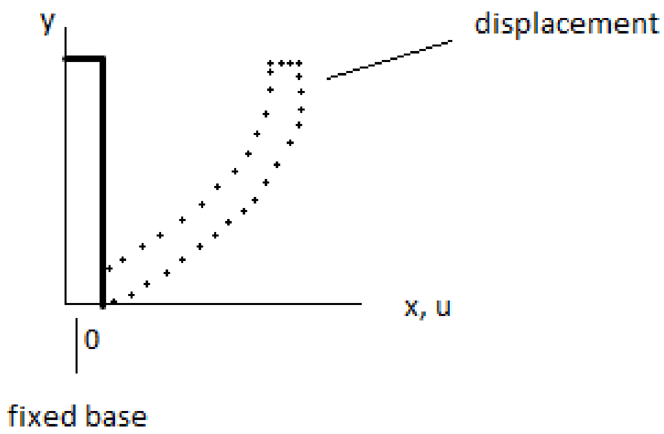

Figure 1 Building model as a one-dimensional element

If $Y(y)$ is a function of space and $T(t)$ is a function of time, then the displacement of the building y at arbitrary time t can be presented as a function $u=u(t,y)$ or

$$u=Y\cdot T \tag{2}$$

Where Y=Y(y) and T=T(t)

It follows that we can write the wave equation in the following form

$$\frac{\partial^2}{\partial t^2}(Y\cdot T)=C^2\frac{\partial^2}{\partial y^2}(Y\cdot T),$$

and after its differentiation it becomes $Y\cdot\frac{\partial^2 T}{\partial t^2}=C^2\frac{\partial^2 Y}{\partial y^2}\cdot T$.

Denoting $\frac{\partial^2 T}{\partial t^2}=\ddot{T}$ and $\frac{\partial^2 Y}{\partial y^2}=Y''$, the equation takes the form

$$Y\cdot\ddot{T}=C^2\cdot Y'',$$

By separation of spatial and temporal variables, to satisfy the equation we must get constant on left and right hand side of the equation

$$\frac{Y''}{Y} = \frac{\ddot{T}}{C^2T} = k \qquad (3)$$

Then, $\frac{Y''}{Y} = k$ and $\frac{\ddot{T}}{C^2T} = k$. Hence $Y'' = kY$ or

$$Y'' - kY = 0$$

and because $Y = e^{ry}$, then

$$r^2e^{ry} - ke^{ry} = 0$$

$$(r^2 - k)e^{ry} = 0/: e^{ry}$$

$$r^2 - k = 0$$

$$r_{1/2} = \pm\sqrt{k}$$

so $Y = Ae^{\sqrt{k}y} + Be^{-\sqrt{k}y}$.

We have the same for T. $\ddot{T} = C^2Tk$ or

$$\ddot{T} - C^2Tk = 0 \qquad (4)$$

and because $T = e^{rt}$, it follows

$$r^2e^{rt} - C^2ke^{rt} = 0$$

$$(r^2 - C^2k)e^{rt} = 0/: e^{rt}$$

$$r^2 - C^2k = 0$$

$$r_{1/2} = \pm C\sqrt{k}$$

From here we can define T as $T = Ae^{c\sqrt{k}t} + Be^{-c\sqrt{k}t}$.

We analyze the last equation. If the constant k is positive, the first term exponentially increases and goes to infinity as the time increases. Physically this is not possible, because our building oscillates with time elapsing. If the constant k is zero, from the last equation we get T = A + B, meaning that the temporal function is constant, so the solution of the equation (2) does not depend on t, which is not possible because the building moves (oscillates) with time elapsing. So, we conclude that the constant k must be negative. To get negative k in our equation (3), we denote $k = -\alpha^2$ where α is some other constant.

$$\frac{Y''}{Y} = \frac{\ddot{T}}{C^2T} = -\alpha^2$$

Hence $Y'' = -\alpha^2 Y$ or $Y'' + \alpha^2 Y = 0$ and because $Y = e^{rt}$, we have

$$r^2 e^{rt} + \alpha^2 e^{rt} = 0$$

$$(r^2 + \alpha^2)e^{rt} = 0/: e^{rt}$$

$$r^2 + \alpha^2 = 0$$

$$r_{1/2} = \pm\sqrt{-\alpha^2} = \pm\sqrt{(-1)^2\alpha^2} = \pm\alpha\sqrt{-1} = \pm i\alpha$$

Hence it follows that $Y = C_1 e^{r_1 y} + C_2 e^{r_2 y} = C_1 e^{i\alpha y} + C_2 e^{-i\alpha y}$

Starting from Euler's formula $e^{\pm iy} = cosy \pm siny$ we obtain

$$Y = C_1 cosy + C_1 i\, siny + C_2 cosy - C_2 i\, siny = (C_1 + C_2)cosy + (C_1 - C_2)i\, siny$$
$$= Acosy + Bsiny = Acos(\alpha y) + Bsin(\alpha y)$$

In the same way we can express the value of T. $\ddot{T} = -\alpha^2 C^2 T$ or $\ddot{T}+\alpha^2 C^2 T = 0$ and because $T = e^{rt}$, it follows

$$r^2 e^{rt} + \alpha^2 C^2 e^{rt} = 0$$

$$(r^2 + \alpha^2 C^2)e^{rt} = 0/: e^{rt}$$

$$r^2 + \alpha^2 C^2 = 0$$

$$r_{1/2} = \pm\sqrt{\alpha^2 C^2} = \pm iC\alpha$$

From here $T = C_1 e^{iC\alpha t} + C_2 e^{-iC\alpha t} \rightarrow T = Acos(C\alpha t) + Bsin(C\alpha t)$

If we refer to the first boundary condition in equation (1a), for $y = 0$ we will have $u(y,t) = Y(y) \cdot T(t) = 0$. As soon as $y = 0$, then $Y(y = 0) = 0$, yielding $A = 0$.

Then $Y = Acos(\alpha y) + Bsin(\alpha y)\ or$

$$Y = Bsin(\alpha y) \quad (5)$$

To satisfy the second boundary condition of equation (1a) we take

$\frac{\partial u}{\partial y}(y = H) = \frac{\partial}{\partial y}(Y \cdot T)(y = H) = 0$. Plugging the value of Y(y) obtained in (5) and having in mind that T is a function of time, i.e. constant with respect to differentiation with respect to y, we get

$$\frac{\partial u}{\partial y}|_{y=H} = B \cdot T \cdot \alpha \cdot \cos(\alpha y)\,|_{y=H} = 0 \quad (6)$$

The equation (6) is satisfied for B = 0, but then from (5) we get Y = 0, which related with equation (2) means that the points on the beam do not move. This is physically not possible because during excitation the points of the beam move.

This means that we should search for solution of (6) which satisfies

$\cos(\alpha y)\,|_{y=H} = 0$ or

$$\cos(\alpha H) = 0 \tag{7}$$

The equation (7) will be satisfied if the argument of the cosine function, αH, is a product of odd integer and $\frac{\pi}{2}$ so we write

$\alpha H = (2n-1)\cdot\frac{\pi}{2}$ from where we get

$$\alpha = \frac{2n-1}{2H}\cdot\pi \tag{8}$$

This number is also known as a wave number.

Setting $B = 1$, we will have infinitely many solutions $Y = Y_n(y)$ one for each n$\in N$

Hence, solutions of (1) satisfying boundary conditions are $u_n(t,y) = T_n(t)Y_n(y) = Y_n(y)T_n(t)$. Plugging the values for $Y_n(y)$ and $T_n(t)$ we get

$$u_n(t,y) = [A_n\cos(C\alpha_n t) + B_n\sin(C\alpha_n t)]\cdot\sin(\alpha_n y) \tag{9}$$

The functions $\sin(\alpha_n y)$ are called the eigenfunctions, or characteristic functions, and the values $C\alpha_n$ are called the eigenvalues, or characteristic values. The set $\{\,C\alpha_1\,,C\alpha_2\,\ldots\}$ is called the spectrum.

The solutions (9) satisfy the wave equation (1) and the boundary conditions. A single u_n will generally satisfy the initial conditions. The initial conditions are

$$\begin{cases} for\ t = 0, u(y) = f(y) \\ for\ t = 0, \dot{u}(y) = g(y) \end{cases}$$

where $f(y)$ is the initial displacement, and $g(y)$ is the initial velocity.

Supposing we have a given initial velocity

$g(y) = V_0\ for\ each\ value\ of\ y$

To obtain the solution that also satisfies the initial conditions, we consider the infinite series

$$u(t,y) = T \cdot Y = \sum_{n=1,3,5..}^{\infty} [A_n \cos(C\alpha_n t) + B_n \sin(C\alpha_n t)] \cdot sin(\alpha_n y)$$

For a given initial displacement and $t = 0$ we obtain

$$u(0,y) = \sum_{n=1,3,5..}^{\infty} [A_n \cos(C\alpha_n t) + B_n \sin(C\alpha_n t)] \cdot sin(\alpha_n y)$$

$$u(0,y) = \sum_{n=1,3,5...}^{\infty} A_n sin(\alpha_n y) = f(y) \tag{10}$$

When initial velocity is given and $t = 0$, we obtain

$$\frac{\partial u}{\partial t}|_{t=0} = \left[\sum_{n=1,3,5..}^{\infty} (-A_n\, C\alpha_n \sin(C\alpha_n t) + B_n C\alpha_n \cos(C\alpha_n t)) sin(\alpha_n y)\right]_{t=0}$$

$$= \sum_{n=1,3,5..}^{\infty} B_n\, C\alpha_n \sin(\alpha_n y) = g(y) \tag{11}$$

Hence, we have to choose the $B_n{}'$s so that for $t = 0$ the derivative $\frac{\partial u}{\partial t}$ becomes the Fourier sine series of $g(y)$. Thus,

$$B_n C\alpha_n = \frac{2}{H}\int_0^H g(y)\sin(\alpha_n y)dy, \quad n = 1,3,5, \dots$$

Because from eq. (8), $\alpha_n = \frac{2n-1}{2H} \cdot \pi$, we obtain

$$B_n = \frac{2}{HC\alpha_n}\int_0^H g(y)\sin(\alpha_n y)\,dy$$

$$= \frac{4}{C(2n-1)\pi}\int_0^H g(y)\sin(\alpha_n y)\,dy, \qquad n = 1,3,5, \dots$$

When $g(y) = V_0$ for each value of y, then

$$B_n = \frac{2}{HC\alpha_n}\int_0^H V_0 \sin(\alpha_n y)\,dy = -\frac{2V_0}{HC\alpha_n^2} \cdot \left(cos\frac{(2n-1)\pi}{2} - 1\right) \tag{12}$$

Proof of equation (9):

Because $g(y) = V_0$, then $V_0 = \sum_{n=1,3,5..}^{\infty} B_n\, C\alpha_n \sin(\alpha_n y)$. To find B_n , we first have to multiple with $\sin(\alpha_m y)$ and then to integrate from -H to H. So, we have

$$\int_{-H}^{H} \sin(\alpha_n y) * \sin(\alpha_m y) = \int_{-H}^{H} \frac{1}{2}[\cos(\alpha_n y - \alpha_m y) - \cos(\alpha_n y + \alpha_m y)]dy$$
$$= \frac{1}{2}\left[\frac{1}{\alpha_n - \alpha_m}\sin(\alpha_n - \alpha_m)\right] * H - \frac{1}{2}\left[\frac{1}{\alpha_n - \alpha_m}\sin(\alpha_n - \alpha_m)\right]$$
$$* (-H) - \frac{1}{2}\left[\frac{1}{\alpha_n + \alpha_m}\sin(\alpha_n + \alpha_m)\right] * H$$
$$- \frac{1}{2}\left[\frac{1}{\alpha_n + \alpha_m}\sin(\alpha_n + \alpha_m)\right] * (-H)$$

For $n \neq m \Longrightarrow\ \alpha_n \neq \alpha_m$ it follows that $\sin(\alpha_n + \alpha_m) * H = sin\left(\frac{(2n-1)\pi}{2H} + \frac{(2m-1)\pi}{2H}\right) * H = sin\left(\frac{2\pi(n-1+m)}{2H}\right) * H = \sin[(n-1+m)\pi] = 0$, and also $\sin(\alpha_n - \alpha_m) * H = sin\left(\frac{(2n-1)\pi}{2H} - \frac{(2m-1)\pi}{2H}\right) * H = sin\left(\frac{2\pi(n-1-m)}{2H}\right) * H = \sin[(n-1-m)\pi] = 0.$

For $n = m \Longrightarrow\ \alpha_n = \alpha_m$ it follows that

$$\int_{-H}^{H} \sin(\alpha_m y) * \sin(\alpha_m y) = \int_{-H}^{H} \frac{1}{2}[\cos(\alpha_m y - \alpha_m y) - \cos(\alpha_m y + \alpha_m y)]dy$$
$$= \frac{1}{2}\int_{-H}^{H} dy - \frac{1}{2}\int_{-H}^{H} cos2\,\alpha_m y dy$$
$$= \frac{1}{2}2H - \frac{1}{4\alpha_m}\sin\left[2\frac{(2m-1)\pi}{2H} * H - 2\frac{(2m-1)\pi}{2H} * (-H)\right] = H$$

This is H according to $\sin[(2m-1)\pi] = 0$ for each value of m.

The natural frequency of a one-dimensional element, in our case a building, can be determined using angular frequency $\omega_n = C \cdot \alpha_n$. Because $\omega = 2\pi f = \frac{2\pi}{T}$ we get $\frac{2\pi}{T_n} = C \cdot \alpha_n => T_n = \frac{2\pi}{C\alpha_n} = \frac{2\pi}{C\cdot\frac{2n-1}{2H}\cdot\pi} = \frac{4H}{(2n-1)\cdot C}$. The last equation proves that the period T_n depends on the height H and on the propagation speed of the waves, i.e. on the material properties. When we already know T_n and C , we can also calculate the corresponding wavelengths λ_n as $\lambda_n = C \cdot T_n$.

3. Results

We consider the solution of 1D building problem. Initially the building is positioned correctly (vertically). The problem is governed by a wave equation, which is a partial differential equation. The building is 20 m high, with the propagation speed of the wave 100 m/s, Δy (high step) is 0.1 m and Δt (time step) is 0.2 s. In the figure below, we present the behavior of the building for five different times from 0.0 to 0.8, because the period of the building is $T_1 = \frac{4H}{(2n-1)\cdot C} = \frac{4*20}{(2*1-1)\cdot 100} = \frac{80}{100} = 0.8\ s$.

In Figure 2, the displacement u along the height of the building is presented when initial displacement $f(y) = \frac{y}{40}$ is applied. This displacement consists only of terms An and we plot it taking the first 21 terms of the series.

$$A_n = \frac{2}{H}\int_0^H f(y)\sin(\alpha_n y)\,dy = \frac{2\cdot sin\alpha_n y - \alpha_n y cos(\alpha_n y)}{40H{\alpha_n}^2}\Bigg|_0^H$$
$$= \frac{sin\alpha_n H - \alpha_n H cos(\alpha_n H)}{20H{\alpha_n}^2}$$

$$u(t,y) = \sum_{n=1,3,5..}^{21} A_n \cos(C\alpha_n t)\cdot sin(\alpha_n y)$$
$$= \frac{sin\alpha_n H - \alpha_n H cos(\alpha_n H)}{20H{\alpha_n}^2}\cdot \cos(C\alpha_n t)\cdot sin(\alpha_n y)$$

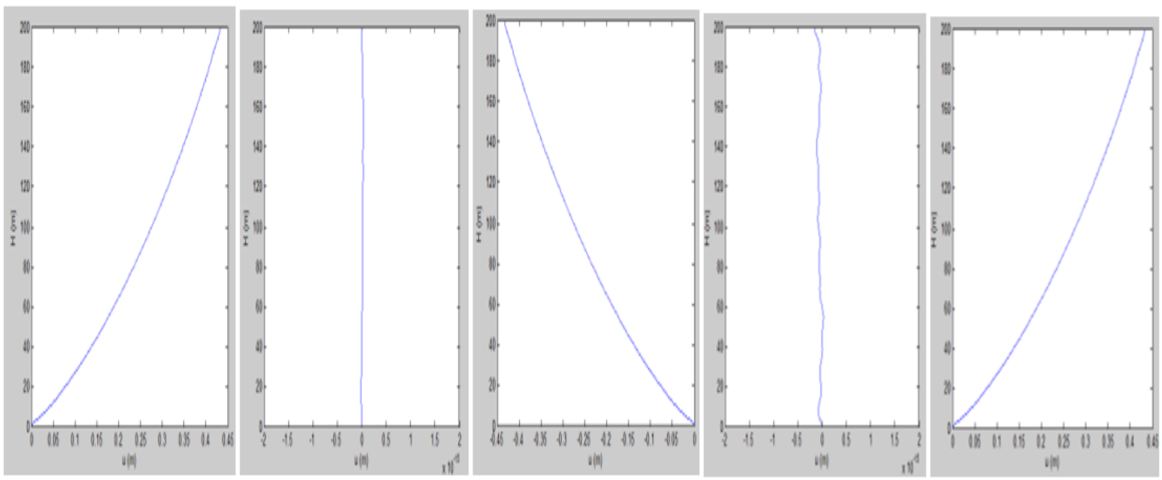

Figure 2 Displacement u at t=0.0, t=0.2, t=0.4, t=0.6 and t=0.8 s, when the initial displacement is applied

When instead initial displacement we apply initial velocity, we get the displacement presented in Figure 3 with a given initial velocity $g(y) = V_0 = 1m/s$. This displacement consists only of terms Bn and again we plot it taking n=21 terms in the series.

$$B_n = \frac{2}{HC\alpha_n}\int_0^H V_0 \sin(\alpha_n y)\,dy = -\frac{2V_0}{HC\alpha_n^2}\cdot\left(cos\frac{(2n-1)\pi}{2} - 1\right)$$
$$= \frac{2V_0}{HC\alpha_n^2}\cdot\left(1 - cos\frac{(2n-1)\pi}{2}\right)$$

$$u(t,y) = \sum_{n=1,3,5..}^{21} B_n \sin(C\alpha_n t)\cdot sin(\alpha_n y)$$
$$= \frac{2V_0}{HC\alpha_n^2}\cdot\left(1 - cos\frac{(2n-1)\pi}{2}\right)\cdot \cos(C\alpha_n t)\cdot sin(\alpha_n y)$$

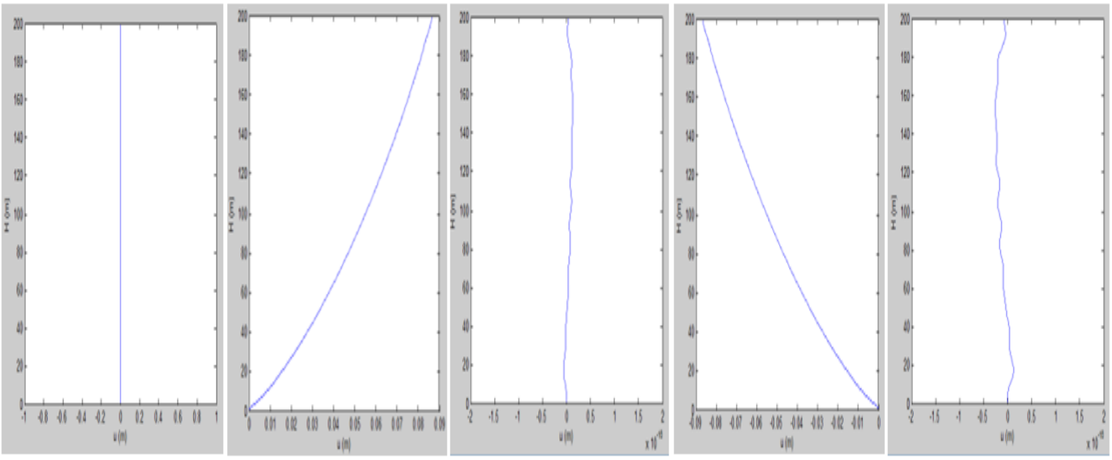

Figure 3 Displacement u at t=0.0, t=0.2, t=0.4, t=0.6 and t=0.8 s, when the initial velocity is applied

4. Conclusions

From the Figures, we can see the difference when for the same boundary conditions we take different initial conditions. When the initial displacement is given, the building for t=0.0 s goes in positive axis, then for 0.2 it goes in the initial position but still vibrating because of the movement. For 0.4 s it is positioned on the left axis with the same amplitude as at 0.0 on the positive side. At 0.6 seconds the building is in the initial position again, and at 0.8 we have a full period. On the other hand, when we have given initial velocity the building for t=0.0 s it is positioned correctly, for 0.2 it goes in positive axis, then again in the initial position but still vibrating because of the velocity. For 0.6 s it is positioned on the left axis with the same amplitude as at 0.2 on the positive side. At the end for 0.8 seconds, we have a full period, so the building is back in its initial position.

References

[1] MANOHAR, AJIT & SENGUPTA, KAUSTAV & TAMIZHARASI.G. (2012). Earthquake Vulnerability Assessment of Buildings in Guwahati. INTERNATIONAL JOURNAL OF EARTH SCIENCES AND ENGINEERING. 5. 618-623.

[2] CHUN, Y. S., YANG, J. S., CHANG, K. K., & LEE, L. H. (2000). Approximate estimations of natural periods for apartment buildings with sehar-wall dominant system. In *12th World Conference on Earthquake Engineering, Nueva Zelanda. Artículo* (Vol. 18).

[3] Goel, R. K., & Chopra, A. K. (1998). Period formulas for concrete shear wall buildings. Journal of Structural Engineering, 124(4), 426-433.

[4] Velani, P. D., & Ramancharla, P. K. (2017, January). New Empirical formula for fundamental period of tall buildings in India by ambient vibration test. In 16th World Conference on Earthquake, 16WCEE.

[5] Lee, L. H., Chang, K. K., & Chun, Y. S. (2000). Experimental formula for the fundamental period of RC buildings with shear-wall dominant systems. The Structural Design of Tall Buildings, 9(4), 295-307.

[6] Kreyszig, E. (2019) Advanced Engineering Mathematics, 10th edition.

[7] Knut A. Lie. The Wave Equation in 1D and 2D, 2005

[8] Kocaleva, Mirjana and Risteska, Aleksandra (2017) Практична примена на едно – димензионалната бранова равенка. Yearbook of the Faculty of Computer Science, 5 (5). pp. 5-12. ISSN 1857- 8691

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