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CONTENT

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GAUSSIAN METHOD FOR COMPUTING THE EARTH'S MAGNETIC FIELD

BLAGICA DONEVA

Abstract. For the study of the geomagnetic phenomena and the solving of numerous tasks from the application of geomagnetic methods, it is very important to find an analytical expression that shows the dependence of the magnetic field, or of its components, on the coordinates of the points placed on the Earth's surface. To find such an analytic expression, one can start from the assumption of a homogeneously magnetized Earth, or, based on the measured values of the elements of the Earth's magnetic field at a set point, an expression can be found that shows the distribution of magnetization inside the Earth corresponding to the measured field. The elements of the geomagnetic field are: declination D, inclination I, horizontal H, eastern Y, northern X and vertical Z component, as well as the vector T of the geomagnetic field. The intensities of the geomagnetic field elements are expressed in [nT] nanotesla, and the values of declination D and inclination I of the geomagnetic field are expressed in degrees. [5]

1. Introduction

The magnetic field of a homogeneously magnetized sphere can be approximated by a straight quadruple magnetic prism with a magnetic moment M that causes it at the measuring point P, and then the magnetic potential at the measuring point is:

$$U = \frac{M}{r^2} \cos\theta \qquad (1)$$

M - magnetic moment;

r - radius - vector of the measurement point P on the Earth's surface,

θ - angle that the axis of the ONm magnet occupies with the radius vector, r = OP at the measuring point.

In order to consider the distribution of the elements of a magnetic field on a sphere, it is assumed that the beginning of the spherical coordinate system is located in the center of the Earth and that the center of the magnet (O) coincides with it, and that axis ON_g represents the axis of the Earth's rotation (fig. 1). [4]

As the points P, N_m and N_g form a spherical triangle PN_mN_g, the angle that the radius vector of the point P and the axis of the magnet intersect can be expressed as

$$\cos\theta = \sin\varphi \cdot \sin\varphi_m + \cos\varphi \cdot \cos(\lambda - \lambda_m) \qquad (2)$$

φ ; λ - latitude and longitude of the measuring point P,

φ_m ; λ_m - latitude and longitude of the north magnetic pole Nm.

In that case, the potential of the magnetic field on sphere is:

$$U = \frac{M}{r^2}\,[sin\varphi\ \cdot sin\varphi_m + cos\varphi\ \cdot cos\varphi_m\ \cdot \cos(\lambda - \ \lambda_m)] \qquad (3)$$

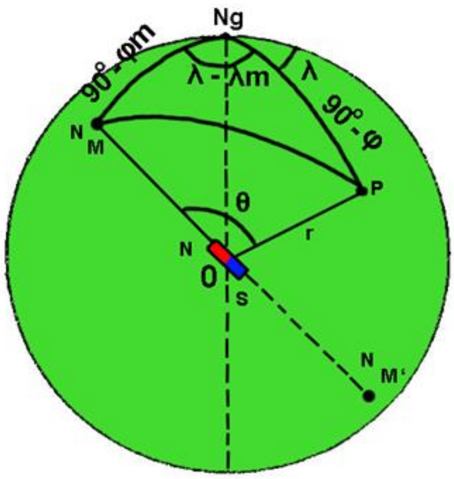

Fig. 1. Sketch of the analysis of magnetic field on a homogeneously magnetized sphere

If the sphere is assumed to be homogeneously magnetized, then the magnetic moment can be expressed by:

$$M = V\ \cdot J = \frac{4}{3}\,R^3\pi J \qquad (4)$$

R - radius of the sphere,
J - magnetic moment per unit volume.

Since the coordinates of the point N_m are invariant, we can write for simplification:

$$g_1^0 = \frac{4}{3}\,\pi \cdot J \cdot sin\varphi_m$$

$$g_1^1 = \frac{4}{3}\,\pi \cdot J \cdot cos\varphi_m\ \cdot cos\lambda_m \qquad (5)$$

$$h_1^1 = \frac{4}{3}\,\pi \cdot J \cdot cos\varphi_m\ \cdot sin\lambda_m$$

In that case, the potential can be expressed by the relation:

$$U = \frac{R^3}{r^2}\,[g_1^0\ sin\varphi + \ (g_1^1\ cos\lambda + \ h_1^1\ sin\lambda)\ cos\varphi] \qquad (6)$$

If the beginning of a rectangular coordinate system is set at the point P such that the positive direction of the Z-axis is along the radius vector r toward the center of the earth, the positive direction of the X-axis is along the direction PN (i.e. it lies in the plane of the meridian of the measuring point), then the Y-axis is perpendicular to the plane XOZ (it lies in a plane parallel to the equator and oriented to the right). In this case, the excerpts of the magnetic field potential U along the coordinate axes are:

$$X = -\frac{1}{r} \cdot \frac{\partial U}{\partial \varphi}$$

$$Y = -\frac{1}{r \cdot cos\varphi} \cdot \frac{\partial U}{\partial \lambda} \qquad (7)$$

$$Z = -\frac{\partial U}{\partial r}$$

X component corresponds to the north, Y corresponds to the east, and Z component corresponds to the vertical component of the magnetic field of the homogenous magnetized sphere.

If we take for the initial meridian the one passing through the point Nm, i.e. the point where the axis of the magnet passes through the Earth's surface, in that case it would be $\lambda_m = 0$, and according to the system equation (7) and the coefficient $h_1^1 = 0$, so the equation (7) reduces to

$$X = g_1^0 \cdot cos\varphi - g_1^1 \, cos\lambda \, sin\phi$$

$$Y = -g_1^1 \cdot sin\lambda \qquad (8)$$

$$Z = 2[g_1^0 \cdot sin\varphi + g_1^1 \cdot cos\lambda \cdot cos\varphi]$$

If the axis of the magnet is assumed to coincide with the Earth's axis of rotation, and thus the magnetic and geographic poles coincide, then the coefficient $g_1^1 = 0$, so that:

$$X = g_1^0 \cdot cos\varphi$$

$$Y = 0 \qquad (9)$$

$$Z = 2[g_1^0 \cdot sin\varphi]$$

where the angle φ also represents the angle of the magnetic width. Since in this case there is only a northern X-component of the horizontal components, that component is also a horizontal component of the H field, then:

$$X = H = \frac{M}{R^3} \cdot cos\varphi$$

$$Z = \frac{2M}{R^3} \cdot sin\varphi \qquad (10)$$

These equations correspond to the field of a dipole magnet whose axis coincides with the Earth's axis of rotation, and such field is also called the axial dipole field. It is obvious that for the axial dipole field the declinations at all points on the Earth will be equal to zero, while the magnitude of the inclination angle I will be determined by the relation:

$$\frac{Z}{H} = tgI = 2tg\varphi \tag{11}$$

where the angle φ is both the geographical and the magnetic latitude of the measuring point. The expression shows that the magnitude of the inclination angle increases with increasing latitude.

For the intensity of the total field T, which is the vector sum of the horizontal H and the vertical Z-component, the expression is obtained: [4]

$$T = \frac{M}{R^3}\,(1 + 3\,sin^2\varphi)^{\frac{1}{2}} \tag{12}$$

Based on numerous analyses of the Earth's magnetic field measurement data, the magnetic moment M of a hypothetical dipole magnet is $8.3 \cdot 10^{25}$ SI units [T].

According to the previous expressions, the intensities of the magnetic field on the equator are:

$$Z = 0 \qquad and \qquad H = T = \frac{M}{R^3} \tag{13a}$$

and on the poles are:

$$H = 0 \qquad and \qquad Z = T = \pm\frac{2M}{R^3} \tag{13b}$$

Consequently, the intensity of the Earth's magnetic field on its surface changes from 0.000033 Teslas at the equator to 0.000066 Teslas at the poles. In other words, the intensity of the horizontal component at the equator is twice the intensity of the vertical component at the poles. If we consider such a field as a normal magnetic field of the Earth, any field values that would deviate from the values determined by expressions (10) and (12) would be anomalous.

For an axial dipole field, it is possible to calculate the values of the gradients of that field, which are usually expressed in γ/km. [6]

Horizontal gradients of the magnetic field of an axial dipole would be:

$$\begin{aligned} \frac{\partial Z}{r \cdot \partial\varphi} &= \frac{2M}{r^4} \cdot cos\varphi = Z \cdot \frac{1}{r}\,ctg\varphi \\ \frac{\partial H}{r \cdot \partial\varphi} &= -\frac{M}{r^4} \cdot sin\varphi = -H \cdot \frac{1}{r}\,tg\varphi \end{aligned} \tag{14}$$

and would represent the increments of the corresponding components in the north direction while the vertical gradients would be:

$$\frac{\partial Z}{\partial r} = -\frac{6M}{r^4} \cdot sin\varphi = -3Z \cdot \frac{1}{r}$$
$$\frac{\partial H}{\partial r} = -\frac{3M}{r^4} \cdot cos\varphi = -3H \cdot \frac{1}{r} \tag{15}$$

If for the territory of the Balkans an example is given by a point whose latitude is 45° S, then the values for the gradient of the magnetic field for the axial dipole will be:

$$\frac{\partial Z}{r \cdot \partial \varphi} = \frac{Z}{r} ctg\varphi = 6{,}36\, \gamma/km$$
$$\frac{\partial X}{r \cdot \partial \varphi} = \frac{H}{r} tg\varphi = 3{,}52\, \gamma/km$$
$$\frac{\partial Z}{\partial r} = -3\frac{Z}{r} = -19{,}08\, \gamma/km$$
$$\frac{\partial H}{\partial r} = -3\frac{H}{r} = -10{,}56\, \gamma/km \tag{16}$$

2. Gaussian method of analysis of the Earth's magnetic field

In his analysis of the Earth's magnetic field, Gauss [3] assumed that the cause of the magnetic field was inside the Earth and such a field must satisfy the Laplace equation and that, for it, the valid formula is:

$$\vec{T} = -gradU \tag{17}$$

T – magnetic field vector
U- magnetic field potential.

If a magnetic mass dm corresponds to each point in the Earth's interior , then the potential U of the magnetic field at any point P out of the sphere is:

$$U = \int_V \frac{dm}{\rho} \tag{18}$$

where integration is done by volume (V) of the entire sphere.

If it is assumed that the elementary magnetic mass dm is within the point M with coordinates r', Θ', λ' (fig. 2) and that the point P is far away from the point M so $\rho = \overline{MP}$ and radius vectors of the point M and P occupy an angle γ, then

$$U = \int_V \frac{dm}{\left(r^2 + r'^2 - 2r \cdot r' \cdot \cos\gamma\right)^{1/2}} \tag{19}$$

where

r - radius vector to point P;

r' - radius vector to point a M.

Θ', λ' - colatitude and longitude to the points Q and M;

Θ, λ - colatitude and longitude to the point P_1 and P.

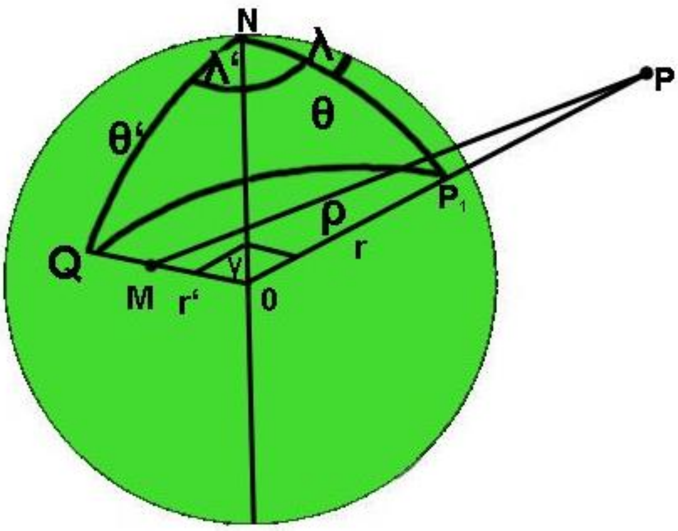

Figure 2. Sketch of the Gaussian method of analysis of the magnetic field [2]

Since $r = \overline{OP} = const.$, then it can be put in front of the integral, because all the magnetic masses dm are inside the sphere, the condition r < r' is always satisfied, then the expression for the sub-integral function can be developed into a row using Newton's binomial expression:

$$\frac{1}{\rho} = \frac{1}{r}\left[1 + \left(\frac{r'}{r}\right)^2 - 2\left(\frac{r'}{r}\right)\cos\gamma\right]^{-\frac{1}{2}} = \tag{20}$$

$$= \frac{1}{r}\left\{1 - \frac{1}{2}\left[\left(\frac{r'}{r}\right)^2 - 2\left(\frac{r'}{r}\right)\cos\gamma\right] + \frac{3}{8}\left[\left(\frac{r'}{r}\right)^2 - 2\left(\frac{r'}{r}\right)\cos\gamma\right]^2 - \ldots.\right\}$$

The expressions that appear in the Newton's formula as coefficients of the members $\left(\frac{r\prime}{r}\right)^n$ are Legendre polynomials after $cos\,\gamma$ argument, and the expression $P_n(cos\,\gamma)$ is a function $cos\,\gamma$ of degree n. The properties of these functions are studied in the theory of spherical functions and they allow to calculate the values of the polynomials of the order (n+1) when the polynomials of n and (n-1) rows are known, or

$$P_{n+1}(cos\,\gamma) = \frac{2n+1}{n+1} cos\,\gamma \cdot P_n(cos\,\gamma) - \frac{n}{n+1} \cdot P_{n+1}(cos\,\gamma) \quad (21)$$

The values of the first two polynomials are obtained by developing in order after Newton's binomial equation and based on equation (21) their values are

$$P_0(cos\,\gamma) = 1 \quad \text{and} \quad P_1(cos\,\gamma) = cos\,\gamma$$

Using the equation (21) the expressions can be calculated

$$\begin{aligned} P_2(cos\,\gamma) &= \tfrac{3}{2} cos^2\,\gamma - \tfrac{1}{2} \\ P_3(cos\,\gamma) &= \tfrac{5}{2} cos^3\,\gamma - \tfrac{3}{2} cos\,\gamma \\ P_4(cos\,\gamma) &= \tfrac{35}{8} cos^4\,\gamma - \tfrac{15}{4} cos^2\,\gamma + \tfrac{3}{8} \quad \text{etc.} \end{aligned} \quad (22)$$

Thus the expression for the potential (U) expressed by equation (19) can be written in the form:

$$U = \frac{1}{r} \sum_{n=0}^{\infty} \int_V \left(\frac{r'}{r}\right)^n P_n(cos\,\gamma) dm \quad (23)$$

that is, if we extract the value $1/r^n$ in front of the integral

$$U = \sum_{n=0}^{\infty} \frac{A_n}{r^{n+1}} \quad (24)$$

where

$$A_n = \int_V r'^n P_n(cos\,\gamma) dm \quad (25)$$

The expression (24) for the potential of the magnetic field can be written in the form

$$U = \frac{A_o}{r} + \frac{A_1}{r^2} \ldots. \quad (26)$$

if we would only dwell on the first two members of the infinite convergent order in which the expressions A_n occur. In that case it is obvious that

$$A_0 = \int_V dm = 0$$

arising from the bipolar nature of magnetism while the expression

$$A_1 = \int_V r' \cos\gamma \, dm$$

is the projection of the magnetic moment (M) to the axis sphere, so the expression for the potential (U) is

$$U = \frac{M}{r^2} \cos\varphi$$

which is actually an expression of the magnetic field potential of the dipole.

The above analysis would make sense if we could consider the Earth's magnetic field as a short-dipole field whose center is equally distant from all points on the Earth's surface. But, for a more complete analysis, we have to assume that there are magnetizations in the Earth that are differently distributed, so the value of the expression A_n should be determined. Therefore, the magnitude of the angle that intersects the radius vectors of points M and R should be expressed by the spherical coordinates of those points, which, on the basis of Fig. 2 can be written as

$$\cos\gamma = \cos\theta \cos\theta' + \sin\theta \sin\theta' \cos(\lambda - \lambda')$$

Based on the theorems that are proven when analyzing spherical functions, we can write

$$P_n(\cos\gamma) = \sum_{m=0}^{n} \frac{(n-m)!}{(n+m)!} C_n P_n^m(\cos\theta) \cdot P_n^m(\cos\theta') \cos m(\lambda - \lambda') \qquad (27)$$

where C = 1 when m = 0 and C = 2 when m > 0

$$P_n^m(\cos\theta) = \sin^m\theta \cdot \frac{d^m P_n(\cos\theta)}{d(\cos\theta)^m}$$

$$P_n^m(\cos\theta') = \sin^m\theta' \cdot \frac{d^m P_n(\cos\theta')}{d(\cos\theta')^m}$$

The functions $P_n^m(\cos\theta)$ are called Legendre associative functions and for values n = 1, 2, 3 and m = 2, 3 they have the following forms:

$$P_1^1(\cos\theta) = \sin\theta, \quad P_2^1(\cos\theta) = 3\cos\theta\sin\theta, \quad P_2^2(\cos\theta) = 3\sin^2\theta$$

$$P_3^1(\cos\theta) = -\frac{15}{2}\sin^3\theta + 6\sin\theta, \quad P_3^2(\cos\theta) = 15\cos^2\theta\sin\theta$$

$$P_3^3(\cos\theta) = 15\cos^3\theta$$

Based on the above expressions we get

$$A_n = \sum_{m=0}^{n} (a_n^m \cos m\lambda + b_n^m \sin m\lambda) \cdot P_n^m(\cos\lambda) \tag{28}$$

where

$$\begin{aligned} a_n^m &= \frac{(n-m)!}{(n+m)!} C_n . r'^n P_n^m(\cos\theta') \cdot \cos m\lambda' dm \\ b_n^m &= \frac{(n-m)!}{(n+m)!} C_n . r'^n P_n^m(\cos\theta') \cdot \sin m\lambda' dm \end{aligned} \tag{29}$$

If the expression for An in the equation (28) is substituted into the expression for a magnetic field potential, we will get

$$U = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{1}{r^{n+1}} (a_n^m \cos m\lambda + b_n^m \sin m\lambda) \cdot P_n^m(\cos\theta) \tag{30}$$

Since for a certain sphere expresses a_n^m, b_n^m are constant, assuming that the radius of the selected sphere is R, tags can be inserted

$$\begin{aligned} a_n^m &= R^{n+2} \cdot g_n^m \\ b_n^m &= R^{n+2} \cdot h_n^m \end{aligned} \tag{31}$$

Inserting the expression (31) in the equation (30) for the potential U the following is obtained

$$U = R \cdot \sum_{n=1}^{\infty} \left(\frac{R}{r}\right)^{n+1} \cdot \sum_{m=0}^{n} (g_n^m \cos m\lambda + h_n^m \sin m\lambda) \cdot P_n^m(\cos\theta) \tag{32}$$

And if it is assumed that r = R, or if the potential is observed on the surface of the sphere, then we obtain

$$U = R \cdot \sum_{n=1}^{\infty} \sum_{m=0}^{n} (g_n^m \cos m\lambda + h_n^m \sin m\lambda) \cdot P_n^m(\cos\theta) \tag{33}$$

From the expression (33) it can be said that the magnetic potential of the surface of the sphere, caused by the magnetic masses located inside the sphere, is expressed as a double sum of infinitely many members, and each of those members is a function

$P_n^m(\cos\theta) \cos m\lambda$ from θ and λ

$P_n^m(\cos\theta) \sin m\lambda$ from θ and λ

with constant coefficients g_n^m, h_n^m, which is called a spherical function. Therefore, the Gaussian method is also called the method of spherical harmonic analysis, because the magnetic field is represented by its harmonics.

The number of members g_n^m, h_n^m theoretically can be infinite, but if m is never greater than n and if m = 0 all members of type h are equal to zero, then it is obvious that the number of members (N) of type g_n^m, h_n^m m can be expressed through

$$N = n\,(n + 2)$$

Therefore, depending on n, the number of members N can be, for example

n =	1	2	3	4	5	6
N =	3	8	15	24	35	48 etc.

In order to find expressions for the components of the Earth's magnetic field in the selected directions, one has to differentiate the potential (U) given by the expression (33) along the specified directions. If we differentiate along the axes of the coordinate system whose x-axis is oriented in the plane of the geographic meridian, the z-axis is in the vertical direction, and the y-axis is perpendicular to them, then we get the north (X), the vertical (Z) and the eastern (Y) component, and through them all other elements of the Earth's magnetic field. The expressions for the X, Y and Z components would be

$$X = -\frac{1}{r}\cdot\frac{\partial U}{\partial \theta} = -\sum_{n=1}^{\infty}\sum_{m=0}^{n}(g_n^m \cos m\lambda + h_n^m \sin m\lambda)\frac{dP_n^m(\cos\theta)}{d\theta}$$

$$Y = -\frac{1}{r\sin\theta}\cdot\frac{\partial U}{\partial \lambda} = \sum_{n=1}^{\infty}\sum_{m=0}^{n}(mg_n^m \sin m\lambda - mh_n^m \cos m\lambda)\frac{P_n^m(\cos\theta)}{\sin\theta} \qquad (34)$$

$$Z = -\frac{\partial U}{\partial r} = \sum_{n=1}^{\infty}\sum_{m=0}^{n}[(n+1)g_n^m \cos m\lambda + (n+1)h_n^m \sin m\lambda]\cdot P_n^m(\cos\theta)$$

The Gaussian method of spherical harmonic analysis consists in the fact that, based on the values of the elements of the Earth's magnetic field or in a particular territory, appropriate equations are formed by applying the Gaussian algorithm to determine the g_n^m and h_n^m coefficients . How many coefficients will be determined depends on the choice of the number n. Gauss, in his work, limited himself to n = 4 and determined 24 coefficients using the data from the measurement of the elements of the Earth's magnetic field at 12 points. Thanks to modern computing machines, today this problem can be easily solved using a large number of points and the coefficients g_n^m and h_n^m can also be determined when n is a two-digit number, but the physical meaning of those values is not easy to determine.

With the solution of the system of equations (34), the normal magnetic field of a selected territory can be represented by the expression [1]

$$E(\Delta\varphi, \Delta\lambda) = a_1 + a_2\Delta\varphi + a_3\Delta\lambda + a_4\Delta\phi\varphi^2 + a_5\Delta\lambda^2 + a_6\Delta\varphi\Delta\lambda$$

where

$E(\Delta\varphi, \Delta\lambda)$- the value of the normal field of the point whose geographical coordinates are φ_1 and λ_1;

φ_1 and λ_1- latitude and longitude of the point;

φ_0 and λ_0- latitude and longitude of the point relative to which the measurements are made;

$\Delta\varphi = \varphi_1 - \varphi_0$- difference in latitude in minutes;

$\Delta\lambda = \lambda_1 - \lambda_0$- difference in longitude in minutes;

a_i - coefficients of the corresponding difference in γ / minute, i.e. minutes / minutes or gamma and minutes.

It is common for differences in latitude and longitude to be calculated with respect to the coordinates of the geomagnetic observatory located on the territory for which the coefficients a_i for the normal field are calculated.

Since the Republic of North Macedonia has no geomagnetic observatory, coefficients are not the coefficients ai are not published, but the equations for the normal field of the neighboring countries, such as Bulgaria or Serbia can be used.

3. Conclusion

The geomagnetic field of the Earth at any of its points or in the domain of the magnetosphere can be represented by a vector that is the tangent to the magnetic lines of force at the measuring point. A common geomagnetic field vector designation is $\vec{T}$, although a H_T designation is often used. The vector module defines the geomagnetic field intensity at the observation point. The vertical plane in which the vector of the geomagnetic field lies is also called a magnetic meridian.

For the analysis of the magnetic field, the Earth is approximated to a sphere. In order to consider the distribution of the elements of a magnetic field on a sphere, it is assumed that the beginning of the spherical coordinate system is located in the center of the Earth and that the center of the magnet coincides with it.

Gauss assumed that the cause of the magnetic field was inside the Earth and such a field must satisfy the Laplace equation. The Gaussian method is also called the method of spherical harmonic analysis, because the magnetic field is represented by its harmonics.

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