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BASIC PRINCIPLES OF GEOMETRIC SEISMICS

BLAGICA DONEVA, MARJAN DELIPETREV, GJORGI DIMOV

Abstract. In a homogeneous and isotropic medium, the waves excited in the source propagate, so that the wave fronts form surface spheres.
In order to represent the wavefront in an inhomogeneous medium, the Huygens principle is applied, which indicates that every point of the wavefront can, at some moment, be considered as a source of a new wavelet. Elemental spherical waves of radius r = VΔt are formed around these points, where V is the speed of wave propagation in a given area, and Δt is the time interval that is small enough so the medium corresponding to that interval can be considered as homogeneous. The envelope of a large number of elemental wave waves gives a new position of the wavefront after the time Δt..
Fig. 1 represents the Huygens principle.

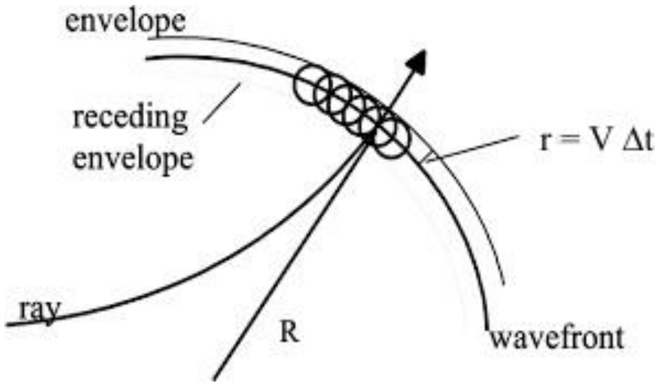

Fig. 1. Explanation of Huygens principle

1. Introduction

Law of wave reflection and refraction

When elastic waves encounter surfaces that separate the media of different elastic properties, they reflect and refract from these, similarly to light waves.

If the source of the spherical wave is quite distant, it can be considered that at least a part of the spherical surface of the wavefront is plane; in that case, these are plane waves. The rays that are perpendicular to the wavefront are parallel to each other.

Law of wave reflection

Huygens' principle is applied to a plane longitudinal wave, which during propagation encounters a plane MN that separates two media of different elastic properties, one in which the longitudinal wave propagates at velocity V_{p1} and the transverse at velocity V_{s1}, and the other in which the longitudinal wave propagates at V_{p2}, and transverse velocity V_{s2} (Fig. 2). Let's consider the wavefront AB encountering the mentioned plane MN. Point A of the wavefront becomes the source of new and longitudinal and transverse waves that propagate hemispherically in both media. Now, we observed just the waves that return to the upper medium. During the time that the seismic beam, passing through point B, reaches the boundary plane MN in point C, passing the path x, the longitudinal sphere wave from point A also passes the path x, and the transverse wave es the path $x\,\frac{V_{s1}}{V_{p1}}$. The tangent drawn through point C to the first sphere CD, represents the wavefront of the reflected transverse wave. [6]

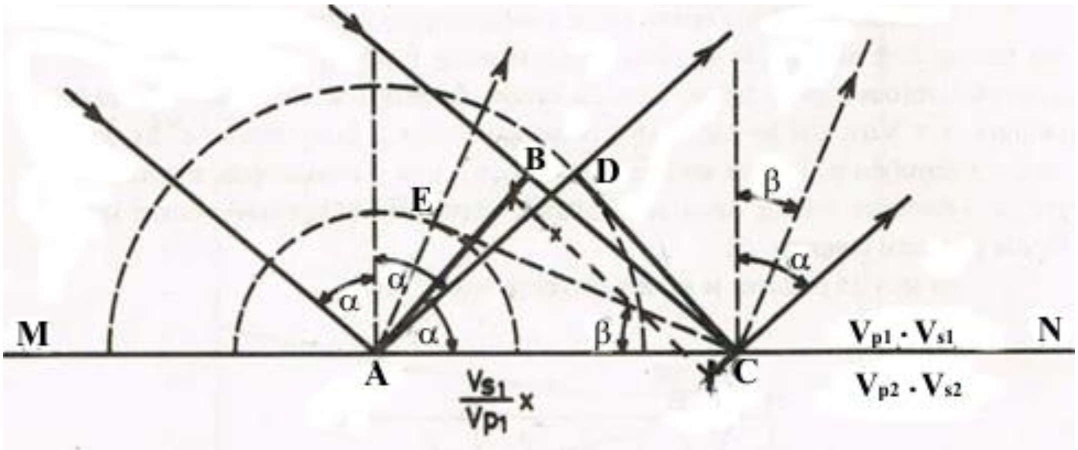

Fig. 2. Law of wave reflection

It is not difficult to determine, from Figure 2, that the angle of the wavefront of the reflected longitudinal wave is equal to the angle of the wavefront of the incident longitudinal wave. Namely, ΔABC ≅ ΔCDA since $\overline{AC} = \overline{CA}$, $\overline{BC} = \overline{AD} = x$, $\measuredangle ABC = \measuredangle ADC = 90°$. So $\measuredangle BCA = \measuredangle ACD$, so the angles of incidence and reflection of the corresponding rays are equal. The angle of the wavefront of the reflected transverse wave β can be determined as follows:

From the triangle AEC $\qquad \sin\beta = \dfrac{\overline{AE}}{\overline{AC}}$

$$\tag{1}$$

From the triangle ABC $\qquad \overline{AC} = \dfrac{\overline{BC}}{\sin\alpha}$

Since $\overline{BC} = x,$ and $\overline{AE} = \frac{V_{s1}}{V_{p1}}\,x$, then by substituting these values in the equation (2) the following is obtained

$$\sin\beta = \frac{V_{s1}}{V_{p1}}\sin\alpha \tag{2}$$

Law of wave refraction - Snellius law

Let us observe, similarly as during wave reflection, the encounter of the wave front of a plane longitudinal wave AB (Fig. 3) on the plane MN. Of interest are the waves whose source is at point A, and which pass into the lower medium (V_{p2}, V_{s2}). While the seismic ray, which passes through point B, has reached the boundary plane at point C, i.e. passed the path x, the longitudinal spherical wave from point A passed the path $\frac{V_{p2}}{V_{p1}}x$ in the lower medium, and transverse spherical wave passed the path $\frac{V_{s2}}{V_{p1}}x$. If a tangent is drawn from point C, the wavefront of the refracted longitudinal wave is obtained on the sphere CD, and the tangent to the other sphere CE represents the wavefront of the refracted transverse wave.

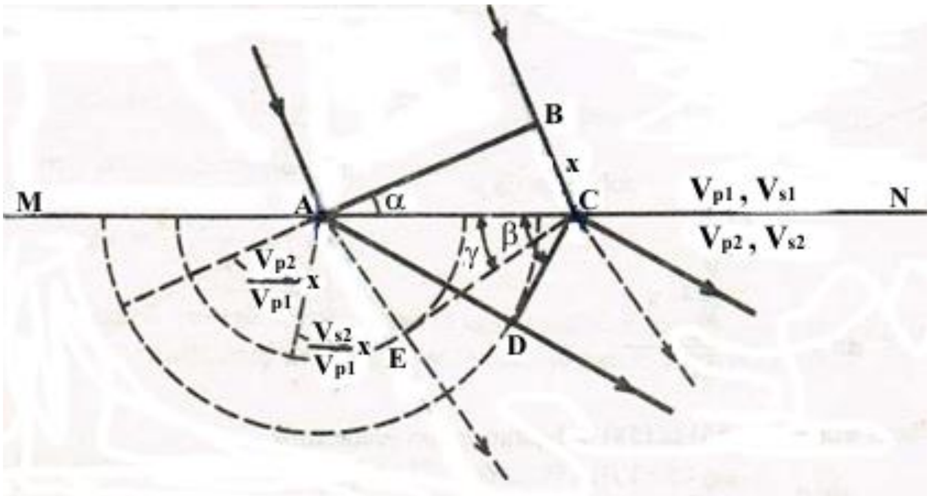

Fig. 3. Description of Snellius' law

From the triangle ABC $\qquad \sin\alpha = \frac{x}{\overline{AC}}$

From the triangle ADC $\qquad \sin\beta = \frac{\overline{AD}}{\overline{AC}}$ (3)

Since $\overline{AD} = \frac{V_{p2}}{V_{p1}}x$, it is obtained that

$$\sin\beta = \frac{\frac{V_{p2}}{V_{p1}}x}{\overline{AC}} \tag{4}$$

Dividing the equations (3) and (4) the law of refraction, or Snellius law is obtained.

$$\frac{\sin\alpha}{\sin\beta} = \frac{V_{p1}}{V_{p2}} \tag{5}$$

If the angle formed by the wavefront of the refracted transverse wave with the plane MN is marked as γ, then, according to Figure 3 it follows:

$$\sin\gamma = \frac{\overline{AE}}{\overline{AC}} \tag{6}$$

Because $\overline{AE} = \frac{V_{s2}}{V_{p1}}\, x$, it is obtained that:

$$\sin\gamma = \frac{\frac{V_{s2}}{V_{p1}}\, x}{\overline{AC}} \tag{7}$$

Dividing the equations (3) and (7), the law of refraction is obtained

$$\frac{\sin\alpha}{\sin\gamma} = \frac{V_{p1}}{V_{s2}} \tag{8}$$

From the recent presentations, it is clear that when a longitudinal wave encounters a boundary plane, which separates two media of different elastic properties, four new waves appear: reflected and refracted longitudinal wave, and reflected and refracted transverse wave. Similarly, four waves would appear if a transverse wave encountered the boundary surface.

It is assumed that there are two media in which the waves propagate with the velocities V_1 and V_2, so that $V_2 > V_1$ and there is a plane MN that separates the two media (Figure 4).

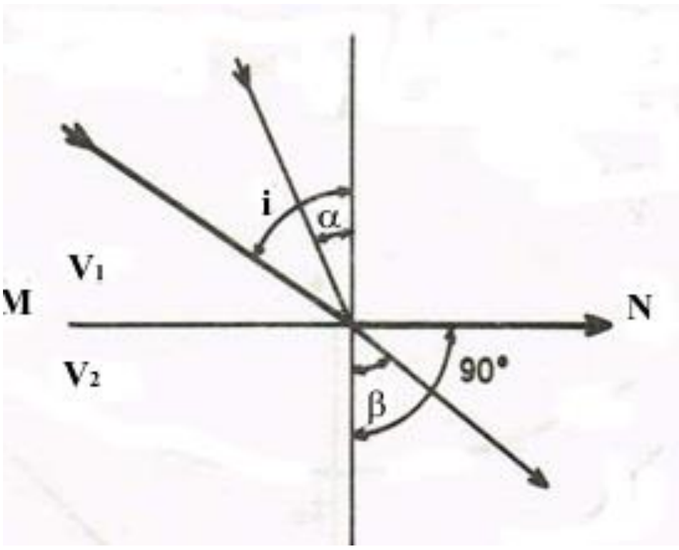

Fig. 4. Total wave refraction and critical angle

There is a certain angle of incidence i of the ray at which the angle of refraction β is equal to 90°.

Replacing the angle α with i in the equation $\frac{\sin\alpha}{\sin\beta} = \frac{V_1}{V_2}$, and since sin β = sin 90º = 1, this equation can be written as

$$\sin i = \frac{V_1}{V_2} \tag{9}$$

The angle i is called the *critical angle* or the angle of total reflection. Any ray whose angle of incidence is greater than the critical cannot be refracted, but only reflected. [5]

2. Path of reflected and refracted waves

Horizontal two-layer medium

The main problem of research by the reflective seismic method is to determine the position of the layer from which the wave is reflected. First, a two-dimensional problem is considered, assuming that the profile is set at a right angle to the layer orientation. First of all, let us consider the case of the horizontal layer. [2]

From Figure 5 it is seen that h is the thickness of the layer given by the expression

$$h = \frac{Vt_0}{2}$$

where t_0 is the time for which the wave travels from the source to the boundary plane and back to the receiver.

Let O be the source of the wave, and S the place of the receiver and denote the average velocity of the wave propagation from the ground to the boundary plane with V, then (Figure 5) the propagation time of the reflected waves "t" is equal to:

$$t = \frac{OC + OS}{V}$$

or, if by R the image of the point of the wave source is denoted, the time "t" is given with the relation

$$t = \frac{SR}{V}$$

Finding RS from a right triangle (Fig. 5), the time t can be expressed depending on the distance x between the points of the wave source and the depth of the boundary h

$$t^2 = \frac{x^2}{V^2} + \frac{4h^2}{V^2} \qquad or \qquad V^2t^2 = x^2 + 4h^2 \tag{10}$$

Dividing this equation with $4h^2$ we obtain

$$\frac{V^2\,t^2}{4h^2} - \frac{x^2}{4h^2} = 1 \tag{11}$$

This is the hyperbola equation.

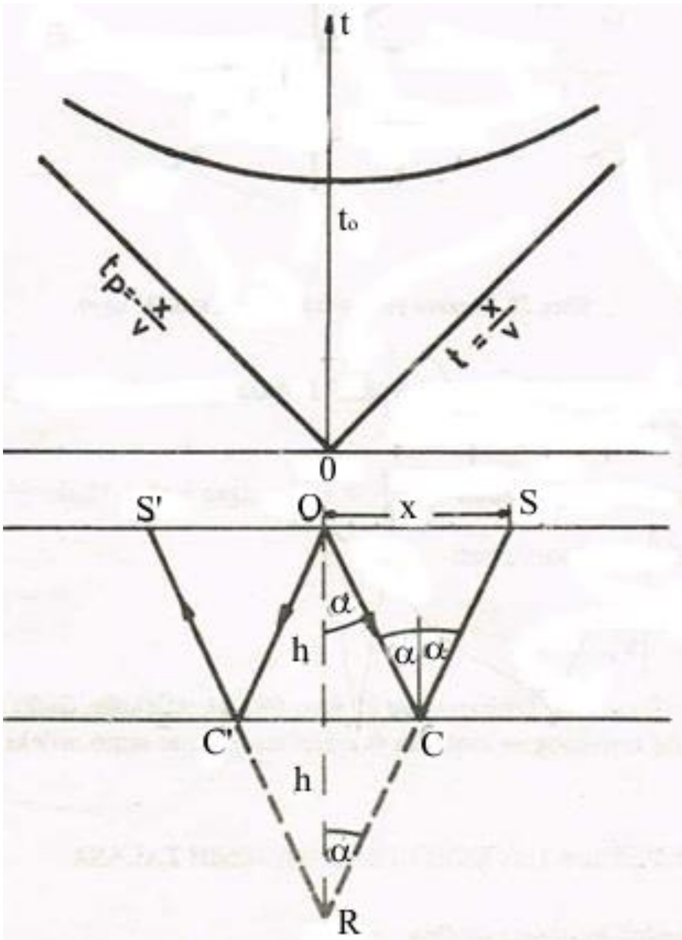

Fig. 5. The curve time – distance for reflected waves in a two – layer medium

By developing equation (11) in order, the following expression is obtained

$$t = \frac{2h}{V}\left[1 + \left(\frac{x}{2h}\right)^2\right]^{1/2} = t_0\left[1 + \left(\frac{x}{Vt_0}\right)^2\right]^{1/2}$$
$$= t_0\left[1 + \frac{1}{2}\left(\frac{x}{Vt_0}\right)^2 - \frac{1}{8}\left(\frac{x}{Vt_0}\right)^4\right] \tag{11a}$$

If we hold on the first two terms in equation (11a), the times t_1 and t_2, which correspond to the encounters of two successive rays reflected from the boundary of the two-layer medium, it can be expressed by equations

$$t_1 = t_0 + \frac{x_1^2\, t_0}{2V^2 t_0^2} \qquad \text{and} \qquad t_2 = t_0 + \frac{x_2^2\, t_0}{2V^2 t_0^2} \tag{12}$$

The difference $t_2 - t_1$ is

$$t_2 - t_1 = t_0 + \frac{x_2^2\, t_0}{2V^2 t_0^2} - t_0 - \frac{x_1^2\, t_0}{2V^2 t_0^2}$$

or

$$t_2 - t_1 = \frac{x_2^2 - x_1^1}{2V^2 t_0}$$

the difference in the time of occurrence of the reflected waves at the position points of the two receivers is determined by the expression

$$\Delta t = t_2 - t_1 = \frac{x_2^2 - x_1^1}{2V^2 t_0}$$

In the case when one geophone (receiver) is at the flash point, Δt is called the normal time difference and is expressed by the expression

$$\Delta t_N = \frac{x^2}{2V^2 t_0}$$

The layer from which the wave is reflected has an incline

If the boundary of the two layers from which the wave is reflected has an incline (Figure 6), the difference in the time of arrival of the wave at two successive points can be expressed by

$$\Delta t\, V = x \sin\varphi$$

$$\sin\varphi = \frac{\Delta t\, V}{x}$$

$$\varphi = arc\ \sin\frac{\Delta t\, V}{x}$$

If we connect the position of the receiver S with the point R, which is the image of the point O, the distance RS = OC + CS.

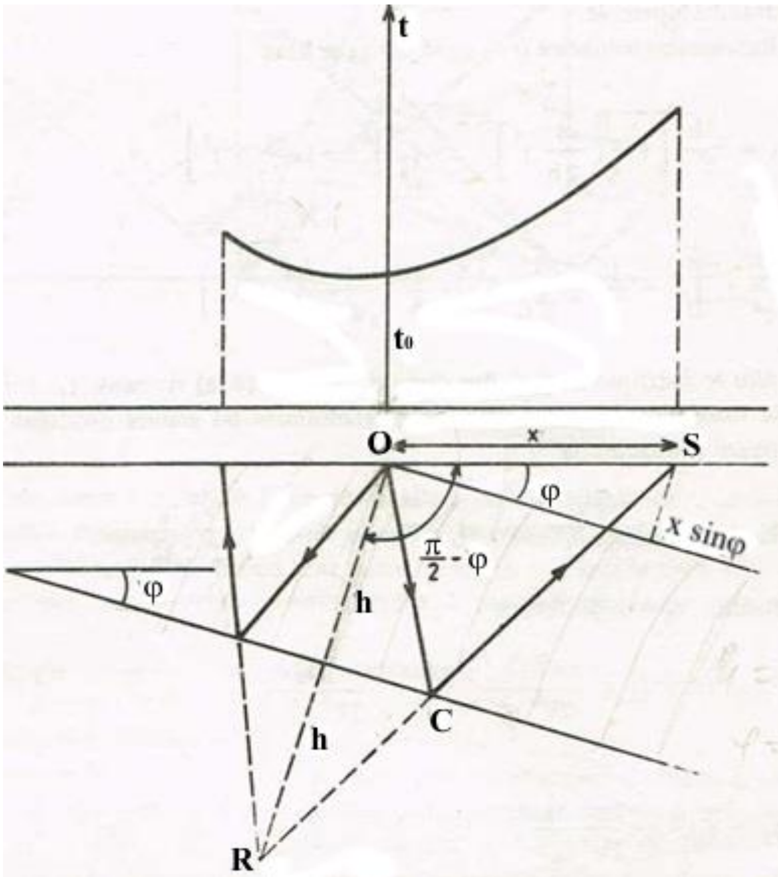

Fig. 6. Time - distance curve for reflected waves in a two-layer medium with an incline

Applying the cosine theorem, from the triangle ORS the following is obtained

$$V\,t = \overline{RS}$$

$$V^2\,t^2 = x^2 + 4h^2 - 4hx\left(\frac{\pi}{2} + \varphi\right) \tag{13}$$

$$V^2\,t^2 = x^2 + 4h^2 - 4hx\,\sin\varphi$$

If the equation (13) is divided with $(2h\cos\varphi)^2$ we obtain

$$\frac{V^2\,t^2}{(2h\,cos\varphi)^2} - \frac{(x + 2h\cos\varphi)^2}{(2h\,cos\varphi)^2} = 1 \tag{14}$$

This is the hyperbola equation.

If we set x = 0 in equation (14), the same expression is obtained for x as in the case of the horizontal layer. It should be noticed that the new value of h does not represent the thickness of the layer vertically below the flash point, but it is the length of the normal from point O to the boundary plane. [3]

If equation (14) is solved by t assuming that 2h is greater than x and develops into a row, based on the first two members of the row, it is obtained that

$$t = t_0 \left(1 + \frac{x^2 + 4hx \sin \varphi}{8h^2}\right)$$

If we express the times to the first S and the second S' geophone with

$$t_1 = t_0 \left(1 + \frac{x_1^2 + 4hx_1 \sin \varphi}{8h^2}\right)$$

$$t_2 = t_0 \left(1 + \frac{x_2^2 - 4hx_2 \sin \varphi}{8h^2}\right)$$

then the difference between the times Δt is

$$\Delta t = t_1 - t_2 = t_0 \left(\frac{\Delta x \sin \varphi}{h}\right) = \frac{2\Delta x}{V} \sin \varphi$$

$$\sin \varphi = V \frac{\Delta t}{\Delta x}$$

When testing downhill, the layer is assumed to be ip positive, and when testing uphill, the layer the angle φ is negative.

If the profile line is not in the direction of the incline of layers, on the seismic section the apparent inclines are expressed. In order to determine the true incline, it is necessary to perform tests along two mutually perpendicular profiles. From the components of the apparent inclines obtained along these profiles, the true incline and the orientation of the layer can be determined.

Wave refraction in a two-layer horizontal medium

It can be seen from Figure 7 that the time t of the wave propagation, from the excitation point to the reception point, after refraction at the boundary that divides the mean velocities V_1 and V_2 ($V_1 < V_2$), is represented by the equation

$$t = \frac{OA}{V_1} + \frac{AB}{V_2} + \frac{BS_2}{V_1} = \frac{AB}{V_2} + 2\frac{OA}{V_1}$$

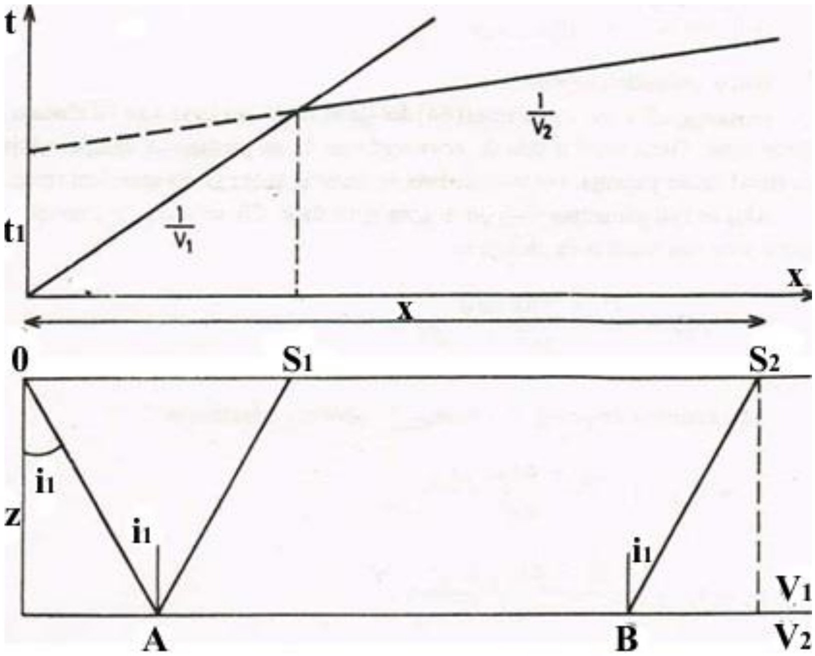

Fig. 7. Time – distance curve for refracted waves in a two – layer medium

By substituting the indicated distances with appropriate relations (Figure 7), it was obtained that

$$t = \frac{x - 2z\,\mathrm{tg}i}{V_2} + \frac{2z}{V_1 \cos i}$$

or

$$t = \frac{x}{V_2} + \frac{2z}{V_1 \cos i}\left(1 - \frac{V_1}{V_2}\sin i\right)$$

Knowing that sin $i = V_1 / V_2$, it is obtained that

$$t = \frac{x}{V_2} + \frac{2z}{V_1 \cos i}(1 - \sin^2 i)$$

or

$$t = \frac{x}{V_2} + \frac{2z \cos i}{V_1}$$

If $\frac{2z \cos i}{V_1}$ is denoted with t_1, it is obtained that

$$t = \frac{x}{V_2} + t_1$$

$$t_1 = \frac{2z\cos i}{V_1} \tag{15}$$

represents a section of the extended second branch of the hodochron on the t-axis.
From equation (15) it is obtained that

$$z = \frac{1}{2} \cdot \frac{V_1 t_1}{\cos i}$$

The case of wave refraction at several horizontal boundaries

The case of three layers with wave propagation velocities V_1, V_2, V_3 is considered, where the relation among the velocities is $V_1 < V_2 < V_3$ (Fig. 8).

As in the case of a two-layer medium, we will observe the time t required for wave to cross the path from the source to the receiver, refracting on the path at the boundary of the second and first layer, as well as the third and the second layer.

So, $$t = \frac{OA' + B'S_1}{V_1} + \frac{A'A" + B'B"}{V_2} + \frac{A"B"}{V_3}$$

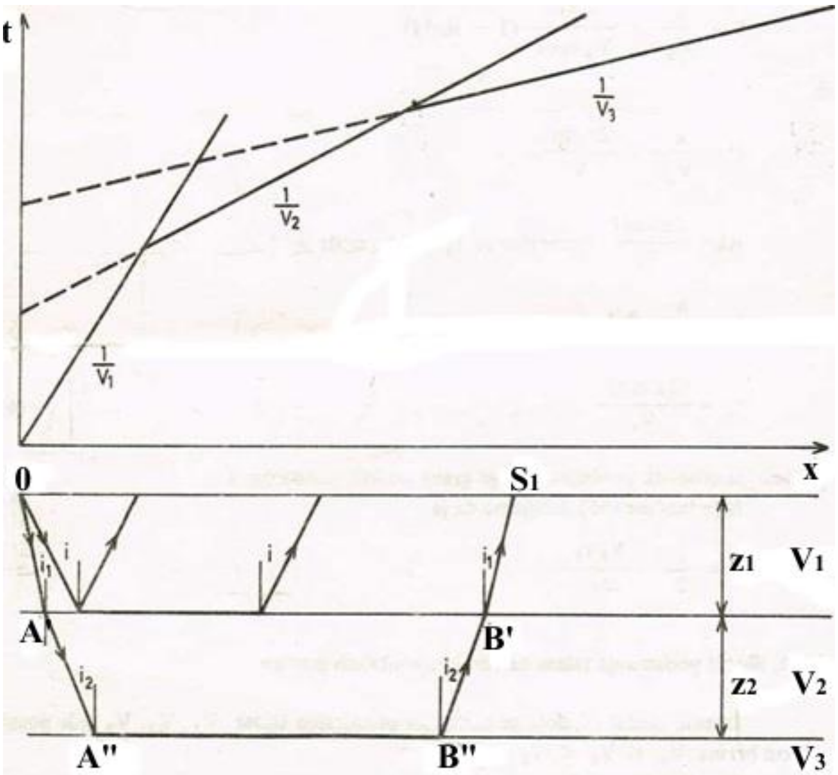

Fig. 8. Curve time – distance for refracted waves in a multi – layer medium

By substituting the lengths with the help of Figure 8 and arranging the obtained values, the following expression was obtained

$$t = \frac{x}{V_3} + \frac{2z_1}{V_1 \cos i_1}\left(1 - \frac{V_1}{V_3}\sin i_1\right) + \frac{2z_2}{V_2 \cos i_2}\left(1 - \frac{V_2}{V_3}\sin i_2\right)$$

Knowing that $\frac{V_1}{V_3} = \sin i_1$, and $\frac{V_2}{V_3} = \sin i_2$, it follows that

$$t = \frac{x}{V_3} + \frac{2z_1}{V_1}\cos i_1 + \frac{2z_2}{V_2}\cos i_2$$

The case of the layer with incline

Changing the inclination of the wave refraction boundary will affect the change in the time t of wave propagation from the source to the receiver. [4]

The case of a two – layer medium separated by boundary with inclination angle φ is represented in fig. 9.

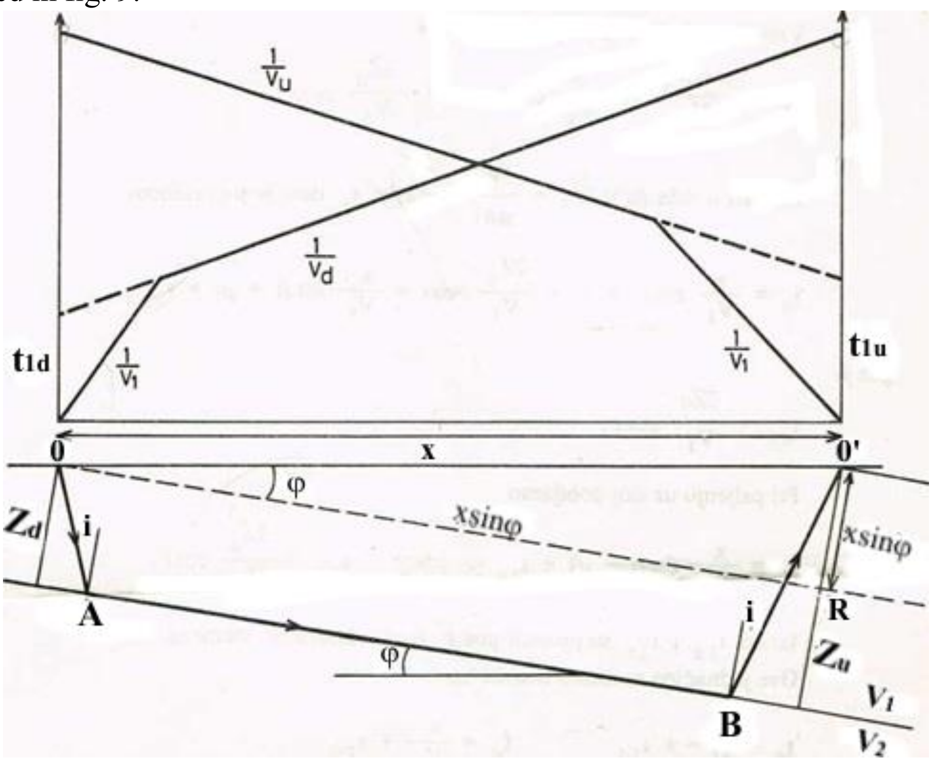

Fig. 9. The time - distance curve for refracted waves in a two-layer medium with an incline

The time required for the wave to reach receiver 0’ from source 0 is

$$t = \frac{OA + O'B}{V_1} + \frac{AB}{V_2}$$

or

$$t = \frac{Z_d + Z_u}{V_1 \cos i} + \frac{OR - (Z_d + Z_u)\mathrm{tg}i}{V_2}$$

or

$$t = \frac{x \cos\varphi}{V_2} + \frac{Z_d + Z_u}{V_1} \cos i$$

From the fig. 9 it is seen that

$$Z_u = Z_d + x \sin\varphi$$

The time t_d is obtained by ignition along the layer

$$t_d = \frac{x \cos\varphi}{V_2} + \frac{x}{V_1} \cos i \sin\varphi + \frac{2Z_d}{V_1} \cos i$$

Bearing in mind that $V_2 = \frac{V_1}{\sin i}$, the time t_d is given by the equation

$$t_d = \frac{x}{V_1} \sin(i + \varphi) + \frac{2Z_d}{V_1} \cos i = \frac{x}{V_1} \sin(i + \varphi) + t_{1d}$$

where

$$t_{1d} = \frac{2Z_d}{V_1} \cos i$$

When the ignition is upward, the layer it is obtained

$$t_u = \frac{x}{V_1} \sin(i - \varphi) + \sin(i + \varphi) + t_{1u}$$

where

$$t_{1u} = \frac{2Z_u}{V_1} \cos i$$

Expressions t_{1d} and t_{1u} are known as *intercept* of time.
These equations can be expressed as

$$t_d = \frac{x}{V_d} + t_{1d} \qquad t_u = \frac{x}{V_u} + t_{1u}$$

where

$$V_d = \frac{V_1}{\sin(i + \varphi)} \qquad\qquad V_u = \frac{V_1}{\sin(i - \varphi)} \tag{16}$$

V_d and V_u are apparent velocities downward and upward the layer.

For a profile tested from two directions as in Figure 9, equations (16) can be solved by φ and i.

$$\begin{aligned} i &= \frac{1}{2}\left[arc\ \sin\left(\frac{V_1}{V_d}\right) + \text{arc}\ \sin\left(\frac{V_1}{V_u}\right)\right] \\ \varphi &= \frac{1}{2}\left[arc\ \sin\left(\frac{V_1}{V_d}\right) - \text{arc}\ \sin\left(\frac{V_1}{V_u}\right)\right] \end{aligned} \tag{17}$$

The distances from the refractive surfaces Z_d and Z_u can be found using the intercept of time, using the previously given equations.

If both, i and V_1 are known, the true velocity V2 can be calculated from the hodochron.

$$V_2 = \frac{V_1}{\sin i} \tag{18}$$

The equations (16) can be written in the following form

$$\sin i\ \cos\varphi + \cos i\ \sin\varphi = \frac{V_1}{V_d}$$

$$\sin i\ \cos\varphi - \cos i\ \sin\varphi = \frac{V_1}{V_u}$$

By summing the above equations, it is obtained that

$$2\sin i\cos\varphi = V_1\left(\frac{1}{V_d} + \frac{1}{V_u}\right) \qquad\qquad \frac{\sin i}{V_1} = \frac{\frac{1}{V_d} + \frac{1}{V_u}}{2\cos\varphi}$$

Based on the equation (18), the above equation can be written as

$$\frac{1}{V_2} = \frac{\frac{1}{V_d} + \frac{1}{V_u}}{2\cos\varphi} \tag{19}$$

If the angle φ does not exceed the value of 10° it can be assumed that cos φ = 1 and equation (19) takes a simpler form

$$\frac{1}{V_2} = \frac{\frac{1}{V_d} + \frac{1}{V_u}}{2} \tag{20}$$

The error made by this simplification does not exceed 1.5 %.

3. Conclusion

The basic principle of all seismic methods is the controlled generation of elastic waves by a seismic source in order to obtain an image of the subsurface. Seismic waves are pulses of strain energy that propagate in solids and fluids. Seismic energy sources, whether at the Earth's surface or in shallow boreholes, produce wave types known as:

- body waves, where the energy transport is in all directions, and
- surface waves, where the energy travels along or near to the surface.

Two main criteria distinguish these wave types from each other – the propagation zones and the direction of ground movement relative to the propagation direction.

The velocity of seismic waves is the most fundamental parameter in seismic methods. It depends on the elastic properties as well as bulk densities of the media and varies with mineral content, lithology, porosity, pore fluid saturation and degree of compaction. P-waves have principally a higher velocity than S-waves. S-waves cannot propagate in fluids because fluids do not support shear stress.

During their propagation within the subsurface seismic waves are reflected, refracted or diffracted when elastic contrasts occur at boundaries between layers and rock masses of different rock properties (seismic velocities and/or bulk densities) or at man-made obstacles. The recording of seismic waves returning from the subsurface to the surface allows drawing conclusions on structures and the lithological composition of the subsurface. By measuring the traveltimes of seismic waves and determining their material specific velocities, a geological model of the subsurface can be constructed. [1]

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