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## COMPUTATIONAL METHODOLOGY IN DETERMINING SHADING AMONG PHOTOVOLTAIC PANELS

Biljana Chitkusheva Dimitrovska, Roman Golubovski,  
Hristina Spasevska, Jasmina Veta Buralieva

**Abstract.** Photovoltaic industries perform a lot of research and development in their technological aspects, all of which more or less tackle the efficiency aspects. This paper proposes methodology in support of shading estimation among the panels in a rectangular grid. Knowing the inter-shading in any moment throughout the year or within a certain day can allow simulation and analysis of the power issues and the energy production.

**Keywords:** model of shadow, power, electrical energy, land efficiency

### 1. Introduction

Energetic issues are fundamental for global development trends. The energy consumption and requirements raise along with the growing population and its consequential impact on the supporting industries. The nonrenewable (fossil) resources are depleting, and humanity is already focusing on the renewable resources, which can be defined as geothermal (resulting from earth's core activity); sun related (direct irradiation, wind), and gravitationally related (tide, water flow). Nuclear energy is also of current interest, but predominantly still in the ecologically dangerous domain of nuclear fission, as nuclear fusion is far from routine exploitation [1],[2].

The Sun is one of the most used renewable energy resources. The process of photovoltaic conversion is one of the ways to get electric energy. The photovoltaic conversion is a quantum-mechanical process where the sun's radiation is directly transformed into electric energy in devices called solar cells. The manufacturers of photovoltaic panels that consist of many solar cells provide a guarantee of their efficiency and the expected working life, which is about 20 years, but they can be used longer than 25 years with reduced efficiency [3].

One of the factors that significantly affects the efficiency of panels is shade. In previous research, the shadow effect has been described in terms of how it affects the characteristics of photovoltaic panels, without taking into account the surface efficiency of a given land, which in the future would be an important element in the design of photovoltaic power plants. Due to insufficiently researched area, the contribution of this paper is aimed at analyzing the effect of the shadow and its impacts in the terms of the efficiency of the occupied land and the advantages of the produced energy. The shadow model is implemented in a model for power and energy calculation. The model changes parameters that are of major interest: the angle of inclination  $\beta$ , the distances between panels in x and y direction ( $r_x$ ,  $r_y$ ), the geographical position, day, and time. This type of analysis is necessary in order to obtain a greater efficiency of land utilization, i.e., optimal produced electric energy of a photovoltaic plant.

This paper is organized as follows. In Section 2 a model of geometry of the solar position and the main solar angles that describe the solar position are presented. In Section 3 a model of shading among nine photovoltaic panels is developed. The implementation of the shadow model in an application tool for calculating power and energy is given in Section 4. Section 5 concludes this paper.



## 2. Model of geometry of the Sun's position

It is both necessary and of essential meaning to know the correct position of the Sun at the given moment of a day, for developing a model of shadow. The moving of the Sun, the solar angles and all the basic equations necessary for modelling the solar position are described in this chapter, based on figure 2.1. The altitude angle  $\alpha$  and the angle of the solar azimuth  $z$  are the solar angles which are most important in the model that follows [1],[4].

The altitude angle  $\alpha$  is the angle between the direction of the solar rays and the horizontal plain. This angle is in correlation with the angle of the solar zenith  $\Phi$ , which is an angle between the direction of the solar rays and the vertical plain. The correlation between these two angles is given with the following equation:

$$\Phi + \alpha = \pi/2 = 90^\circ \quad (2.1)$$

The mathematical expression for the altitude angle  $\alpha$  is the following:

$$\sin(\alpha) = \cos(\Phi) = \sin(L)\sin(\delta) + \cos(L)\cos(\delta)\cos(h) \quad (2.2)$$

where  $L$  is the local geographic latitude,  $\Phi$  is the zenith angle,  $\delta$  is the angle of declination, and  $h$  is the hour angle.

The angle of the solar azimuth  $z$  is the angle of the solar rays measured on the horizontal plain from the south for the north hemisphere, or from the north for the south hemisphere. The west is considered with positive value.

The mathematical expression for the angle of a solar azimuth  $z$  is the following:

$$\sin(z) = \frac{\cos(\delta)\sin(h)}{\cos(\alpha)} \quad (2.3)$$

This equation is correct if the condition  $\cos(h) > \tan(\delta)/\tan(L)$  is fulfilled. If the condition is not fulfilled, then the Sun will be between the east-west line. The angle of the solar azimuth can be presented as  $-\pi + |z|$  in the morning hours, but it can be presented as  $\pi - z$  in the afternoon hours.

At the solar noon, according to definition, the Sun is exactly on the meridian, which contains the north-south line. So, the solar azimuth has  $0^\circ$  value. That is why, the altitude angle  $\alpha_n$  at noon is:

$$\alpha_n = 90^\circ - L + \delta \quad (2.4)$$

where  $L$  is the geographic latitude, and  $\delta$  is the angle of solar declination.

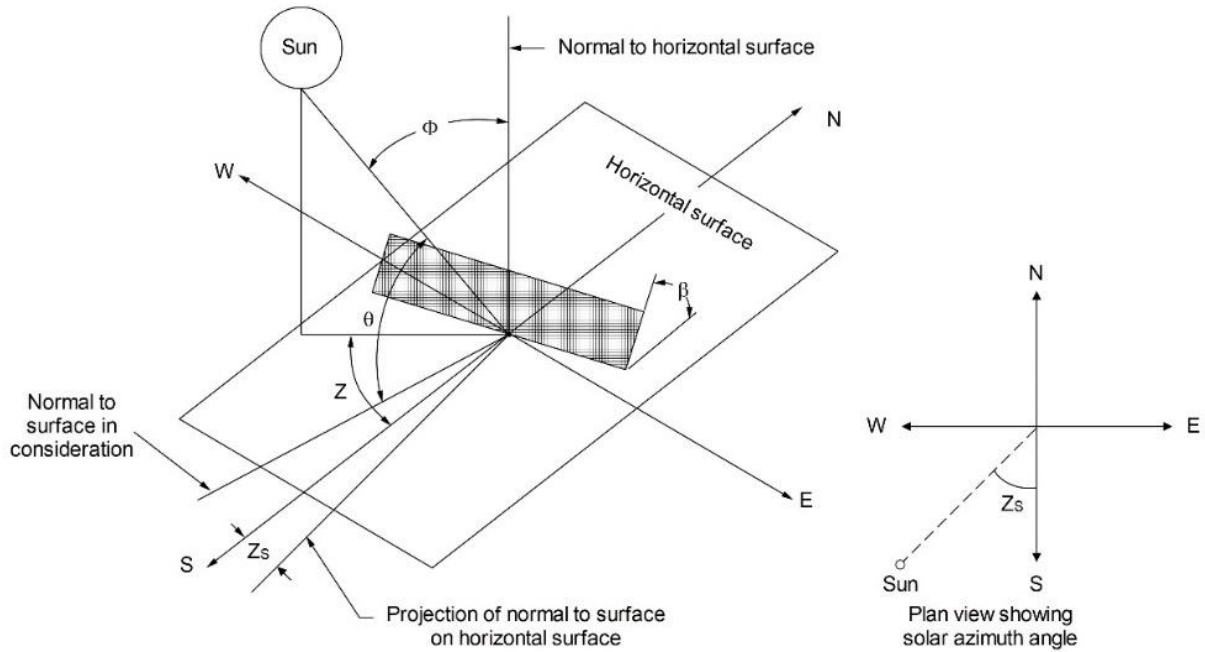
The incidence angle  $\theta$  is evaluated after the determination of the solar angles ( $\alpha, z$ ) and all parameters which are necessary for describing the solar position.

The incidence solar angle  $\theta$  is the angle between the solar rays and the vertical on the surface. The incidence angle  $\theta$  and the zenith angle  $\Phi$  have the same value for the horizontal plain. The equation (2.5) gives the relation between the incidence angle  $\theta$  and the other important parameters in the solar geometry: geographic latitude  $L$ , angle of solar declination  $\delta$ , angle of inclination on the surface in relation to the horizontal surface  $\beta$ , hour angle  $h$  and angle of azimuth of the orientation on surface  $Z_s$ .

$$\begin{aligned}
 \cos(\theta) = & \sin(L) \sin(\delta) \cos(\beta) - \cos(L) \sin(\delta) \sin(\beta) \cos(Z_s) + \\
 & + \cos(L) \cos(\delta) \cos(h) \cos(\beta) + \sin(L) \cos(\delta) \cos(h) \sin(\beta) \cos(Z_s) \\
 & + \cos(\delta) \sin(h) \sin(\beta) \sin(Z_s)
 \end{aligned}
 \tag{2.5}$$

### 3. Model of shadow

The model of shadow determines the inter-shading among the photovoltaic panels in their grid. This model is made for a photovoltaic grid of 9 panels. The photovoltaic panels are put on the flat field fixed on the ground, arranged into three rows with a distance  $r_y$  among them and three columns with a distance  $r_x$  among them presented in Figure 3.1.



*Fig. 2.1 Diagram of solar angles*

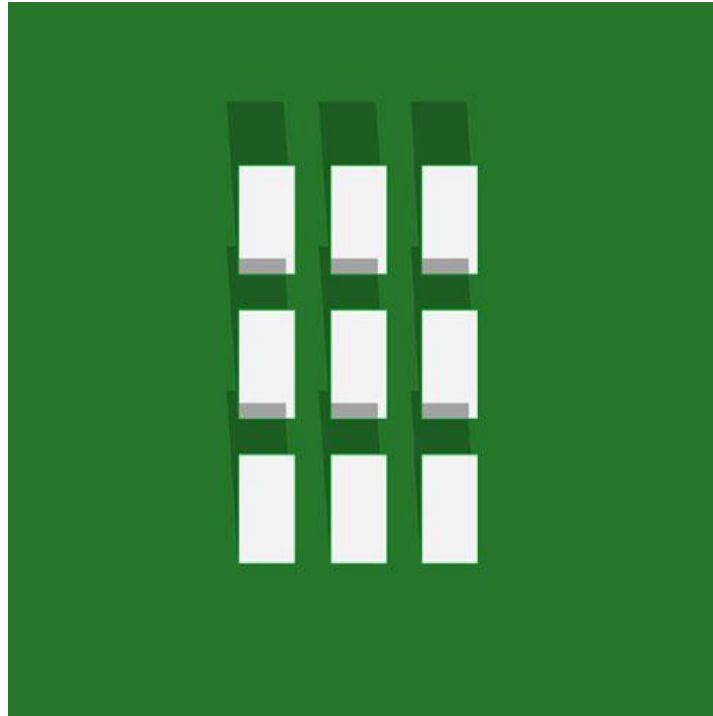


Figure 3.1 PV grid with 3x3 photovoltaic panels

Photovoltaic panels are characterized with the following parameters: angle of inclination  $\beta$ , width of the panel  $P_x$  and length of the panel  $P_y$ . If the panel thickness is neglected, the shadow which falls on a specific panel has a rectangle form. The proposed model of shadow calculates the shading on every panel in the defined grid. The model considers two configurations:

1.  $P_x \leq r_x$ , the horizontal distance between the panels is greater than the panel width,
2.  $P_x > r_x$ , the distance is bigger than the panel width.

If there is no inclination east-west, and the sun rays are considered parallel, then the width of the shadows behind panels can be approximated equal to the panel width  $P_x$ . The horizontal shadow overlap  $Psh_x$  is calculated according to the specific configuration and it depends on the azimuth displacement  $\Delta x$ , which always has an absolute value moving from  $-90^\circ$  to  $+90^\circ$  during daylight. The azimuth displacement  $\Delta x$  is calculated according to Fig. 3.2:

$$\Delta x = (P_{proj\_y} + r_y) \cdot \tan(Z) \quad (3.1)$$

$$P_{proj\_y} = P_y \cdot \cos\beta \quad (3.2)$$

$$Z = z - Z_s \quad (3.3)$$

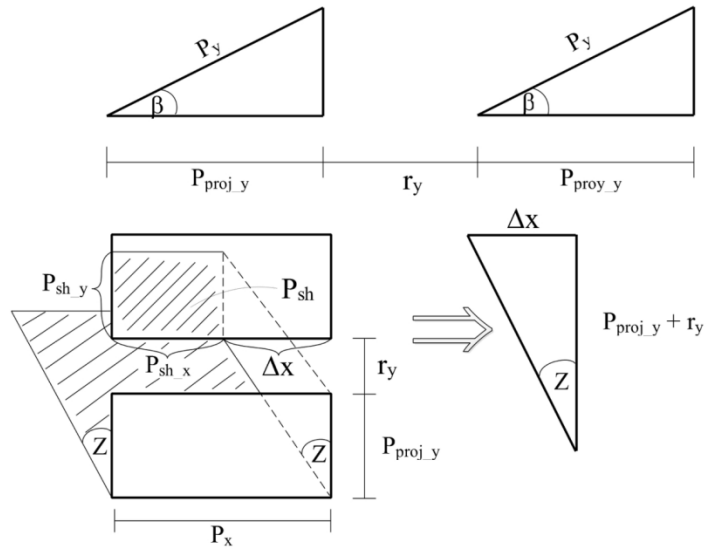


Figure 3.2 Elements of a shadow

where  $z$  is the solar azimuth, and  $Z_s$  is the azimuth angle of the panel orientation. The effective area that is under shadow of a specific panel is calculated as:

$$P_{sh} = P_{sh,x} \cdot P_{sh,y} \quad (3.4)$$

In the first configuration (Fig. 3.3), the panel width  $P_x$  is less or equal to the distance between the columns  $r_x$ . Only one type of shadows (named "single") appears in this configuration.

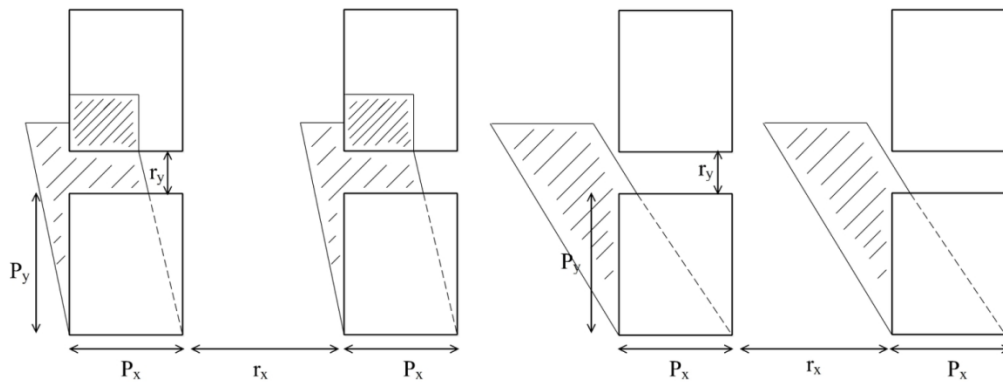


Figure 3.3 Presentation of shadows in configuration 1

In the second configuration (Fig. 3.4), the shadow of one panel can fall on one or two panels behind, resulting in a "single" or a "double" shadow.

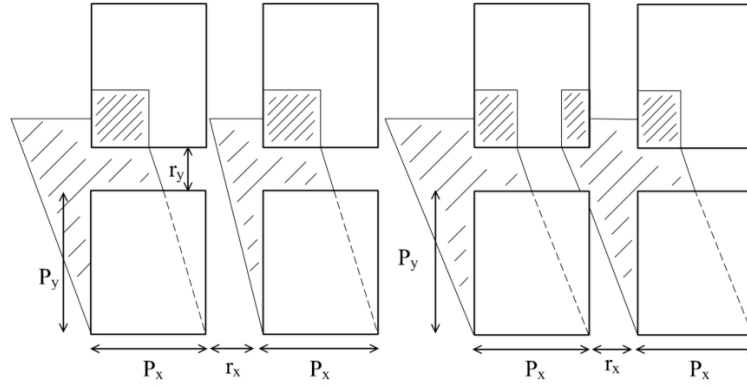


Figure 3.4 Presentation of shadows in configuration 2

The cases of both configurations are visually simulated in CAD software. Three, two, one or no columns from the grid can be shaded depending on the shadow movement (calculated azimuth displacement  $\Delta x$ ) [2],[5].

All described cases for configuration 1 and configuration 2 are covered by the algorithm that determines the number of shaded panels as well as their corresponding shaded surfaces. The algorithm is described with the flowchart in figure 3.5. This algorithm can be generalized for a grid of photovoltaic panels composed of  $M \times N$  panels.

The algorithm for determining shaded surfaces gives the relations for each shaded surface  $P_{sh}$  or  $S_{dif}$  appropriate for cases 1 to 8 for configuration 1 and cases 1 to 9 for configuration 2. It results in the equations (3.5) and (3.6) for configuration 1 and configuration 2 accordingly.

$$S_{dif} = P_{sh} = \left\{ \begin{array}{l} 6 \cdot [(P_x - \Delta x) \cdot P_{shy}], \text{ case 1} \\ 0, \text{ case 2} \\ 4 \cdot [(\Delta x - r_x) \cdot P_{shy}], \text{ case 3} \\ 4 \cdot [(2 \cdot P_x + r_x - \Delta x) \cdot P_{shy}], \text{ case 4} \\ 0, \text{ case 5} \\ 2 \cdot [(\Delta x - P_x - 2 \cdot r_x) \cdot P_{shy}], \text{ case 6} \\ 2 \cdot [(3 \cdot P_x + 2 \cdot r_x - \Delta x) \cdot P_{shy}], \text{ case 7} \\ 0, \text{ case 8} \end{array} \right\} \quad (3.5)$$

$$S_{dif} = P_{sh} = \left\{ \begin{array}{l} 6 \cdot [(P_x - \Delta x) \cdot P_{shy}], \text{ case 1} \\ 4 \cdot [(P_x - r_x) \cdot P_{shy}] + 2 \cdot [(P_x - \Delta x) \cdot P_{shy}], \text{ case 2} \\ 4 \cdot [(\Delta x - r_x) \cdot P_{shy}], \text{ case 3} \\ 4 \cdot [(2 \cdot P_x + r_x - \Delta x) \cdot P_{shy}], \text{ case 4} \\ 2 \cdot [(P_x - r_x) \cdot P_{shy}] + 2 \cdot [(2 \cdot P_x + r_x - \Delta x) \cdot P_{shy}], \text{ case 5} \\ 2 \cdot [(\Delta x - P_x - 2 \cdot r_x) \cdot P_{shy}], \text{ case 6} \\ 2 \cdot [(3 \cdot P_x + 2 \cdot r_x - \Delta x) \cdot P_{shy}], \text{ case 7} \\ 2 \cdot [(3 \cdot P_x + 2 \cdot r_x - \Delta x) \cdot P_{shy}], \text{ case 8} \\ 0, \text{ case 9} \end{array} \right\} \quad (3.6)$$

The surface exposed to direct solar radiation  $S_{dir}$  is the remaining difference from the total panels surface:

$$S_{dir} = 9 \cdot P_x \cdot P_y - S_{dif} \quad (3.7)$$

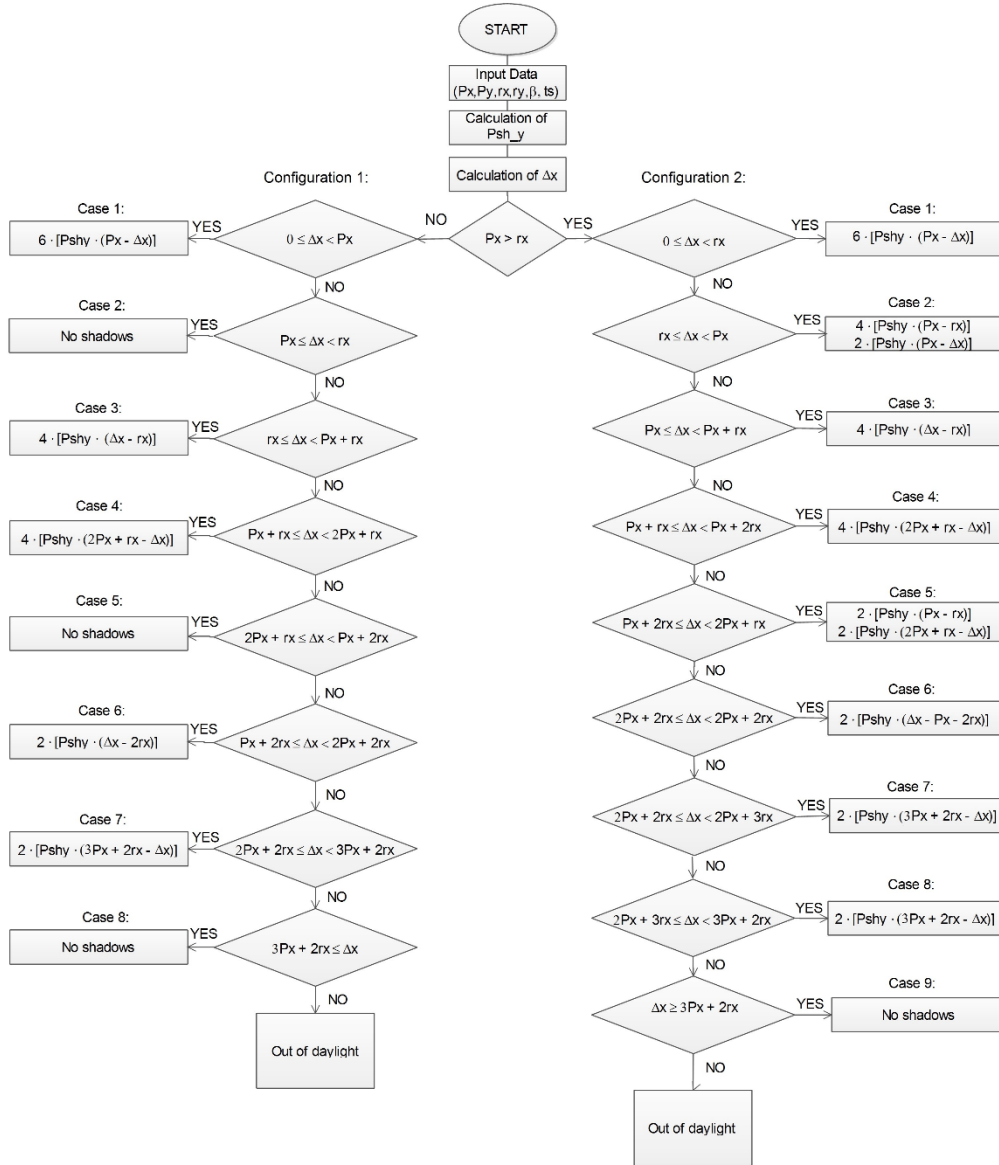


Figure 3.5 Flowchart for shadow determination

#### 4. Results from simulations and discussion

The proposed model of shadow is verified with commercial CAD software for visualization - PV Sol and ARCHICAD. Randomly chosen moments of date and time are evaluated and visualized in the CAD, and they all perfectly match the corresponding model's configuration and case.

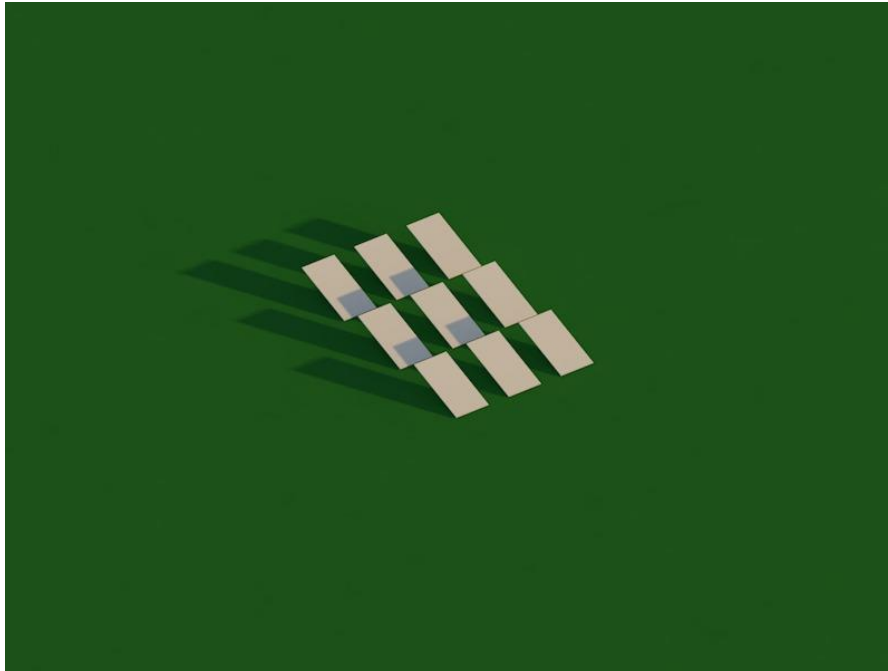
The following are examples of cases given at specific moment in a given day with a visual representation of the same in ARCHICAD. An example analysis is given for 10<sup>th</sup> January (N=10) with the following parameters:

- geolocation: Skopje, Macedonia: *latitude* = 42° and *longitude* 21.43°
- grid geometry:  $P_x = 1$  m,  $P_y = 1.64$  m,  $r_x = 0.5$ m,  $r_y = 0.5$ m, panel azimuth

$az = 0^\circ$ , tilt angle  $\beta = 15^\circ$

- grid occupied surface:  $(3 \cdot P_x + 2 \cdot r_x) \times (3 \cdot P_y \cdot \cos\beta + 2 \cdot r_y)$

Fig. 4.1 gives the visualization of the photovoltaic grid at 8:30h in the morning.



*Figure 4.1 Case 4, Configuration 2 (8:30h)*

Shadows fall behind on the own and neighboring column. There are 4 single shadows.

Based on equation (3.1), the azimuth displacement is calculated  $\Delta x = 1.86$  m. According to the shadow model,  $\Delta x$  belongs to case 4, Configuration 2 ( $P_x > r_x$ ). The total shadow surface  $S_{dif} = 1.52$  m<sup>2</sup> or 0.32 m<sup>2</sup> per photovoltaic panel. Each of the four panels is shaded with 20% of its total area, which is 1.64 m<sup>2</sup>.

In Fig. 4.2 the visualization of the photovoltaic grid at 9:30h in the morning is given. At this moment, shadows fall behind on the neighboring columns. Based on equation (3.1), the azimuth displacement is calculated  $\Delta x = 2.28$  m. According to the shadow model,  $\Delta x$  belongs to case 5, Configuration 2 ( $P_x > r_x$ ). The total shadow surface  $S_{dif} = 1.11$  m<sup>2</sup> or 7.5% is shaded of its total area, which is 14.76 m<sup>2</sup>.

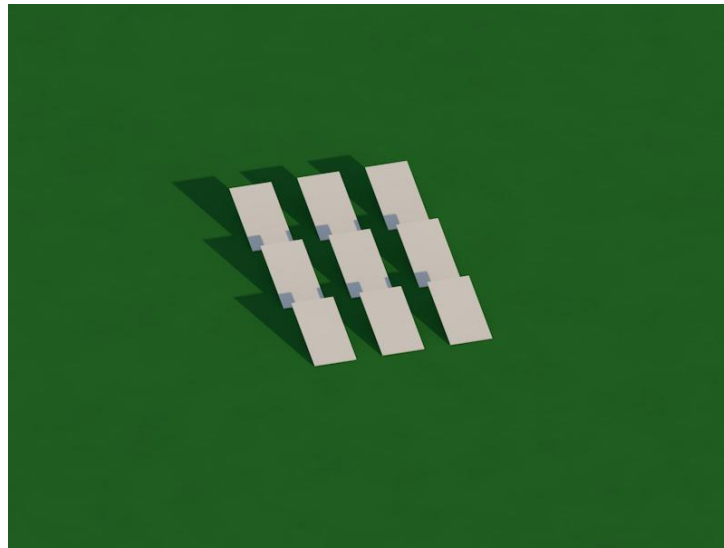


Figure 4.2 Case 5, Configuration 2 (9:30 h)

In Fig. 4.3 the visualization of the photovoltaic grid at 14.30h in the afternoon is given. At this moment, shadows fall behind on the neighboring columns. At this moment on 10<sup>th</sup> January, the azimuth displacement is calculated  $\Delta x = 2.28$  m. According to the shadow model,  $\Delta x$  belongs to case 5, Configuration 2 ( $P_x > r_x$ ). The total shadow surface  $S_{dif} = 1.2$  m<sup>2</sup> or 8 % is shaded of its total area, which is 14.76 m<sup>2</sup>.

## 5. Conclusion

The proposed methodology in determining shading among the panels in the PV grid provides an accurate way of calculating the total directly irradiated and the total shaded surfaces. This can then allow the implementation of a PV conversion energy model for calculating the grid power and energy production for certain time intervals within the year.

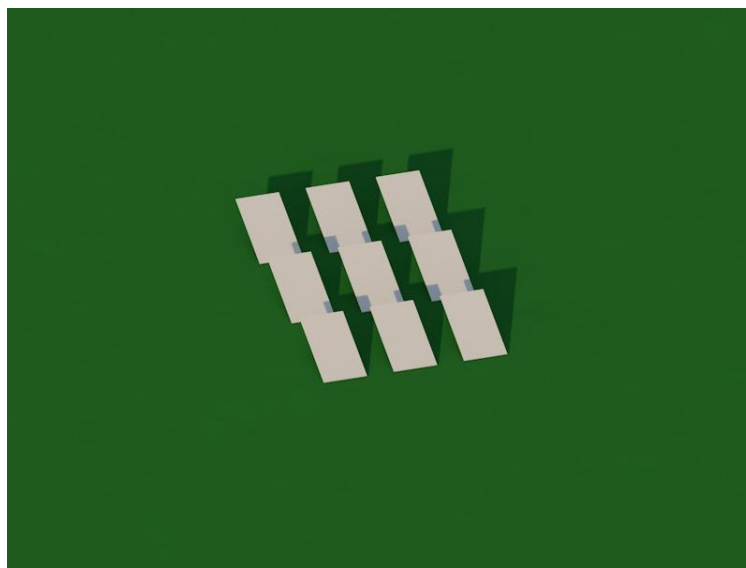


Figure 4.3 Case 5, Configuration 2 (14.30 h)



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