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The Appendix

In honor of the first Doctor of Mathematical Sciences Acad. Blagoj Popov, a mathematician dedicated to differential equations, the idea of holding the "Day of Differential Equations" was born, prompted by Prof. Ph.D. Boro Piperevski, Prof. Ph.D. Borko Ilievski, and Prof. Ph.D. Lazo Dimov. Acad. Blagoj Popov presented his doctoral dissertation on 05.05.1952 in the field of differential equations. This is the main reason for holding the " Day of Differential Equations" at the beginning of May.

This year on May 7th, the "Day of Differential Equations" was held for the sixth time under the auspices of the Faculty of Computer Sciences at "Goce Delcev" University in Stip and Dean Prof. Ph.D. Cveta Martinovska - Bande, organized by Prof. Ph.D. Biljana Zlatanovska. The event was organized online via the platform Microsoft Teams and with the selfless help and support of Prof. Ph.D. Natasa Stojkovik, Ass. Prof. Ph.D. Limonka Koceva Lazarova, Ass. Prof. Ph.D. Marija Miteva, Ass. Prof. Ph.D. Mirjana Kocaleva, Ass. Prof. Ph.D. Aleksandra Stojanova.

The participants of this event were:

1. Prof. Ph.D. Boro Piperevski, Prof. Ph.D. Sanja Atanasova and Stefan Boshkovski (student) from the Faculty of Electrical Engineering and Information Technology at Ss. Cyril and Methodius, University in Skopje;
2. Prof. Ph.D. Aleksa Malcheski from the Faculty of Mechanical engineering at Ss. Cyril and Methodius, University in Skopje;
3. Prof. Ph.D. Slagjana Brsakoska from the Faculty of Natural Sciences and Mathematics at Ss. Cyril and Methodius, University in Skopje;
4. Prof. Ph.D. Natasa Stojkovik, Prof. Ph.D. Martin Lukarevski, Ass. Prof. Ph.D. Limonka Koceva Lazarova, Ass. Prof. Ph.D. Marija Miteva, Ass. Prof. Ph.D. Mirjana Kocaleva, Ass. Prof. Ph.D. Aleksandra Stojanova, Ass. Prof. Ph.D. Jasmina Buralieva Veta, Ass. Prof. Ph.D. Elena Karamazova, Prof. Ph.D. Biljana Zlatanovska from the Faculty of Computer Sciences at "Goce Delcev" University in Stip.

Acknowledgments to Prof. Ph.D. Boro Piperevski, Prof. Ph.D. Borko Ilievski and Prof. Ph.D. Lazo Dimov for the wonderful idea and the successful realization of the event this year and in previous years.

Acknowledgments to the Dean of the Faculty of Computer Sciences, Prof. Ph.D. Cveta Martinovska - Bande for her overall support of the organization and implementation of the "Day of Differential Equations".

The papers that emerged from the "Day of Differential Equations" are in the appendix to this issue of BJAMI.

UDC: 517.926:517.982.22

SPACE OF SOLUTIONS OF A LINEAR DIFFERENTIAL EQUATION OF THE SECOND ORDER AS 2-NORMED SPACE

Slagjana Brsakoska[1], Aleksa Malcheski[2]

Abstract. In [1] the notion of 2-norm is introduced, and in [2] an equivalent definition for 2-norm, and with its help the number of axioms is reduced to the equal number as in the regular norm of a vector space. This paper gives the connection between *n*-norms and the solutions of a differential equation of the second order when the solutions are in a trigonometric shape in the sense of the solutions of prof. d-r D. Dimitrovski.

1. INTRODUCTION

The notion of a 2-norm is introduced in [1]. Let X be a vector space over the field Φ and $\dim X > 1$. Under the term field Φ we mean a field of real numbers, and a field of complex numbers as well.

Definition. Let X be a vector space over the field Φ and $\dim X > 1$. The mapping $\|\ ,\ \|: X^2 \to$ which satisfies the conditions

i) $\|x_1, x_2\| = 0$, if and only if $\{x_1, x_2\}$ is a linearly dependent set in X

ii) $\|x_1, x_2\| = \|x_2, x_1\|$ for any $x_1, x_2 \in X$.

iii) $\|\alpha x_1, x_2\| = |\alpha|\,\|x_1, x_2\|$, for any scalar α and for any $x_1, x_2 \in X$,

iv) $\|x_1 + x_1^{'}, x_2\| \le \|x_1, x_2\| + \|x_1^{'}, x_2\|$ for any $x_1, x_1^{'}, x_2 \in X$.

is called the 2-norm of the vector space X, and the ordered pair $(X, \|\cdot,\cdot\|)$ is called a 2-normed space.

It can be shown that the 2-norm is nonnegative. Also, as a corollary from the definition of the 2-norm, there is the following property:

Lemma. *Let X be a 2-normed space. For every $x, y \in X$ and a scalar $\alpha \in \Phi$ the following equality holds*

$$\|x, y\| = \|x, y + \alpha x\|.$$

Applying this lemma, we get that for every matrix $A \in M_2(\Phi)$ and for every $(x_1, x_2) \in X^2$ the following equality is fulfilled

$$\|A \cdot (x_1, x_2)^T\| = |\det A|\,\|x_1, x_2\|,$$

where $A \cdot (x_1, x_2)^T$ is an operation the same as the operation multiplication of a matrix with a vector column.

In [2], with the help of the last equation, an equivalent definition for the 2-norm is introduced.

Definition. Let X be a vector space over the field Φ and $\dim X > 1$. The mapping $\|\ ,\ \|: X^2 \to$ which satisfies the conditions

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(P1) If $\| x, y \| = 0$ then the set of vectors $\{ x, y \}$ is a linearly dependent set.

(P2) $\left\| A(x, y)^T \right\| = |\det A| \cdot \| x, y \|$, for every $x, y \in X$ and $A \in M_2(\Phi)$.

(P3) $\| x + x', y \| \le \| x, y \| + \| x', y \|$.

is called the 2-norm, and the ordered pair $(X, \|\cdot,\cdot\|)$ is called the 2-normed space.

Most of the papers on functionals in 2-normed spaces refer to the class of bilinear functionals. In [3] an analysis is made of one of its subclasses that best corresponds to the properties of the 2-norm, and that is the class of alternative 2-forms (skew-symmetric linear 2-forms).

This definition is even more justified by the attempt to introduce a class of limited functionals in a 2-normed space. By analogy, with the definition of a norm of linear functional, the bilinear functional is limited if

$$\sup_{\{x,y\} \text{ e lnzm}} \frac{|\Lambda(x,y)|}{\| x, y \|} < +\infty .$$

If the bilinear functional Λ is limited and $K = \sup\limits_{\{x,y\} \text{ e lnzm}} \dfrac{|\Lambda(x,y)|}{\| x, y \|}$, then from $\| x, y \| = 0$ we get that

$$| \Lambda(x,y) | \le K \| x, y \| = 0 ,$$

$|\Lambda(x,y)| = 0$, i.e., $\Lambda(x,y) = 0$. Therefore, for $\{x,y\}$ - a linearly dependent set, we have that $\Lambda(x,y) = 0$. So, in the class of limited bilinear functionals regarding the 2-norm belong those for which the pairs (x,y) for which $\{x,y\}$ is a linearly dependent set are annulated. For these reasons and the linearity, instead of limited bilinear functionals, we will consider the class of alternative linear functionals. The alternative 2-linear forms are called 2-linear functionals. Therefore, we will consider the class of alternative linear 2-forms. Its definition can be written in the following form.

Definition. Let X be a vector space with $\dim X > 1$. The mapping $\Lambda : X \times X \to \Phi$ which satisfies the conditions

$$\Lambda(x+y, z) = \Lambda(x,z) + \Lambda(y,z) ,$$

$$\Lambda(A(x,y)^T) = (\det A)\Lambda(x,y) ,$$

is called a 2-linear functional.

It is not hard to check that this definition is equivalent to the definition for an alternative 2-linear form.

2. *n*-NORM IN THE SPACE OF LIMITED ANALYTIC FUNCTIONS

Let the space of limited analytic functions $A(D)$ be considered as a set of functions determined on the set $D = (a,b) \times$. The functions are analytic and at the same time they are limited. For two arbitrary functions f and g from the set of all analytic functions determined on D, we define a function $\| \bullet, \bullet \| : A(D) \times A(D) \to$ determined with

$$\| f,g \| = \sup_{x\in(a,b)} \left| \begin{vmatrix} f(x) & g(x) \\ f(y) & g(y) \end{vmatrix} \right| .$$

It is clear that the function $\| \bullet,\bullet \|$ is well defined on the set $A(D)\times A(D)$. From the fact that f and g are analytic, they are continuous functions. Additionally, because they are limited functions, we get that

$$\left| \begin{vmatrix} f(x) & g(x) \\ f(y) & g(y) \end{vmatrix} \right| = | f(x)g(y) - g(x)f(y) | \leq | f(x)g(y) | + | g(x)f(y) | \leq$$
$$\leq M_1M_2 + M_1M_2 < 2M_1M_2 < +\infty$$

where we directly get that the function $\| \bullet,\bullet \|$ is well defined, i.e.

$$\| f,g \| = \sup_{x\in(a,b)} \left| \begin{vmatrix} f(x) & g(x) \\ f(y) & g(y) \end{vmatrix} \right| \leq 2M_1M_2 < +\infty .$$

We will consider the general properties of the function $\| \bullet,\bullet \|$.

a) $\| f,g \| = 0$ if and only if $\{f,g\}$ is a linearly dependent set.

Let (f,g) be an ordered pair in which $\{f,g\}$ is a linearly dependent set. Therefore, there is a scalar $\alpha \in$, such that $f = \alpha g$ for every $x \in (a,b)$. Then we have that

$$\left| \begin{vmatrix} f(x) & f(y) \\ g(x) & g(y) \end{vmatrix} \right| = \left| \begin{vmatrix} f(x) & f(y) \\ (\alpha f)(x) & (\alpha f)(y) \end{vmatrix} \right| = | \alpha f(x)f(y) - \alpha f(x)f(y) | = | 0 | = 0 ,$$

so, it is clear that

$$\sup_{x\in(a,b)} \left| \begin{vmatrix} f(x) & f(y) \\ g(x) & g(y) \end{vmatrix} \right| = \sup_{x\in(a,b)} \left| \begin{vmatrix} f(x) & f(y) \\ (\alpha f)(x) & (\alpha f)(y) \end{vmatrix} \right| = \sup_{x\in(a,b)} | \alpha f(x)f(y) - \alpha f(x)f(y) | = \sup_{x\in(a,b)} | 0 | = 0$$

Therefore, we have that $\| f,g \| = 0$.

Vice versa, let us assume that $\| f,g \| = 0$. Then, according to the definition of the function $\| \bullet,\bullet \|$, we have that

$$\left| \begin{vmatrix} f(x) & f(y) \\ g(x) & g(y) \end{vmatrix} \right| \leq \sup_{x\in(a,b)} \left| \begin{vmatrix} f(x) & f(y) \\ g(x) & g(y) \end{vmatrix} \right| = \| f,g \| = 0 ,$$

i.e., $| f(x)g(y) - g(x)f(y) | = 0$, and from the properties of real numbers we have that $f(x)g(y) - g(x)f(y) = 0$. If $f(x) = 0$ for any real number $x \in (a,b)$, then the set $\{f,g\}$ is a set in which one function is the zero function, therefore it is a linearly dependent set. If there exists a scalar $x \in (a,b)$ such that $f(x\ ) \neq 0$, then $g(y) = \frac{g(x\ )}{f(x\ )} f(y)$, for any $y \in (a,b)$. Therefore, $g = \alpha f$, where $\alpha = \frac{g(x\ )}{f(x\ )}$. So, in any case the set $\{f,g\}$ is a linearly dependent set.

b) Let $A \in M_2(\Phi)$ (in this case $\Phi =$) and (f,g) be an arbitrary ordered pair of two functions from the space of limited analytic functions.

$$\|A(f,g)\| = \left\|\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}(f,g)\right\| = \|(a_{11}f + a_{12}g, a_{21}f + a_{22}g)\| =$$

$$= \sup_{x\in(a,b)} \left|\begin{vmatrix} a_{11}f(x)+a_{12}g(x) & a_{11}f(y)+a_{12}g(y) \\ a_{21}f(x)+a_{22}g(x) & a_{21}f(y)+a_{22}g(y) \end{vmatrix}\right| =$$

$$= \sup_{x\in(a,b)} \left|\begin{vmatrix} a_{11}f(x)+a_{12}g(x) & a_{21}f(x)+a_{22}g(x) \\ a_{11}f(y)+a_{12}g(y) & a_{21}f(y)+a_{22}g(y) \end{vmatrix}\right| =$$

$$= \sup_{x\in(a,b)} \left|\begin{vmatrix} a_{11}f(x) & a_{21}f(x)+a_{22}g(x) \\ a_{11}f(y) & a_{21}f(y)+a_{22}g(y) \end{vmatrix} + \begin{vmatrix} a_{12}g(x) & a_{21}f(x)+a_{22}g(x) \\ a_{12}g(y) & a_{21}f(y)+a_{22}g(y) \end{vmatrix}\right| =$$

$$= \sup_{x\in(a,b)} \left|\begin{vmatrix} a_{11}f(x) & a_{21}f(x) \\ a_{11}f(y) & a_{21}f(y) \end{vmatrix} + \begin{vmatrix} a_{11}f(x) & a_{22}g(x) \\ a_{11}f(y) & a_{22}g(y) \end{vmatrix} + \begin{vmatrix} a_{12}g(x) & a_{21}f(x) \\ a_{12}g(y) & a_{21}f(y) \end{vmatrix} + \begin{vmatrix} a_{12}g(x) & a_{22}g(x) \\ a_{12}g(y) & a_{22}g(y) \end{vmatrix}\right| =$$

$$= \sup_{x\in(a,b)} \left| a_{11}a_{21}\begin{vmatrix} f(x) & f(x) \\ f(y) & f(y) \end{vmatrix} + a_{11}a_{22}\begin{vmatrix} f(x) & g(x) \\ f(y) & g(y) \end{vmatrix} + a_{12}a_{21}\begin{vmatrix} g(x) & f(x) \\ g(y) & f(y) \end{vmatrix} + a_{12}a_{22}\begin{vmatrix} g(x) & g(x) \\ g(y) & g(y) \end{vmatrix}\right| =$$

$$= \sup_{x\in(a,b)} \left| a_{11}a_{22}\begin{vmatrix} f(x) & g(x) \\ f(y) & g(y) \end{vmatrix} + a_{12}a_{21}\begin{vmatrix} g(x) & f(x) \\ g(y) & f(y) \end{vmatrix}\right| = \sup_{x\in(a,b)} \left| (a_{11}a_{22} - a_{12}a_{21})\begin{vmatrix} f(x) & g(x) \\ f(y) & g(y) \end{vmatrix}\right| =$$

$$= \sup_{x\in(a,b)} \left|\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}\begin{vmatrix} f(x) & g(x) \\ f(y) & g(y) \end{vmatrix}\right| = \sup_{x\in(a,b)} \left| (\det A)\right|\left|\begin{vmatrix} f(x) & g(x) \\ f(y) & g(y) \end{vmatrix}\right| =$$

$$=|(\det A)| \sup_{x\in(a,b)} \left\|\begin{vmatrix} f(x) & f(y) \\ g(x) & g(y) \end{vmatrix}\right\| = |\det A| \, \| f,g \|$$

c) Let f,g,h b three arbitrary functions from the space which we consider. Then, from the inequality

$$\left|\begin{vmatrix} f(x)+g(x) & f(y)+g(y) \\ h(x) & h(y) \end{vmatrix}\right| = \left|\begin{vmatrix} f(x) & f(y) \\ h(x) & h(y) \end{vmatrix} + \begin{vmatrix} g(x) & g(y) \\ h(x) & h(y) \end{vmatrix}\right| \le \left|\begin{vmatrix} f(x) & f(y) \\ h(x) & h(y) \end{vmatrix}\right| + \left|\begin{vmatrix} g(x) & g(y) \\ h(x) & h(y) \end{vmatrix}\right|$$

and from the properties of the set function $\sup$ we get that

$$\sup_{x,y\in(a,b)} \left|\begin{vmatrix} f(x)+g(x) & f(y)+g(y) \\ h(x) & h(y) \end{vmatrix}\right| = \sup_{x,y\in(a,b)} \left|\begin{vmatrix} f(x) & f(y) \\ h(x) & h(y) \end{vmatrix} + \begin{vmatrix} g(x) & g(y) \\ h(x) & h(y) \end{vmatrix}\right| \le$$

$$\le \sup_{x,y\in(a,b)} \left|\begin{vmatrix} f(x) & f(y) \\ h(x) & h(y) \end{vmatrix}\right| + \sup_{x,y\in(a,b)} \left|\begin{vmatrix} g(x) & g(y) \\ h(x) & h(y) \end{vmatrix}\right|$$

i.e., the following inequality holds

$\| f+g,h \| \le \| f,h \| + \| g,h \|$.

With this, it is finally proven that the space of limited analytic functions with the defined function $\| \bullet,\bullet \| : A(D)\times A(D) \to$ is a 2-normed space.

As in every 2-normed space and in this space, we can choose a set of two linearly independent functions. Let that be, for example, the set $\{h_1,h_2\}$. Then for an arbitrary function f we have that

$\| f \| = \| f,h_1 \| + \| f,h_2 \|$

and with that a norm on the space of limited analytic functions is determined.

3. SOLUTIONS OF A HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION OF THE SECOND ORDER

Let us consider the homogeneous linear differential equation of the second order

$$y''+a(x)y'+b(x)=0 \tag{1}$$

where the functions $y=f(x)$ and $y=g(x)$ are (limited) analytic functions. The last equation with the substitution $y=e^{-\frac{1}{2}\int_0^u f(x)dx}\cdot z$ reduces to a linear differential equation of the second order which is in a canonical form, i.e.

$$z''+A(x)z=0, \tag{2}$$

where

$$A(x)=b(x)-\frac{1}{4}a^2(x)-\frac{a'(x)}{2}. \tag{3}$$

For the last equation, it is enough to find two particular solutions which are linearly independent. This is given in the following theorem.

Theorem. *The functions defined with*

$$y_1=e^{-\frac{1}{2}\int_0^u a(x)dx}\sum_{k=0}^{\infty}(-1)^k\underbrace{\int_0^x\int_0^x A(x)dx^2\int_0^x\int_0^x A(x)dx^2\ldots\int_0^x\int_0^x A(x)dx^2}_{k-\text{double integrals}}$$

and

$$y_2=e^{-\frac{1}{2}\int_0^u a(x)dx}\left[x+\sum_{k=0}^{\infty}(-1)^k\underbrace{\int_0^x\int_0^x A(x)dx^2\int_0^x\int_0^x A(x)dx^2\ldots\int_0^x\int_0^x xA(x)dx^2}_{k-\text{double integrals}}\right]$$

are linearly independent solutions of the equation (1), where $A(x)$ is given with (3).

These solutions are given in the book [5] of prof. d-r Dragan Dimitrovski.

Further on, as a development of this theory and this presentation of these solutions, arise the solutions that are called generalized sine and generalized cosine of the differential equation 1. These functions are given with:

Definition. Generalized sine is a solution of the differential equation (1) ($a(x)>0$) with initial conditions

$$y(0)=1,\ y'(0)=0,$$

given with

$$y_1(x)\equiv\cos_{a(x)}x=1-\int_0^x\int_0^x a(x)dx^2+\int_0^x\int_0^x a(x)dx^2\int_0^x\int_0^x a(x)dx^2-\ldots.$$

Definition. Generalized cosine is a solution of the differential equation (1) ($a(x)>0$) with initial conditions

$$y(0)=0,\ y'(0)=1,$$

given with

$$y_2(x) \equiv \sin_{a(x)} x = x - \int_0^x \int_0^x xa(x)dx^2 + \int_0^x \int_0^x a(x)dx^2 \int_0^x \int_0^x xa(x)dx^2 - ...$$

The last two functions are linearly independent functions. They have a number of properties that are analogous to the ordinary sine and cosine.

4. DIFFERENTIAL EQUATION OF THE SECOND ORDER IN THE SPACE OF LIMITED ANALYTIC FUNCTIONS AND 2-NORMED SPACE

As it is done in part 1, for the functions $h_1(x) = \cos_{a(x)} x$ and $h_2(x) = \sin_{a(x)} x$, they are linearly independent functions. Therefore with

$$\| f \| = \| f, h_1 \| + \| f, h_2 \| = \| f, \cos_{a(x)} x \| + \| f, \sin_{a(x)} x \|$$

a norm is determined in regarding to which the same space can be considered. For example, we can consider a function that will have a minimum norm in relation to the solutions of the differential equation that are of trigonometric shape, h_1, h_2.

Analogously, in a given space we can have two linearly independent functions $\{f, g\}$ that play a significant role in a given process or are essential in a given process in terms of determining solutions and the distance between the functions in that space. In that case, the differential equation (1) is also considered. Then with

$$\| h \| = \| h, f \| + \| h, g \|$$

a function h can be required which will be a solution of equation (1) and at the same time will have the lowest possible norm in relation to the previously introduced norm. In other words, in terms of a suitable norm, introduced by means of a 2-norm, to determine a function of a given shape that is technically the most suitable in the process, which at the same time will be a solution of the differential equation, at least in approximate form, and at the same time be "close enough" to the norm to two given functions of the process, whether or not they are solutions of equation (1).

Analogously, a nonlinear second order differential equation can be considered. In other words, we have an equation of the form

$$y'' = f(x, y, y').$$

For it we can have solutions of the form $y_1(x)$ and $y_2(x)$, which are of course linearly independent. With the help of these two functions, we can determine the norm in the space of limited analytical functions in the same way as in the previous part of this point.

It remains a key requirement of solutions that they have a predetermined performance, that suits the applicant, and at the same time meets some of the conditions of this paper.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

AUTHOR'S CONTRIBUTIONS

All authors contributed equally and significantly to writing this paper. All authors read and approved the final manuscript.

References

[1] Gähler S.: *Lineare 2-normierte Raume,* Math. Nach. 28(1965)
[2] Malčeski, A.: *Zabeleška za definicijata na 2-normiran prostor,* Matematički bilten, Tom 26 (2002)
[3] Malčeski, A.: *Za edna klasa na 2-linearni preslikuvawa*, Tret kongres na SMM, Struga 2005
[4] Malčeski, A.: $L^p(\mu)$-kako 2-normiran prostor, Matematički Bilten, 29(LV)2005 (71-76)
[5] Dimitrovski D., *A new approach of theory of ordinary differential equations*, издавачка куќа НУМЕРУС, ул Пиринска бб

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