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CONTENT

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ANALYSIS OF THE DEFORMATION DISTRIBUTION IN THE SYSTEM DEPENDING ON THE YIELD DEFORMATION

ALEKSANDRA RISTESKA-KAMCHESKI AND VLADO GICEV

Abstract. We study the response of nonlinear soil-linear foundation- linear building system to pulse – like a wave with arbitrary incidence. The soil is assumed as an ideally elastoplastic material where the nonlinear property of the material is defined by the yielding strain ε_m. Depending on the yielding strain, the soil responds to external excitation in a linear or nonlinear manner. As the yielding strain is lower, larger permanent strains occur during the soil response. The creation and development of these strains spends energy carried by the wave so less energy enters the building. In this way, the nonlinear soil response acts as a passive base isolator, i.e., the smaller the yielding strain, the larger the nonlinear soil response, a smaller amount of seismic energy entering the building.

1. Introduction

During a seismic event which can be an earthquake or an underground explosion, due to stress drop in the source, seismic waves are generated. These waves are due to the occurrence of cracks in the rock causing a sudden drop of the stress in the source of the seismic event. Because of the law of conservation of energy, the large amount of potential energy in the rock which is released with the rupture is transformed in mechanical energy for crashing the surrounding area in the vicinity of the source, heat energy and wave energy which propagate from the source. The propagation of seismic waves can be seen as a propagation of energy from the source through a semi-infinite medium in all directions. The energy along each ray, per unit area perpendicular to the ray per unit of time, is defined by the expression ([1]):

$$E = V \cdot \rho \cdot C^2,$$

where V is the velocity of propagation of the seismic wave, ρ is the density of the medium in which the seismic wave propagates, and C is particle velocity of material particles on the ground. This equation is derived from the general equation (1.5), for a constant particle velocity per unit of time, i.e., $C^2 = \int_0^1 v^2\, dt$. Since it is a unit area, in (1.5), $A_{sn} = 1$, while β_s in (1.5) in the last equation is denoted by V. The interest of the researchers in the field of soil-structure interaction started in 1970s ([2], [3], [4], [5]) and this topic is still popular. For these studies researchers usually use numerical models of seismic wave propagation which are bounded with the so-called artificial boundaries which allow the outgoing waves to exit the numerical models with as low as possible spurious reflections ([6], [7], [8]). While the earlier work until the beginning of 21st century was exclusively on linear soil-structure systems, in the last decade the authors have started to study the nonlinear response. There is ample observational evidence that soil response can become highly nonlinear in the near-field of strong earthquakes. This is manifested by the visible effects on the ground surface, by the damage to water and gas pipes ([9], [10]), which is later confirmed with numerical simulations on 2D models ([11], [12], [13]).

The purpose of this paper is to show illustratively how the level of the nonlinear soil response influences the amplitude of the pulse and energy entering the structure. For that reason, in the following we briefly recall some definitions and relations important for this study ([14], [15]).

We assume soil material with the ideally elastoplastic constitutive law $\sigma(\varepsilon)$ where σ is the shear stress in x or y direction and ε is the corresponding shear strain. The property of this law is the so-called shear strain ε_m which defines the strain for which the corresponding shear stress σ is maximum. With increasing ε in the same direction (positive or negative), σ remains constant (Figure 1).

The question that arises is how to choose the yield strain ε_m (Figure 1) to study the strain distribution in the system.

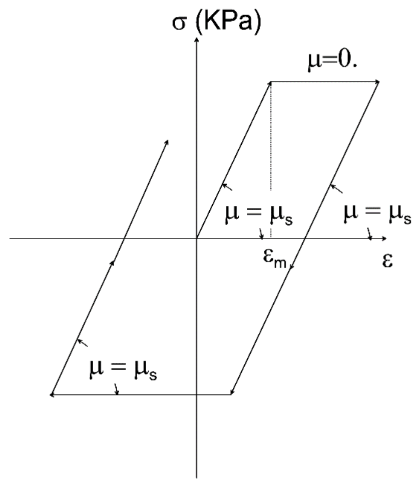

Figure 1. Constitutive law σ-ε, for the soil and foundation

Displacement, velocity, and linear deformation in the soil during the passage of the wave in the plane xOy, in the form of a half-sinusoidal pulse, are:

$$p = A\sin\left(\pi \cdot t / t_{d0}\right) \tag{1.1}$$

$$v = \dot{p} = \left(\pi / t_{d0}\right) A\cos\left(\pi \cdot t / t_{d0}\right) \tag{1.2}$$

$$\left|\varepsilon_p\right|_{\max} = \left|\dot{p}\right|_{\max} / \alpha_s = \pi \cdot A / \left(\alpha_s \cdot t_{d0}\right) \tag{1.3}$$

where p is a displacement in direction z, v is particle velocity, A is the amplitude, t_{d0} is the duration of the half-sinusoidal pulse and $|\dot{p}|_{max}$ is the maximum value (amplitude) of the particle velocity of the input pulse.

If for a given input wave we choose a yielding deformation ε_m, given by equation (1.3), multiplied by a constant C with a value between 1 and 2, the deformations in both directions will remain linear before the wave reaches the free surface or the foundation for each input angle. This case can be called "intermediate nonlinearity". If we want to analyze only the nonlinearity due to scattering and reflection from the foundation we should avoid the occurrence of nonlinear deformations caused by reflection from the free surface of the half-space. Then we can choose $\varepsilon_m \geq \max\left(\frac{2\pi A\sin\theta}{\beta_s t_{d0}}; \frac{2\pi A\cos\theta}{\beta_s t_{d0}}\right)$, where θ is the input angle. This case is called "small nonlinearity". If the soil is exposed to nonlinear deformations during the passage of the input wave in full space, then we can choose $\varepsilon_m < \max\left(\frac{\pi A\sin\theta}{\beta_s t_{d0}}; \frac{\pi A\cos\theta}{\beta_s t_{d0}}\right)$.

This condition guarantees that in direction x or y, the soil will be exposed to nonlinear deformations during the passage of the wave through it. In general, the resulting deformation can be written as:

$$\varepsilon_m = C \cdot v_{\max} / \beta_s = C \cdot \pi \cdot A / \left(\alpha_s \cdot t_{d0}\right) \tag{1.4}$$

where C is a constant that controls the resulting stress (or strain) in the soil.

We consider the following cases of nonlinearity, depending on C:

- $C \geq 2$ - Small nonlinearity, permanent deformation does not occur until the wave hits the foundation.

- $1 \leq C < 2$ - Intermediate nonlinearity, permanent deformation does not occur until the wave is reflected from the free surface or scattered through the foundation. Permanent deformation will occur or not after the reflection of the incoming wave from the free surface, depending on its angle.
- $C < 1$ - Large nonlinearity, permanent deformation occurs after the wave bounces off the free surface. It may or may not occur before the wave is reflected from the foundation surface.

The flow of energy through a given area can be defined in relation to the passage of the wave through the surface A_{sn}:

$$E_{in}^{a} = \rho_s \cdot \beta_s \cdot A_{sn} \cdot \int_0^{t_{d0}} v^2 \cdot dt \qquad (1.5)$$

where ρ_s is the density of the soil, β_s is the velocity of propagation of the wave through the soil, v is the velocity of displacements of soil particles, and A_{sn} is the area normal to the direction of wave propagation. From the geometry of our calculation model (Figure 2), the area normal to the passing wave is:

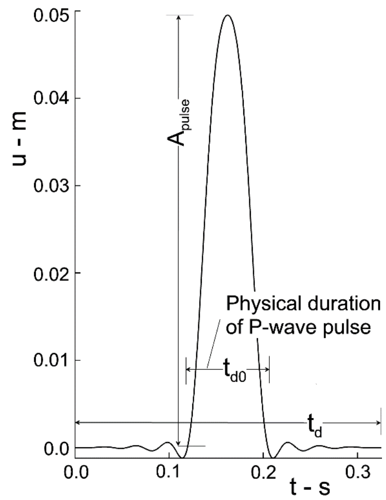

Figure 2. Input pulse (filtered) in function of time
(taken from Gicev et al, 2015)

$$A_{sn} = 2 \cdot H_m \cdot \sin\gamma + L_m \cdot \cos\gamma = L_m \cdot (\sin\gamma + \cos\gamma), \qquad (1.6)$$

where H_m is the height and L_m the width of the soil section in our model (Figure 2), respectively. By inserting and integrating equations (1.2) in (1.5), we get the analytical solution for the wave energy input in the model, as follows:

$$E_{in}^{a} = \rho_s \cdot \beta_s \cdot L_m \cdot (\sin\gamma + \cos\gamma) \cdot (\pi A / t_{d0})^2 \cdot t_{d0} / 2 \tag{1.7}$$

As can be seen from equation (1.7), for the defined length of the soil section L_m and the defined input angle γ, the input energy is reciprocal of the pulse duration, which means that it is a linear function of the dimensionless frequency η.

Due to the law of conservation of energy, the input energy is balanced by the following:

- Cumulative energy E_{out} goes out of the model, and is calculated by equation (1.5),
- Cumulative (hysteretic) energy, i.e., the energy consumed for the creation and development of permanent deformations in the soil, is calculated by:

$$E_{hys} = \sum_{t=0}^{T_{end}} \Delta t \cdot \sum_{i=1}^{N} \left(\sigma_{xi} (\Delta\varepsilon_{xpi} + 0.5 \cdot \Delta\varepsilon_{xei}) + \sigma_{yi} (\Delta\varepsilon_{ypi} + 0.5 \cdot \Delta\varepsilon_{yei}) \right) \tag{1.8}$$

where T_{end} is the time at the end of the analysis, *N* is the total number of points, σ_{xi}, σ_{yi} are the stresses at points in *x* and *y* axis, respectively, $\Delta\varepsilon_{xpi} = \varepsilon_{xpi}^{t+\Delta t} - \varepsilon_{xpi}^{t}$ is the increase of the elastic deformation in the direction *x* at point *i*, and $\Delta\varepsilon_{yei} = \varepsilon_{yei}^{t+\Delta t} - \varepsilon_{yei}^{t}$ is the increase of the elastic deformation in the direction *y* at the point *i*.

- The instantaneous energy in a building, which consists of kinetic and potential energy, can be calculated from:

$$E_b = E_k + E_p = 0.5 \cdot \Delta x \cdot \Delta y_b \cdot \sum_{i=1}^{N} \left(\rho \cdot v_i^2 + \mu \cdot (\varepsilon_x^2 + \varepsilon_y^2) \right) \tag{1.9}$$

where Δx and Δy_b are the horizontal and vertical distances of the grid in the building, ρ and μ are the density and shear modulus of the building, respectively, v_i is the velocity of the particles, while ε_x and ε_y are the shear strains at point *i* of the building.

To study only the effect of scattering on the foundation, it is assumed that the building is high enough that the reflected wave from the top of the building cannot reach the building-foundation contact by the end of the analysis. The analysis is stopped when the wave is completely out of the model.

2. Numerical example

We took the Holiday Inn Hotel in Van Nuys, California as a prototype for our two-dimensional numerical model (Figure 2). The hotel is located in the middle of the San Fernando Valley in the metropolitan area of Los Angeles, California, and was fully instrumented. During the Northridge earthquake in California in 1994, the hotel was severely damaged (Figure 3) and its response during this earthquake has been analyzed and described in many articles and reports ([16], [17], [18], [19], [20]). For our two-dimensional SH model, we took the physical-mechanical characteristics of the soil on which the hotel is built, as well as the equivalent physical-mechanical characteristics of the hotel in the east- west direction, obtained by impulse response analysis of a one-dimensional model.

Figure 3. View of Van Nuys Seven Story Hotel (VN7SH) from North-East

We assumed that all contacts in our model, three foundation-soil contacts and one foundation-building contact remain continuous, i.e., no separation or sliding is allowed. The building and the foundation remain linear throughout the analysis. For our example, the velocities of propagation of SH wave in the building and in the soil are $\beta_b = 100\, m/s$, and $\beta_s = 250\, m/s$ respectively. Since we analyzed how the stiffness of the foundation affects the response, we change the propagation velocity of the SH wave in the foundation $\beta_f = 250$ to $1000\, m/s$.

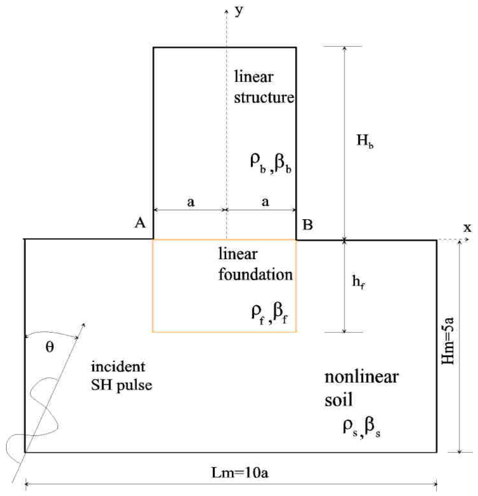

Figure 4. Soil-structure system with linear structure and foundation and nonlinear soil redrawn from Gicev et al, 2015

The width of the foundation is the same as the width of the building $W_b = 2a = 19.1m$, and its depth is half of its width, $h_f = a = 9.55\ m$. The density of the material from which the building is constructed is $\rho_b = 270\ kg/m^3$ for all examples in this study. We took the densities of the foundation and the soil the same $\rho_f = \rho_s = 2000\ kg/m^3$.

We stop the calculation at time T_s, when the complete filtered pulse (Figure 2) passes the right corner of the foundation-structure contact, B (Figure 4).

$$T_s = \frac{H_m}{c_y} + \frac{\frac{L_m}{2} + a}{c_x} + t_d = \frac{H_m}{c_y} + \frac{6a}{c_x} + t_d \qquad (2.1)$$

where $c_y = \frac{\beta_s}{\cos\gamma}$ and $c_x = \frac{\beta_s}{\sin\gamma}$ are the vertical and horizontal phase velocities of the SH wave propagating in the soil, L_m and H_m (Figure 4) are the width and height of the soil section, a is the half-width of the structure, and t_d is the duration of the half-sinusoidal pulse. After this time, we have no energy input in the construction. Since we studied only the energy that enters the construction, we varied the height of the building, H_b. We calculated the height of the building from the condition that the time for which the pulse that reached point A (Figure 4), travels to the top of the structure, reflects and continues to travel backwards, reaches the foundation-building contact, is smaller than the time until the complete pulse has passed point B when we interrupt the numerical simulation. The shortest time for the wave front to travel from the lower left corner of the model to the left corner of the foundation contact, then bounce off the top of the structure and reach the foundation contact again is:

$$T_r = \frac{H_m}{c_y} + \frac{\frac{L_m}{2} - a}{c_x} + \frac{2H_b}{\beta_b} = \frac{H_m}{c_y} + \frac{4a}{c_x} + \frac{2H_b}{\beta_b} \quad . \tag{2.2}$$

Then the required condition for calculating the height of the building is $T_r \geq T_s$, or

$$\frac{H_m}{c_y} + \frac{4a}{c_x} + \frac{2H_b}{\beta_b} \geq \frac{H_m}{c_y} + \frac{6a}{c_x} + t_d . \tag{2.3}$$

From (2.3) and keeping in mind that $c_x = \frac{\beta_s}{\sin\gamma}$, we calculate the required height of the structure (object) $H_b \geq \frac{a \cdot \sin\gamma \cdot \beta_b}{\beta_s} + \frac{t_d \cdot \beta_b}{2}$.

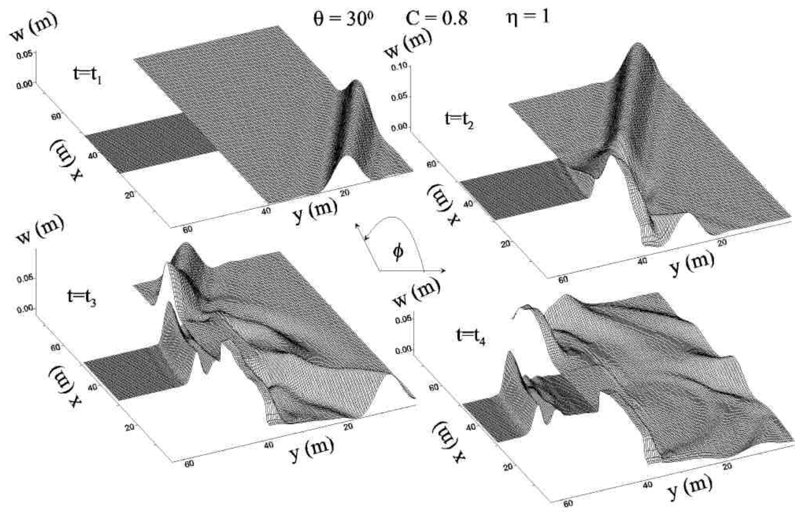

Figure 5. Displacement field of SSI model for angle of incidence *θ = 30⁰, h = 1* and *C=0.8* in four-time instances of propagation of the pulse. The model is rotated *θ = 115⁰* counterclockwise.

t_1: the pulse completely entered the model,

t_2: the front of the pulse just entered the structure,

t_3: the amplitude of the pulse is in the middle of the foundation-structure contact,

t_4: the pulse completely passed the right corner (point B)

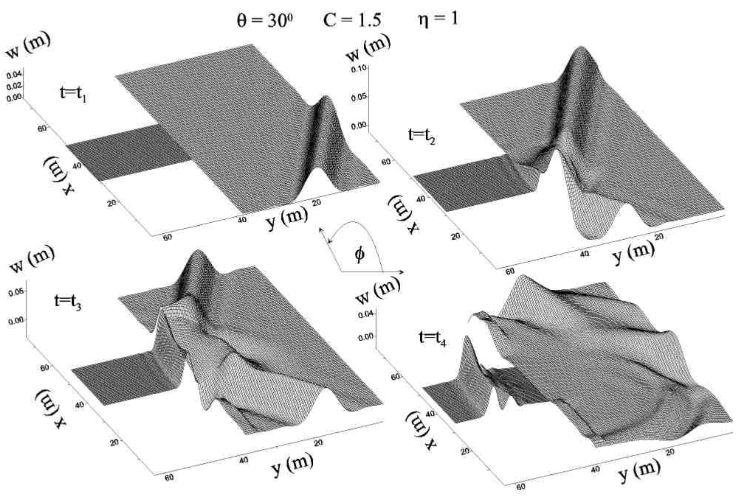

Figure 6. Same as Fig. 5, but for *C=1.5*

Figures 5 and 6 represent a "movie" of the propagation of a pulse with $\eta = \dfrac{a}{\beta_s \cdot t_d} = 1$, in a soil-foundation-structure system with the same geometry and the same characteristics of all elements in the system, except that in the model in Figure 5 we have a soil with a large non-linearity, *C=0.8*, while the model in Figure 6 has medium nonlinearity soil, *C=1.5*. In other words, the yielding deformation in the soil ε_m in the model of Figure 6 is almost two times greater than that of the model in Figure 5. In the first instance, $t=t_1$, the pulse completely entered our model from the lower left side, i.e., the coordinate origin (images are rotated 115 degrees for a better view of the model). In this time instance, there is almost no difference in the field of displacements in both models, because no zones of large permanent deformations have yet formed in the ground. As the wave (pulse) propagates upwards and to the right, it reaches the free surface. The part of the wave that strikes the free surface is reflected back at the same angle of opposite sign as the input angle, θ. In time $t=t_2$ on the left side of the model, $x=0$, comparing with time $t=t_3$ it can be observed that the reflected part of the wave moves downwards, and the part of the wave that has not yet reached the free surface continues to propagate upwards and to the right. Also, in time $t=t_2$ the wave starts entering the building from the left side of the foundation-soil contact (picture 4) $(x_A, y_A) \approx (30m, 40m)$. Also, in this time instance there is no big difference in the field of displacements between the models in Figure 5 and Figure 6, which is the result of the fact that no major permanent deformations have yet developed. In time $t=t_3$, when the amplitude of the input pulse is at the middle of the contact between the foundation and the structure, permanent deformations have already developed in the ground and the field of displacements in the models of Figures 5 (model 1) and 6 (model 2) differs. It can be noted that due to greater deformations in the ground, the foundation is more clearly visible in model 1. A large part of the pulse has entered the building and begins to propagate along the height of the building (comparison with instance $t=t_4$). The soil on the free surface on the left side of the foundation in model 1 has undergone positive permanent displacement, while in model 2, the displacement of the soil on the free surface to the left of the foundation is still elastic (compare with $t=t_4$) and negative.

3. Conclusion

We show that the level of the nonlinear soil response influences the energy propagating through the soil-structure system. Comparing the field of displacements for a time instance $t=t_4$ when the incident wave has completely passed the building-foundation interface for models with large, C = 0.8 and intermediate, C = 1.5 nonlinear soil response, it can be concluded that the displacements in the ground and in the foundation in both models are quite different. The wave in the building propagates along the height and goes towards the top. For both models, the displacements in the linear object are almost equal. Unlike the displacements in the building, the displacements in the soil and foundation for model 1 and model 2 (Figure 5 and Figure 6) differ.

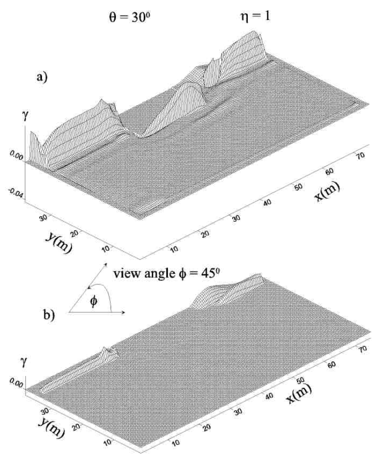

Figure 7. Permanent shear strains in the soil after the pulse has left the model

a) for big nonlinearity in the soil *C=0.8*

b) for small nonlinearity in the soil *C=1.5*

In model 1 (*C=0.8* – large non-linearity in the soil), large permanent displacements remain on the free surface to the left and right of the foundation. Along the entire perimeter around the foundation, permanent deformations have developed in the ground that clearly differentiate the foundation from the ground (Figure 5, Figure 7).

In model 2 (*C=1.5* – medium non-linearity in the soil) smaller permanent displacements remain on the free surface to the left and right of the foundation. On the perimeter around the foundation, no large permanent deformations have developed in the soil, so the displacements in the soil and the foundation are almost elastic (reversible) and do not differ between the foundation and the soil (Figure 6, Figure 7).

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