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CONTENT

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APPLICATION OF GEOSTATISTICS IN THE ANALYSIS AND ADAPTATION OF GEOTECHNICAL PARAMETERS AT COAL DEPOSITS

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Abstract. This paper presents a case study in the application of geostatistics in the analysis and adaptation of representative values of the geotechnical parameters at coal deposits. In order to select the most reliable geotechnical values for the design process of open cast pit excavations of large heights, different pools of laboratory data sets have been processed. The following critical geotechnical parameters were investigated: volume weight, angle of internal friction, and moisture content. By applying the correct geostatistical tools, and in comparison with practical experience and removal of obvious outliers from the datasets, for one of the deposit are prepared the so-called "geotechnical domains" are prepared, shown in the form of spatial distribution maps of the analysed parameters. Advantages of the applied geostatistical approach are seen in enabling higher confidence when performing slope stability analysis, selection of the most optimal geometry of the mines, lesser overburden quantities, protection of the environment, production cost cutting, and time saving.

1. Introduction

Coal as an energetic mineral resource has fundamental importance in N. Macedonia, since it is the dominant source for electricity production. Coal deposits are situated in several so-called sediment basins, characterized by tertiary geological age. Nowadays, only the deposits located in the Pelagonia basin are used for coal exploitation. More specifically, the coal is exploited at two surface mines "Podinska serija" and "Brod-Gneotino". Other deposits such as "Lavci" and "Zvegor-Stamer" and smaller ones, located in other basins, have been investigated to certain extent; however, they are still not with active exploitation.

The application of modern mathematical trends and computer software facilitates the process of interpretation of the obtained data from the performed investigations, by creating different types of geological and geotechnical datasets and models. Using the tools of spatial analysis, developed in the second half of the last century, aided by computer software, fast data processing can be performed. In comparison, manual processing is quite complicated and time consuming.

Geotechnical modelling is fundamental in designing the mines with open pits or underground excavation. A fully defined and representative geotechnical model will provide information about the engineering and geological characteristics of the rock structure, defining its behavior in the domains of mining. The model is composed of individual domains, showing similar geotechnical properties.

Keywords. *coal deposits, geostatistics, geotechnical domains, variograms, kriging.*

The definition of these individual domains and their comprehensive investigation approach is the key for the process of exploitation and managing related hazards, through facilitating the optimal solutions for the mining designs, [1]-[2].

The used mathematical models and specialized software allow better understanding of the complexity of the deposits. We must be careful about the following: the used data for modelling should be appropriate, representative and their quality should be assessed before we start with the mathematical modelling aided by computer software. Geotechnical engineers must have excellent understanding of the hazards and constraints of each separate model, as well as of their influence in creating geotechnical domains, [2].

2. Semivariograms and Kriging

The first step in the geostatistical approach is gathering and structuring a large amount of geotechnical data. The most usual way to define the row of data is to use the basic parameters of descriptive statistics, i.e., mean, standard error, median, mode, standard deviation, simple variance, kurtosis, skewness, range, minimum, maximum, sum, sample size, 95% confidence interval and boxplots, see: for example [3]-[4].

The mathematical principles used to spatially define the variability of geotechnical properties for the case studies are based on the theory from spatial analysis, or, more precisely, on semivariograms and ordinary kriging. Once a theoretical semivariogram is chosen and fixed, we are prepared for spatial prediction. For spatial prediction in geostatistics kriging is usually used. This term is given in honour of the South African mining engineer, Daniel Gerhardus Krige. When we have the data realization, in practice obtained by the sample, the experimental semivariograms derived from the realizations may not satisfy certain requisites in this theory. Because of this reason, we should find the theoretical models (valid models) that do comply with the experimental ones and must be fitted to them. Kriging aims to predict the value of a regularized function, X(s), at non-observed points in a collection of data, obtained by observation at n points, [5].

The semivariogram is defined by the following:

$$\gamma(s_i - s_j) = \frac{1}{2} V(X(s_i) - X(s_j)),$$

$s_i, s_j \in D$, and D is a continuous domain, and $V(\cdot)$ denotes the variance of $\cdot$.

Under the second-order stationary hypothesis:

For the stochastic process (random function) $\{X(s) : s \in D\}$ is said to be second-order stationary, if it has finite second-order moments and the following holds:

a) The mathematical expectation exists and is constant, $E(X(s)) = \mu(s) = \mu$,

b) For every pair $X(s)$ and $X(s+h)$, the covariance exists and depends only on the vector h joining the locations s and $s+h$, i.e. it holds

$$C(X(s), X(s+h)) = C(h), \text{ for all } s \in D \text{ and vectors } h;$$

and the intrinsic hypothesis:

For the stochastic process $\{X(s): s \in D\}$ it is said to be intrinsic if, for any given vector h, the first-order increments $X(s+h)-X(s)$ are second-order stationary, i.e., it holds $E(X(s+h)-X(s))=\mu(s)$, where $\mu(s)$, is linear in h, and

$$C((X(s+h)-X(s)),(X(s+h+h')-X(s+h')))=C(h,h'),$$

which is identical to

$$\frac{1}{2}V(X(s+h)-X(s))=\gamma(h),$$

being only a function of h. Also, this can be written as

$$\gamma(h)=\frac{1}{2}V(X(s+h)-X(s))=\frac{1}{2}E\left((X(s+h)-X(s))^2\right),$$

measuring how the dissimilarity between the points $X(s+h)$ and $X(s)$ develops with the distance h.

The par excellence instrument for the description of the spatial dependence in the regionalized variable is the semivariogram. The reason is that the semivariogram covers a broader spectrum of stochastic processes in comparison to covariograms, which is confined to second-order stationary stochastic processes. The spectrum of semivariograms relies on intrinsically stationary random functions, where the covariance cannot be defined. In the case of second-order stationary framework, the semivariogram and the covariogram are theoretically equivalent, [5]. Indeed, we have

$$C(h)=C(0)-\gamma(h).$$

When we are dealing with certain applications, the mean is unknown and it should be estimated from the observed data, which certainly introduces a bias. The semivariogram of an intrinsically stationary stochastic process depends only on the vector h, connecting the locations (and on the direction). Hence, in general terms, we have an anisotropic situation. Let us mention that, in the case when a semivariogram depends only on distance, it is called an isotropic semivariogram.

The semivariogram as a function is non-decreasing monotone, so that the variability of the first increments of the stochastic process is increasing with distance. The semivariogram corresponding to second-order stationary random functions has a common behavior at intermediate and huge distances. Their behavior is the following: they rise from the origin and increase monotonically as the distance is increased until approaching its limiting value, the a priori variance of the random function, $C(0)$, reaches either exactly or asymptotically, [5].

The semivariogram's slope indicates the change in the dissimilarity of the values of the stochastic process, as the distance is changing. This limit value of the semivariogram is called the variance sill, or just sill, denoted by (m). The distance at which the sill is reached is called the range of the semivariogram, defining the threshold of spatial dependence, i.e., the zone of influence of the stochastic process. In other words, the range is the distance beyond which the values of the stochastic process have no spatial dependence.

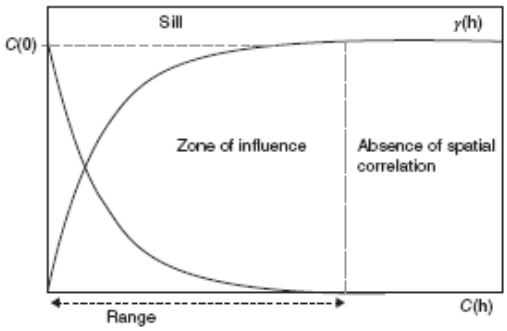


Figure 1. *Theoretical semivariogram and its covariogram counterpart*

Figure 1 shows that if the range is larger, then the zone of influence of the stochastic process will be larger. When the sill is reached asymptotically, there is not a well-marked range, and we use a practical range (the distance at which the semivariogram takes the value $0,95m$). This practical range is closely related to the scale parameter, a, of the semivariogram.

Here we consider a semivariogram that is associated with the second-order stationary hypothesis. Hence, it has a covariogram counterpart. These types of semivariograms are also called transition models. The justification is the following: the distance at which the sill is reached represents the transition from the state of existence of spatial correlation to the state of absence of spatial correlation, [5].

2.1. The spherical model

The spherical model can be defined on $\mathbb{R}$, $\mathbb{R}^2$ and $\mathbb{R}^3$. This model is defined as

$$\gamma(|h|)=\begin{cases} m\left(1,5\frac{|h|}{a}-0,5\left(\frac{|h|}{a}\right)^3\right), & |h|\le k \\ m, & |h|>k \end{cases},$$

where $m=C(0)$ is the value of the semivariogram when it reaches the sill, and k is the range. The spherical semivariogram has a clear linear behavior around the origin, indicating continuity, but a certain degree of irregularity in the stochastic process. When dealing with the behavior at large distances, we have that it reaches the sill at $|h|=k$. We stress that the main reason for its usage is the almost linear behavior up to a certain distance (the range) and then the stabilization matches a large variety of observed regionalizations [5].

Next, we will give the kriging equations, which give us a prediction of the value of the stochastic process $X(s)$ at a non-observed point, as a linear combination of the values of the stochastic process at the observed points or at a group of them that are in the neighborhood to the point subject of prediction. Usually, the kriging prediction is based

on the empirical semivariogram (with a fitted theoretical semivariogram). Furthermore, it is more complicated to measure the consequences not using the original experimental semivariogram. More precisely, in the case of point observation, the point kriging predictor $X^*(s_0)$ at the non-observed point s_0 can be calculated by

$$X^*(s_0) = \sum_{i=1}^{n} \lambda_i X(s_i),$$

where $X(s_i)$ are the values observed at the n points in the close area of s_0, the prediction point, and λ_i are the kriging weights obtained by imposing on the prediction error the classical conditions above referred. For more details, check for example [5] and references therein.

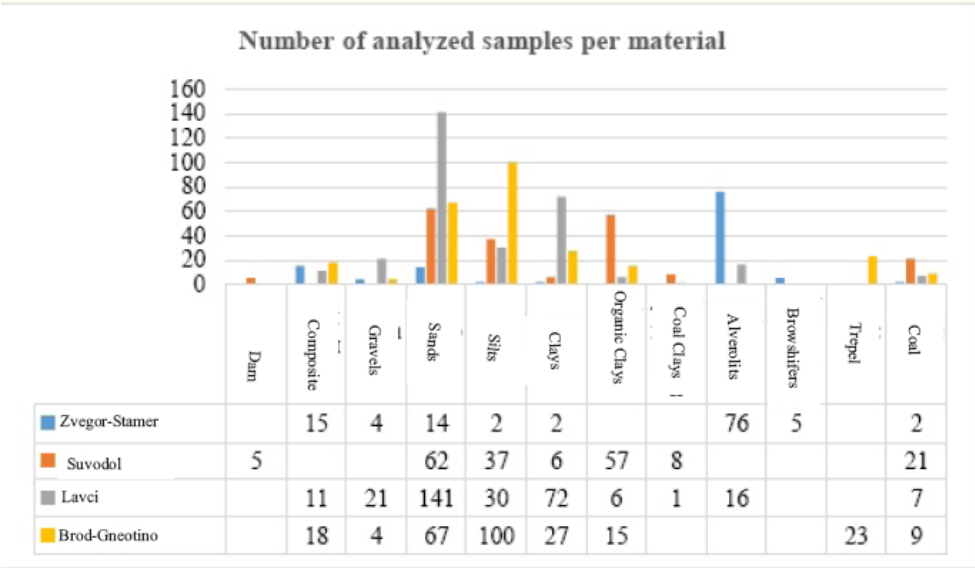
It is clear that the quality of kriging predictions is based on the sample size and the quality of the data. Also, we should pay attention to:

- the observed locations should be uniformly distributed in the studied domain;
- the distance between the observed points and the predicted points, i.e., more confidence should be put in nearby observations than in distant observations;
- the spatial continuity of the stochastic processes should be studied. Clearly, it is easier to predict the value of a stochastic process at a point, compared to a stochastic process that fluctuates considerably.

At the end of this preparatory section, let us make an important remark that the main advantage of kriging over other spatial interpolation techniques, such as inverse distance method (IDW), splines, and polynomial regression is that not only does it take into account the geometric characteristics and the number and organization of locations, but, more importantly, it considers the structure of the spatial correlation. Hence, the weights that kriging is using are not calculated on an arbitrary rule, but on the behavior of the function describing the spatial correlation. Therefore, it is a more refined method than those mentioned above. Additionally, kriging makes it possible to measure how accurate the prediction is; it does not depend on the actual observations, and it is an exact interpolator [5].

3. Case Studies

Coal deposits that are the subject of the example of descriptive statistics analysis have been explored in different periods and extents/conditions. In terms of the number of samples that have been analysed, the largest number is from the Lavci deposits, followed by Brod-Gneotino, Suvodol and finally Zvegor-Stamer. Geotechnical investigations at the Lavci deposit were done in several phases, and within these analyses the results from1985/86, 2008 and 2016/17 are considered. Investigations of the Brod-Gneotino deposit were done in several phases, i.e., 2001, 2014/16 and 2017/18. The results of investigations in 1994 and 2004 were analyzed for the Suvodol deposit. Finally, for the Zvegor-Stamer deposit, the results of the investigations in 2016 are the subject of analysis, [6-8]. A histogram display of the number of analyzed samples per deposit and the type of material is given in the following graph.



	Dam	Composite	Gravels	Sands	Silts	Clays	Organic Clays	Coal Clays	Alvenolits	Brow shifers	Trepel	Coal
Zvegor-Stamer		15	4	14	2	2			76	5		2
Suvodol	5			62	37	6	57	8				21
Lavci		11	21	141	30	72	6	1	16			7
Brod-Gneotino		18	4	67	100	27	15				23	9

Figure 2. *Number of analyzed samples per type of material*

3.1. Natural moisture content analysis

On the given boxplots below, for each of the deposits the minimal and maximal value of the natural moisture, first and third quartile and the median are presented. For the coal deposits Suvodol, Lavci and Brod-Gneotino, the minimal value of the moisture varies around 10%, and the maximal value of the natural moisture varies around 50%. The minimal and maximal values of the natural moisture of the deposit Zvegor-Stamer significantly differ from the previous ones, i.e., the minimal value is around 5% and the maximal value of the natural moisture is around 70%. Generally, we can conclude that, for all four deposits, the samples have natural moisture in the interval of 20-40%.

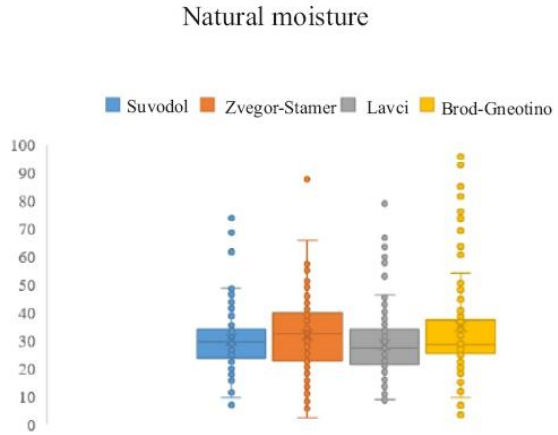


Figure 3.*Natural moisture content for different deposits*

3.2. Quasi-homogeneous domains in the Brod-Gneotino deposit

In this subsection, we are going to use spherical semivariograms, in order give an example of a realistic 3D representation of certain geotechnical parameters. The examples are for sand and coal materials of the coal deposit Brod-Gneotino. With these mathematical tools and respective software, the collected data [6-8] (see also [9-14]) is analysed and the following quasi-homogeneous domains in the Brod-Gneotino deposit are obtained: quasi-homogeneous domains of sand's volume weight, quasi-homogeneous domains of the angle of internal friction of sand determined by the direct shear and quasi-homogeneous domains of the angle of internal friction of coal determined with triaxial tests.

3.2.1. Quasi-homogeneous domains of sand's volume weight

The data used for mathematical modelling of quasi-homogeneous domains belong to the geological age of Quaternary, Pliocene, and Miocene age, hence the created quasi-homogeneous domains will correspond to the part of the geological model with Quaternary, Pliocene, and Miocene age, without the part belonging to the so called material "trepel" and the coal layers.

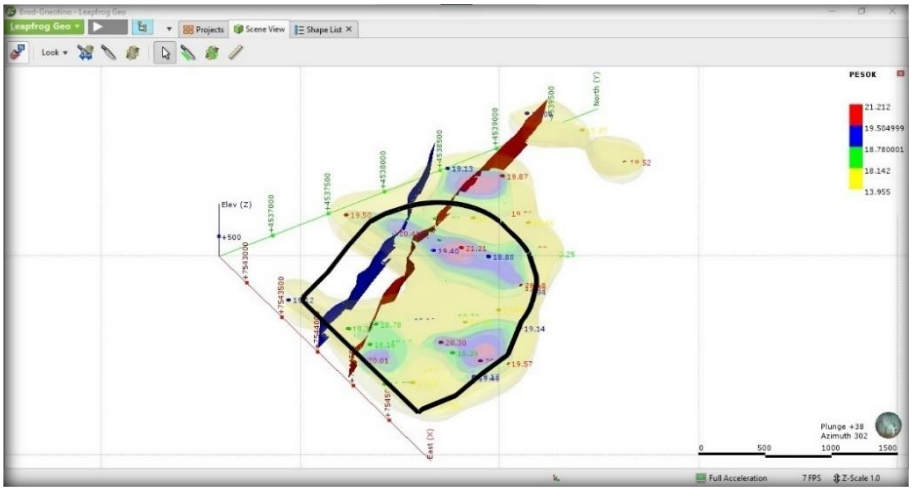


Figure 4. *3D representation of quasi-homogeneous domains of sand's volume weight*

The obtained values for volume weight of the sand in the surface layer in Brod-Gneotino are divided in two parts:

- Gravelly sand (SP, SW) with $\boldsymbol{\gamma}$**=20,10 [kN/m^3]**
- Silty sand (SFc, SFs) with$\boldsymbol{\gamma}$**=18,74 [kN/m^3].**

By the 3D representation of the quasi-homogeneous domains of the volume weight of the sand, we have the following:

✓ The data used for mathematical modelling of these domains are in the range from13,95 to 21,21 [kN/m^3], with the median **18,78 [kN/m^3]**,

✓ The domain with the highest values (19,50 – 21,21 [kN/m^3]) is located in the northern part and in the part of the eastern final slope of the open pit,

✓ The domains with lower values are more represented, i.e., 13,95 – 18,14 [kN/m^3] and 18,14 – 18,78 [kN/m^3]),

✓ Since the obtained values belong in intervals with lower values, we can consider the option, when analyzing the slope stability, to use lower values for volume weight of sand, compared to the obtained values.

3.2.2. Quasi-homogeneous domains of internal friction of sand determined by direct shear

Here, for mathematical modelling of quasi-homogeneous domains of the angle of internal friction of the sand, determined with a direct shear test, we use the data belonging to the geological layers with Pliocene and Miocene age, hence the created quasi-homogeneous domains will correspond to the part of the geological model with Pliocene and Miocene age, without the part belonging to trepel and coal layers.

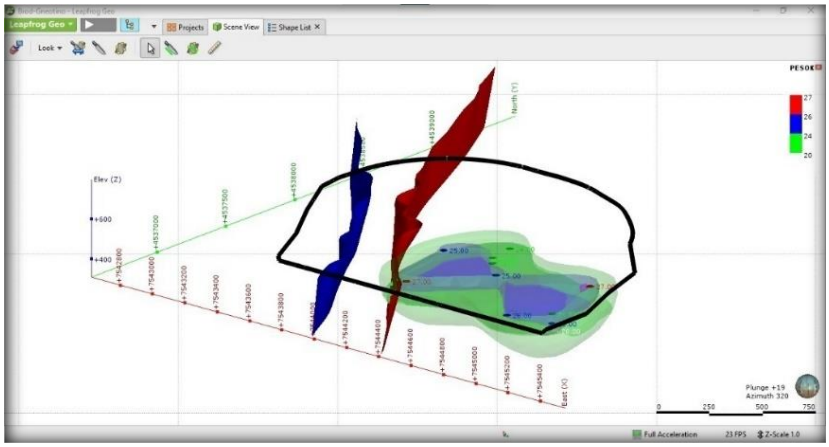


Figure 5. *3D representation of quasi-homogeneous domains of the angle of internal friction of the sand determined with direct shear*

By the 3D representation of the quasi-homogeneous domains of the angle of internal friction obtained with direct shear test of the sand, we have the following:

✓ The data used for mathematical modelling of these domains are in the range from $\varphi = 20°$ to $\varphi = 27°$, with the median $\varphi = 24°$, located in the non-fault domain,

✓ The domain with the highest values ($26° - 27°$) is almost in punctate structure,

✓ The domains with lower values are more represented, i.e., $20° – 24°$ and $24° – 26°$, and they are located in the south-east part of the border of the surface mine,

✓ It can be noticed that the obtained values for this geotechnical parameter are located outside of the domains of the separated quasi-homogeneous domains. Since the domains with the values $20° - 24°$, the mentioned values can be used for analysis of the slope stability of the surface mine.

3.2.3. Quasi-homogeneous domains of the angle of internal friction of coal determined with triaxial tests

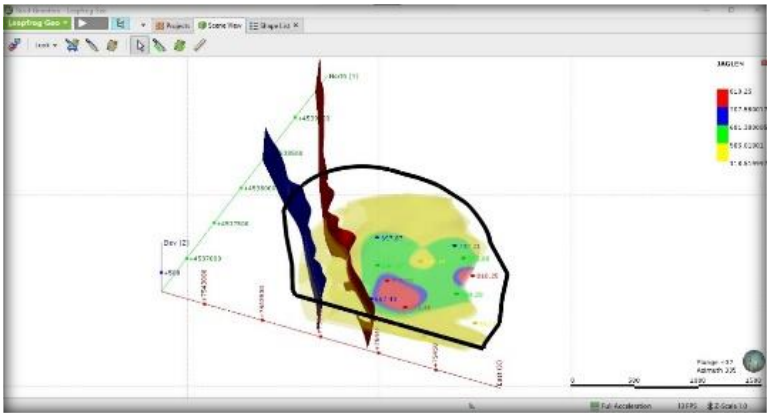


Figure 6. *3D representation of quasi-homogeneous domains of the cohesion of the coal determined with the triaxial tests*

The obtained value for the cohesion of the coal in the surface mine Brod-Gneotino is $c = 50,00\ [kN/m^3]$. From the 3D representation of the quasi-homogeneous domains of the cohesion of the coal, determined with triaxial tests, we can derive the following conclusions:

✓ The data used for creating these domains are in the range from $c = 118,52\ [kN/m^3]$ to $c = 810,25\ [kN/m^3]$, with the median $c = 681,38\ [kN/m^3]$ and are located in the non-fault domain,

✓ The domains with higher values ($681,38 - 707,5\ [kN/m^3]$ and $707,58 - 810,25\ [kN/m^3]$) are the least spatially represented and they are located in the small part in the eastern final slopes, as well as in the middle part in the surface mine,

✓ The domains with lower values ($118,52 - 585,01\ [kN/m^3]$ and $585,01 - 681,38\ [kN/m^3]$) are more spatially represented covering a whole space in the non-fault domain form the surface mine.

4. Results and discussion

The presented examples of the definition of quasi-homogeneous domains and their spatial analysis in the different parts of the surface mine clearly show the significant contribution to adopting certain values for the geotechnical parameters. The studied

domains show that in certain parts of the deposit the obtained laboratory values are appropriate, while in other parts lower or higher values can be used in order to obtain a more realistic analysis of the slope stability, as a process highly important for exploitation and long-term planning and development of the open pit mine.

The used mathematical models, aided by computer techniques, allow a combination of a geological model and numerical models for geotechnical parameters through the definition of geotechnical sections/domains. These models display the distribution of values of the geotechnical parameters in different geological layers/mediums.

All of this emphasizes the importance of the spatial analysis aided with computer programs in geotechnical modelling. The application of these methods is important for the spatial and statistical processing of geotechnical data. Using corresponding interpolations results in a selection of realistic values of the geotechnical parameters in all levels of project design and is thus highly recommended.

This mathematical approach is quite general, easy for application and is of great benefit for the geotechnical characterization of different types of mineral deposits.

Finally, due to the nature of the geological materials, other mathematical approaches can be suggested and are being used for geotechnical characterization in geotechnics such as in kinematic stability analyses for hard rock masses, calculations of bearing capacity, settlement, and others.

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