GOCE DELCEV UNIVERSITY - STIP
FACULTY OF COMPUTER SCIENCE

The journal is indexed in
EBSCO

ISSN 2545-4803 on line
DOI: 10.46763/BJAMI

BALKAN JOURNAL OF APPLIED MATHEMATICS AND INFORMATICS (BJAMI)

YEAR 2024

VOLUME VII, Number 1

AIMS AND SCOPE:
BJAMI publishes original research articles in the areas of applied mathematics and informatics.

Topics:
1. Computer science;
2. Computer and software engineering;
3. Information technology;
4. Computer security;
5. Electrical engineering;
6. Telecommunication;
7. Mathematics and its applications;
8. Articles of interdisciplinary of computer and information sciences with education, economics, environmental, health, and engineering.

BALKAN JOURNAL
OF APPLIED MATHEMATICS AND INFORMATICS
(BJAMI), Vol 7

ISSN 2545-4803 online
Vol. 7, No. 1, Year 2024

EDITORIAL BOARD

C O N T E N T

Balkan Journal of Applied Mathematics and Informatics
Volume VII Number 1 Year 2024

Online ISSN 2545-4803
UDC: 517.93:531.4

MODELLING A MASS-SPRING SYSTEM USING A SECOND-ORDER HOMOGENEOUS LINEAR ORDINARY DIFFERENTIAL EQUATION WITH CONSTANT COEFFICIENTS

VIOLETA KRCHEVA

Abstract. In this paper, a mass-spring system is considered. The system is modelled using a second-order homogeneous linear (ODE) with constant coefficients. Using this model, the behaviour of the system is studied. The most significant factor, the value of the damping, determines whether the case occurs: no damping, underdamping, critical damping, or overdamping. Each case is mathematically analysed to get parameters that impact how the motion system performs. The obtained solution, which demonstrates the behaviour of the system in a diagram plot of a displacement-time graph and a phase plane graph, is graphically presented in MATLAB software.

1. **Introduction**

A mass-spring system is a mechanical system consisting of three components: a mass (m) measured in $[kg]$, a damping coefficient (c) measured in $[Ns/m]$, and a spring constant (k) measured in $[N/m]$. To solve a mass-spring system (of a physical nature), the system must be formulated as a mathematical expression in terms of variables, functions, equations, and so forth. Such an expression is called a mathematical model of the system. The process of setting up the model, solving it mathematically, and interpreting the results in physical or other terms, is known as modelling. [5], [9]

Since many physical concepts, such as velocity and acceleration, are derivatives, the model of the system is very often an equation containing one or more derivatives of an unknown function. Such an equation, which is a relation between unknown functions and their derivatives, is known as an ordinary differential equation (ODE). A first-order ODE involves only the first derivative of the function, while a second-order ODE involves a second derivative of the function. A mass-spring system is a second-order linear ODE that has a variety of applications in science and engineering. The solution to this ODE, its properties, values, and graphs, interpreted in physical terms, lead to understanding the dynamic behaviour of the mass of the system. [3], [13], [15]

Mass-spring systems are extensively studied in a variety of mathematical literatures (for example [1] - [15]). They are often mathematically modelled and analysed using second-order homogeneous linear ODEs with constant coefficients (see [4] and [7]). Such systems can also be modelled in MATLAB software, where appropriate simulations are presented to understand the analysis of the system and predict its behaviour (see [7], [8], [13] and [14]).

Date: September 08, 2023.
Keywords. Mass, damping coefficient, spring constant, modelling, energy.

This paper is structured in five sections. After the introduction, the second section refers to the analysis of the mass-spring system using a second-order homogeneous linear ODE with constant coefficients. In the third section, the model of the system is created, and in the fourth section, the results are presented. According to the obtained results, the fifth section draws a conclusion and a potential application of the research.

2. Analysis of a Mass-Spring System Using a Second-Order Homogeneous Linear ODE with Constant Coefficients

An ordinary spring that resists compression as well as extension is suspended vertically from a fixed point (Fig. 1). At the lower end of the spring, a body of mass m is attached. It is assumed that the mass of the body is so large that the mass of the spring can be neglected. If the body is pulled down a certain distance and then released, it starts moving. That motion is assumed to be strictly vertical.

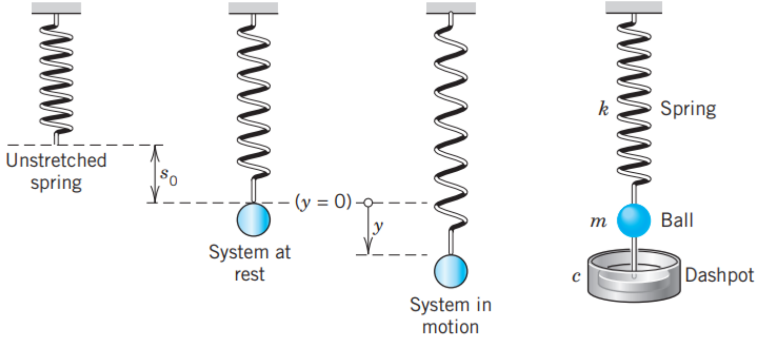


Figure 1. *Mechanical mass-spring system* [10]

The motion of the body, i.e., the displacement $y(t)$ as a function of time t, is determined by Newton's second law:

$$Mass\ x\ Acceleration = my'' = Force \tag{2.1}$$

where $y'' = d^2y/dt^2$ and "$Force$" is the resultant of all the forces acting on the body.

According to Figure 1a), the spring is first unstretched. When the body is attached, the spring stretches by the amount s_0 shown in the figure. This causes an upward force F_0 in the spring, which is proportional to the stretch:

$$F_0 = -ks_0 \tag{2.2}$$

where: k is the spring constant ($k > 0$), and the minus sign indicates that F_0 is directed upward, in the opposite (or negative) direction.

As a result of the extension s_0, the force F_0 in the spring balances the weight ($W = mg$) of the body (where g is the gravitational constant). Consequently: $F_0 + W = -ks_0 + mg = 0$. These forces have no effect on the motion. Therefore, the spring and the body are at rest, and the system is in a state known as 'static equilibrium'. The displacement $y(t)$ of the body is measured from this 'equilibrium point' as the origin $y = 0$ (Fig. 1b).

When the body is pulled down from this position (Fig. 1c), the spring stretches by some amount $y > 0$, which is the appropriate distance to the new position. By principle of Hooke's law, this displacement causes an additional upward force F_1 in the spring:

$$F_1 = -ky \tag{2.3}$$

Therefore, F_1 is also known as a restoring force because it tends to restore the system, i.e., to pull the body back to $y = 0$.

2.1. **Undamped system**

Every system has damping because, without it, it would move forever. But, at a practical level, the impact of damping may often be negligible. The only force in (2.1) that causes the movement is F_1. Accordingly, (2.1) gives the model $my'' = -ky$ or:

$$my'' + ky = 0 \tag{2.1.1}$$

The general solution to this equation is:

$$y(t) = A\cos\omega_0 t + B\sin\omega_0 t \tag{2.1.2}$$

where: $\omega_0 = \sqrt{k/m}$, and the particular motion is called a **harmonic oscillation**.

Given that the trigonometric function in (2.1.2) has an angular frequency ω_0 and a period $T = 2\pi/\omega_0$, the body performs $\omega_0/2\pi$ cycles per second. This is the frequency of the oscillation f, also called the natural frequency of the system. It is expressed as cycles per second, or hertz (Hz).

Considering the sum in the previous formula, it can be combined into a phase-shifted cosine with amplitude $C = \sqrt{A^2 + B^2}$ and phase angle $\delta = arc\tan(B/A)$, expressed as:

$$y(t) = C\cos(\omega_0 t - \delta) \tag{2.1.3}$$

It is obvious that the equation (2.1.2) is simpler in connection with initial value problems, while the equation (2.1.3) is physically more informative because it shows the amplitude and phase of the oscillation. Typical forms of harmonic oscillations (2.1.2) and (2.1.3) are illustrated in Figure 2a, all corresponding to some positive initial displacement $y(0)$ that determines $A = y(0)$ in (2.1.2) and different initial velocities $y'(0)$ that determine $B = y'(0)/\omega_0$.

2.2. **Damped system**

With the addition of a damping force:

$$F_2 = -cy' \tag{2.2.1}$$

to the model $my'' = -ky$, that results in $my'' = -ky - cy'$ or

$$my'' + cy' + ky = 0 \tag{2.2.2}$$

the system becomes a **damping system**.

From a physical point of view, this can be achieved by attaching the body to a dashpot (Fig. 1d). The new force is assumed to be proportional to the velocity $y' = dy/dt$, and the parameter c (called the damping coefficient) is always positive.

When y' is positive, the body is moving downward in the positive direction. Accordingly, the damping force $F_2 = -cy'$, as an upward force always acting in the opposite direction of the motion, i.e., in the negative direction, must be negative, $F_2 = -cy' < 0$, so $-c < 0$ and $c > 0$.

On the other hand, for an upward movement $y' < 0$ and a downward force $F_2 = -cy' > 0$, it follows that $-c < 0$ and $c > 0$ (as in the previous case).

In the case where the damping system is modelled by the second-order homogeneous linear ODE with constant coefficients (2.2.2), λ is a solution to the important characteristic equation (divided (2.2.2) by m):

$$\lambda^2 + \frac{c}{m}\lambda + \frac{k}{m} = 0 \tag{2.2.3}$$

and the exponential function:

$$y = e^{\lambda x} \tag{2.2.4}$$

is a solution to (2.2.2).

Depending on the amount of damping (much, medium, or little), there are three types of motion:

- $c^2 > 4mk$ (distinct real roots λ_1 and λ_2) → **Overdamping,**
- $c^2 = 4mk$ (a real double root) → **Critical damping**,
- $c^2 < 4mk$ (complex conjugate roots) → **Underdamping.**

The roots of the quadratic equation are:

$$\lambda_1 = -\alpha + \beta, \lambda_2 = -\alpha - \beta \tag{2.2.5}$$

where: $\alpha = c/(2m)$, and $\beta = (1/2m)\sqrt{c^2 - 4mk}$.

2.2.1. **Overdamping**

Since the amount of the damping coefficient c is so large that $c^2 > 4mk$, then λ_1 and λ_2 are distinct real roots. The corresponding general solution to (2.2.2) is:

$$y(t) = c_1 e^{-(\alpha-\beta)t} + c_2 e^{-(\alpha+\beta)t} \tag{2.2.6}$$

In a case like this, the body does not oscillate because the damping takes out energy so rapidly. When $t > 0$, both exponents in (2.2.6) are negative because of $\alpha > 0$, $\beta > 0$, and also $\beta^2 = \alpha^2 - k/m < \alpha^2$.

Consequently, both terms in (2.2.6) approach zero when $t \to \infty$. Practically speaking, the mass will be at rest at the static equilibrium position ($y = 0$) after a sufficiently long time. An example of characteristic motions in the overdamped case (2.2.6) for certain typical initial conditions is shown in Figure 2b and Figure 2c.

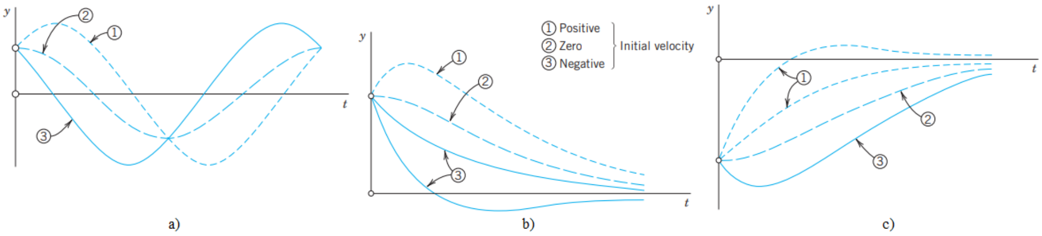


Figure 2. *a) Harmonic oscillations, (b) Positive and (c) Negative initial displacement in the overdamped case* [10]

2.2.2. **Critical damping**

Critical damping is the boundary between non-oscillatory motions and oscillations, i.e., the boundary case between overdamping and underdamping. In this case (when $c^2 = 4mk$), the characteristic equation has a real double root $\lambda_1 = \lambda_2 = -\alpha$, where $\alpha = c/(2m)$ and $\beta = 0$. Then the appropriate general solution is:

$$y(t) = (c_1 + c_2 t)e^{-\alpha t} \tag{2.2.7}$$

This solution can pass through the equilibrium position $y = 0$ once at most because $e^{-\alpha t}$ is never zero and the sum $c_1 + c_2 t$ can have at most one positive zero. If both c_1 and c_2 are positive (or both are negative), it has no positive zero, so y does not pass through zero at all. An example of characteristic motions in the case of critical damping (2.2.7) for certain typical initial conditions is illustrated in Figure 3a).

2.2.3. **Underdamping**

This case occurs when the damping coefficient c is so small that $c^2 < 4mk$. Then β in (2.2.5) is no longer real but imaginary, and:

$$\beta = i\omega^* \tag{2.2.8}$$

where: $\omega^* = (1/2m)\sqrt{4mk - c^2} = \sqrt{(k/m) - (c^2/4m^2)} \;\; > 0$.

The roots of the characteristic equation are complex conjugate:

$$\lambda_1 = -\alpha + i\omega^*, \lambda_2 = -\alpha - i\omega^* \tag{2.2.9}$$

where: $\alpha = c/(2m)$.

Consequently, the corresponding general solution is:

$$y(t) = e^{-\alpha t}(A \cos \omega^* t + B \sin \omega^* t) = Ce^{-\alpha t} \cos(\omega^* t - \delta) \tag{2.2.10}$$

where: $C^2 = A^2 + B^2$, and $\tan \delta = B/A$ as in (2.1.3).

This is a representation of damped oscillations. Their curve lies between the dashed curves $y = Ce^{-\alpha t}$ and $y = -Ce^{-\alpha t}$ in Figure 3b). It touches them when $\omega^* t - \delta$ is an integer multiple of π because at these points $\cos(\omega^* t - \delta)$ equals 1 or -1.

The frequency f in the underdamping case is $\omega^*/2\pi$ Hz. According to (2.2.8), the smaller damping coefficient c (> 0) is, the angular frequency ω^* is larger, and the oscillations become more rapid. If c approaches zero, then ω^* approaches $\omega_0 = \sqrt{k/m}$ and the harmonic oscillation (2.1.2), whose frequency is $\omega_0/2\pi$, becomes the natural frequency of the system.

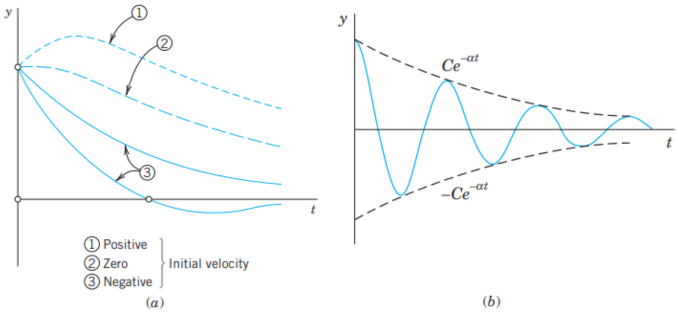


Figure 3. *(a) Critical damping, (b) Damped oscillation in the underdamping case* [10]

3. Setting up the model

The physical model of the mass-spring system and its characteristics were explained in Figure 1. The amount of damping coefficient, as a significant parameter that affects the system and causes various cases of damping oscillations, was also exposed in Figures 2 and 3. Four particular cases of motion in the damping system, depending on the presence and amount of the damping, were modelled and expressed by the ODEs specified above.

Considering that the components of a spring-mass system are mass m, damping coefficient c, and spring constant k, an unforced, damped oscillator can be mathematically modelled by the second-order homogeneous linear ODE with constant coefficients in the form of (2.2.2).

The values of the mass ($m = 1$), and the spring constant ($k = 16$) are assumed to be constant, while the value of the damping coefficient gradually increases in the four following cases:

- $c = 0 \rightarrow$ **Case of no damping,**
- $c = 2 \rightarrow$ **Case of under damping**,
- $c = 8 \rightarrow$ **Case of critical damping,**
- $c = 10 \rightarrow$ **Case of overdamping**.

The ODE (2.2.2) is mathematically analysed and solved for each case, and an appropriate mathematical model based on the values of the components is obtained.

These cases are also modelled and solved in the Matlab software with the function 'Van der Pol (VDP) oscillator', a simulation of the Van der Pol differential equation. In order to use this function, the second-order ODE (2.2.2) is first converted to a system of first-order ODEs, i.e., when:

$$y_1' = y_2,\ y_2' = y_1'' \text{ and } y_1 = y \tag{3.1}$$

then:

$$my_2{}' + cy_2 + ky_1 = 0 \tag{3.2}$$

The equation (3.2) can be written as:

$$my_2' = -cy_2 - ky_1 \tag{3.3}$$

where:

$$y_2' = \frac{-cy_2 - ky_1}{m} = y'' \tag{3.4}$$

Hence, the resulting system of first-order ODEs is the following:

$$\begin{cases} y_1' = y_2 \\ y_2' = \frac{-cy_2 - ky_1}{m} \end{cases} \tag{3.5}$$

The Matlab code for this system is written as:

Matlab Code 1

```
function dydt = vdpt(t,y,c,k,m);
dydt = zeros (2,1);
dydt(1) = y(2);
dydt(2) = (-c*y(2)-k*y (1)) /m;
```

To compute the solution in the software, the initial conditions $y(0) = 1$ and $y'(0) = 0$ over the interval $[0,20]$ are taken. They are the same in the four cases, for which a plot of displacement versus time, i.e., a plot of the current position of the displacement as a function of time, and a plot of y' versus y, i.e., a phase plane plot, are particularly computed.

4. Results and discussion

The behaviour of the spring-mass system with a variable value of the damping coefficient, which is modelled using (2.2.2) for four cases, is explained in following.

- ***Case of no damping*** → $m = 1, c = 0,$ *and* $k = 16$**,**

In the case of no damping, the mass $m = 1$, the damping coefficient $c = 0$, and the spring constant $k = 16$, the mathematical model of (2.1.1) has the form:

$$y'' + 16y = 0 \tag{4.1}$$

According to (2.1.3), the general solution is:

$$y = \cos 4x \tag{4.2}$$

This equation represents the motion of the mass on the spring, graphically shown in Figure 4a) on a plot of the current position of the displacement as a function of time. It is obvious that the mass undergoes harmonic oscillation, which takes between +1 and -1 for the value of the amplitude C, repeats itself over period $T = 1.57$, has the angular frequency $\omega_0 = 4$, the frequency of oscillations $f = 0.64Hz$, and the phase angle $\delta = 0$. This indicates that the oscillation is almost equal to or strictly equal to its natural frequency of the system.

The phase space plot in Figure 4b) clearly shows the phase relationship between velocity and position. As the spring oscillates, the phase diagram creates a clockwise ellipse. This direction is determined by the negative sign on the equation of motion (2.2). Without damping, the system will perform endless motion with a specified velocity in this phase space.

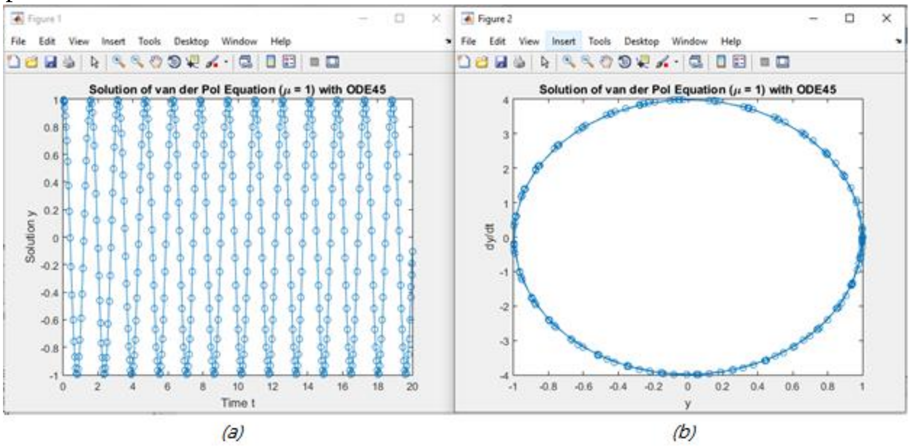


Figure 4. *(a)* Plot of y versus t, *(b)* Plane plot of y' versus y

- ***Case of underdamping*** → $m = 1, c = 2,$ *and* $k = 16$**,**

In the case of underdamping, the mass $m = 1$, the damping coefficient $c = 2$, and the spring constant $k = 16$, the mathematical model of (2.2.2) has the form:

$$y'' + 2y' + 16y = 0 \tag{4.3}$$

The roots of the characteristic equation are complex conjugate (2.2.9):

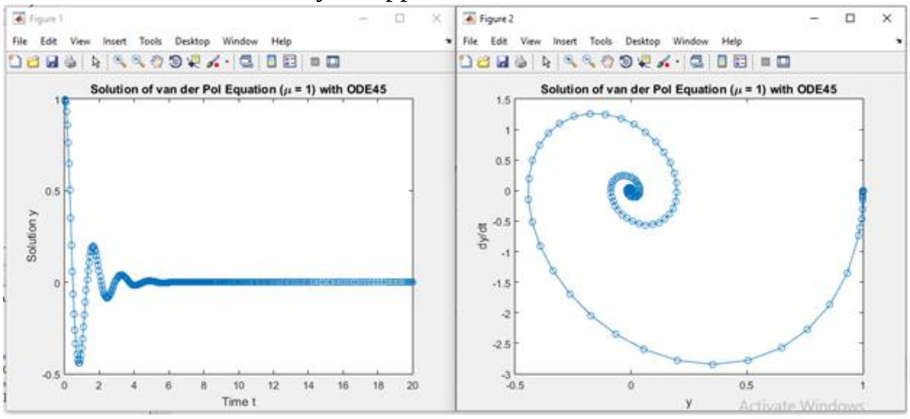
$$\lambda_1 = -1 + i\sqrt{15}, \lambda_2 = -1 - i\sqrt{15} \tag{4.4}$$

Solving the coefficients $A = 1$, $B = 0$, $C = 1$, $\alpha = 1$, $\beta = i\sqrt{15}$, the angular frequency $\omega^* = \sqrt{15}$, and the phase angle $\delta = 0$, the general solution (2.2.10) is:

$$y = e^{-x} \cos \sqrt{15}x \tag{4.5}$$

This equation represents the underdamped motion with a small amount of damping, which is illustrated in Figure 5a) on a plot of the current position of the displacement as a function of time. It is significant that the amplitude gradually decreases (to zero), but the period and the frequency are nearly the same as if the system were not damped at all. This happens because of the system losing energy because of the active damping force.

The phase plane plot in Figure 5b) also shows that the system is losing energy. As the amplitude of the position decreases, the phase diagram spirals inward, suggesting that the motion of the system will be oscillating with decreasing amplitude. The direction of the trajectory, according to the initial conditions, starts from zero and continues in a clockwise direction again (as in the case of no damping), indicating that the energy continuously decreases with time and eventually disappears.



(a) (b)

Figure 5. *(a)* Plot of y versus t, *(b)* Plane plot of y' versus y

When the amount of damping in a system gradually increases, the period and frequency begin to be affected. The damping force opposes the forces in the spring and slows the motion of the spring in both directions. In cases where the amount of damping is larger (the next two cases), the system does not even oscillate, but it slowly moves towards equilibrium.

- ***Case of critical damping*** $\rightarrow m = 1, c = 8,$ *and* $k = 16$**,**

In the case of critical damping, the mass $m = 1$, the damping coefficient $c = 8$, and the spring constant $k = 16$, the mathematical model of (2.2.2) has the form:

$$y'' + 8y' + 16y = 0 \tag{4.6}$$

The roots of the characteristic equation are complex conjugate (2.2.5):

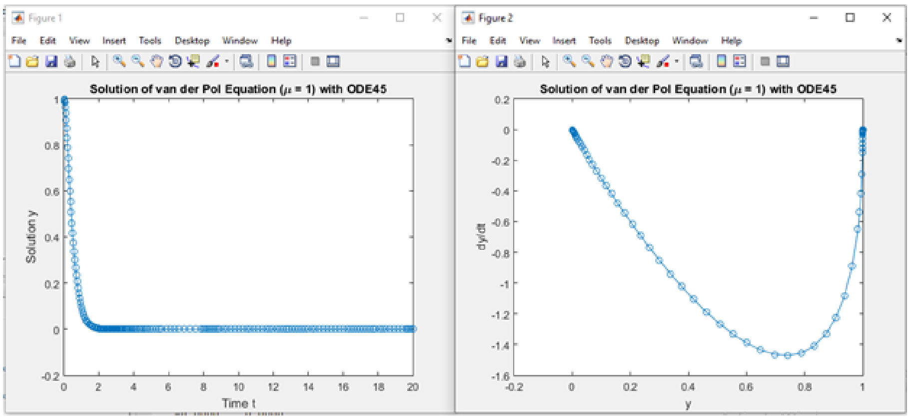
$$\lambda_1 = -4, \lambda_2 = -4 \tag{4.7}$$

Solving the coefficients $C_1 = 1, C_2 = 4, \alpha = 4,\ \beta = 0$, the general solution (2.2.7) is:

$$y = (1 + 4)e^{-4x} \quad (4.8)$$

This equation represents the case of critical damping by the curve in Figure 6a) where a plot of displacement versus time is given. It is obvious that the spring returns to equilibrium (at $y = 0$) rapidly and remains in that position without exceeding or oscillating about it.

The phase plane plot in Figure 6b) is radically different from that of the no damping and underdamping cases. It illustrates that the direction of the trajectory, according to the initial conditions, starts from zero and continues in a clockwise direction (as in the previous cases), indicating that the velocity of the mass starts from zero, reaches its maximum value, and in the shortest possible time, i.e., for the smallest displacement, returns to zero.



(a) *(b)*

Figure. *(a)* Plot of y versus t, *(b)* Plane plot of y' versus y

- ***Case of overdamping*** $\rightarrow m = 1, c = 10,$ *and* $k = 16$**,**

In the case of overdamping, the mass $m = 1$, the damping coefficient $c = 10$, and the spring constant $k = 16$, the mathematical model of (2.2.2) has the form:

$$y'' + 10y' + 16y = 0 \tag{4.9}$$

The roots of the characteristic equation are complex conjugate (2.2.5):

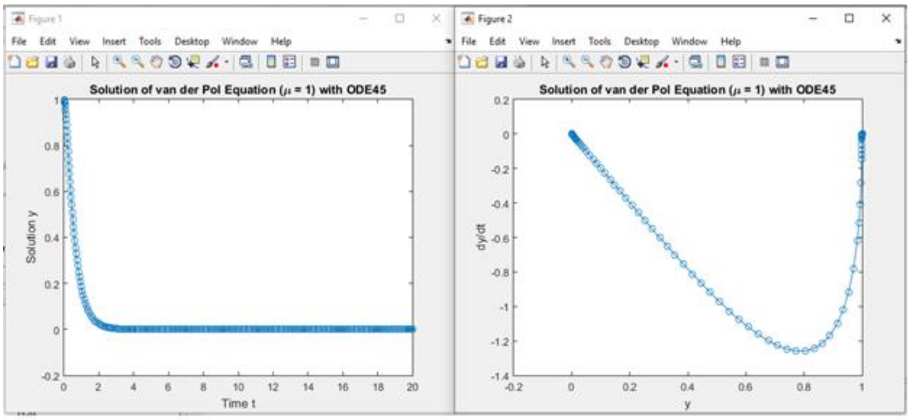
$$\lambda_1 = -2, \lambda_2 = -8 \tag{4.10}$$

Solving the coefficients $C_1 = 4/3, C_2 = -1/3, \alpha = 5,\ \beta = 3,$ the general solution (2.2.6) is:

$$y = \frac{4}{3}e^{-2x} - \frac{1}{3}e^{-8x} \tag{4.11}$$

The curve in Figure 7a) illustrates the function of displacement versus time for the overdamping case. It returns (exponentially decays) to equilibrium without exceeding or oscillating about it. So, the difference between the overdamping case and the critical damping case is that in the critical damping case oscillations do not happen at all, and the amplitude of oscillation in the overdamping case moves more slowly towards equilibrium than in the critically damped system.

The phase plane plot in Figure 7b) can also be considered similar for the critical damping case. The direction of the trajectory in the phase plane starts from zero, continues in a clockwise direction again, reaches its maximum value, and returns to zero, confirming that the mass is slowly reaching equilibrium.



(a) (b)

Figure 7. *(a)* Plot of y versus t, *(b)* Plane plot of y' versus y

5. **Conclusion**

In this paper, an analysis of a mass-spring system using a second-order homogeneous linear ODE with constant coefficients is presented. Considering the value of the damping as the most significant component of the system, four different cases are analysed with exact equations, solved analytically, and computed in MATLAB software.

In real mass-spring systems, the damping slows the motion of the systems. In the case of underdamping, the mass oscillates with amplitude decreasing to zero in comparison to the case of no damping. In the case of critical damping, the mass returns to equilibrium as fast as possible without oscillating, and in the overdamping case, the mass returns to equilibrium for a longer period of time, compared to the critically damped case, without oscillating.

Therefore, the main advantage of the study in this paper is the opportunity to examine the impact of the presence and value of damping in the mass-spring system in order to visualise and monitor the dissipation of the energy stored in the system. Despite the variable value of the damping, with variations in the value of the mass, spring constant, initial conditions, or interval in MATLAB software (using the created code), different mass-spring systems can be examined.

This study is further recommended for analysis and research when discussing the potential energy (which depends on the position) and kinetic energy (which depends on the velocity of the mass) when attempting to make an appropriate application of the mass-spring systems.

References

[1] *Boscolo, I and Loewenstein, R* (2011). The spring-mass experiment as a step from oscillations to waves: mass and friction issues and their approaches, Latin-American Journal of Physics Education, Vol. 5, No. 2, ISSN 1870-9095, pp. 409-417

[2] *Cveticanin, L and Zukovic, M and Cveticanin, D* (2019). Steady state vibration of the periodically forced and damped pure nonlinear two-degree-of-freedom oscillator, Journal of Theoretical and Applied Mechanics, Vol. 57, No. 2, DOI 10.15632/jtam-pl/105463, pp. 445-460

[3] *De la Cruz, S., T and Rodriguez, M., A and Hernandez, V* (2012). Using Spring-Mass Models to Determine the Dynamic Response of Two-Story Buildings Subjected to Lateral Loads, 15th World Conference on Earthquake Engineering, Vol. 31

[4] *Ghosh, S and Talukdar, B and Das, U* (2009). One-way coupled Van der Pol system, DOI 10.48550/arXiv.0910.1700,

[5] *Gomez, J., F and Rosales, J., J and Bernal, J., J* (2012). Mathematical modelling of the mass-spring-damper system – A fractional calculus approach, Acta Universitaria, Vol. 22, No. 5, ISSN 0188-6266, pp. 5-11

[6] *Gowri, P and Deepika, D and Krithika, S* (2017). A Case Study on Simple Harmonic Motion and Its Application, International Journal of Latest Engineering and Management Research, Vol. 2, No. 8, ISSN 2455-4847, pp. 55-64

[7] *Gunawan, M., S* (2017). Analysis of Coupled Mass-Spring-Damper System by Changing Spring Constant, Mass, and Force, Annual Conference on Management and Information Technology, Vol. 4, No. 1, DOI 10.33555/acmit.v4i1.62, pp. 71-91

[8] *Ingale, A and Daga, A and Naik, R* (2018). Modeling Mass-Spring-Damper System using Simscape, International Journal of Engineering Research and Application, Vol. 8, No. 1, ISSN 2248-9622, pp. 30-33

[9] *Kreyszig, E* (2006). Advanced Engineering Mathematics, 9th ed., John Wiley & Sons, Hoboken, USA

[10] *Kreyszig, E* (2011). Advanced Engineering Mathematics, 10th ed., John Wiley & Sons, Hoboken, USA

[11] *Li, K and Darby, A., P* (2009). Modelling a buffered impact damper system using a spring-damper model of impact, The Journal of the International Association for Structural Control and Monitoring, Vol. 16, No. 3, DOI 10.1002/stc.238, pp. 287-302

[12] *Liu, T and Bargteil, A., W and O'Brian, J., F and Kavan, L* (2013). Fast Simulation of Mass-Spring Systems, ACM Transactions on Graphics, Vol. 32, No. 6

[13] *Sharma, R and Pathak, D., K and Dwivedi, V., K* (2014). Modeling & Simulation of Spring Mass Damper System in Simulink Environment, XVIII Annual International Conference of the Society of Operations Management, pp. 205-210

[14] *Skwame, Y and Bakara, A., I and Sunday, J* (2017). Computational Method for the Determination of Forced Motions in Mass-spring Systems, Asian Research Journal of Mathematics, Vol. 3, No. 1, ISSN 2456-477X, pp. 1-12

[15] *Sunday, J* (2017). Systems Theory: An Approach to Mass-Damper-Spring and Mass-Nondamper-Spring Systems, General Letters in Mathematic, Vol. 3, No. 3, ISSN 2519-9277, pp. 169-176

Violeta Krcheva
"Goce Delchev" University
Faculty of Mechanical Engineering,
"Krste Misirkov", No.10-A, Stip
Republic of North Macedonia
E-mail address: violeta.krceva@ugd.edu.mk