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The Appendix

In honor of the first Doctor of Mathematical Sciences Acad. Blagoj Popov, a mathematician dedicated to differential equations, the idea of holding the "Day of Differential Equations" was born, prompted by Prof. Ph.D. Boro Piperevski, Prof. Ph.D. Borko Ilievski, and Prof. Ph.D. Lazo Dimov. Acad. Blagoj Popov presented his doctoral dissertation on 05.05.1952 in the field of differential equations. This is the main reason for holding the "Day of Differential Equations" at the beginning of May.

This year on May 5th, the "Day of Differential Equations" was held for the seventh time under the auspices of the Faculty of Computer Sciences at "Goce Delcev" University in Stip and Dean Prof. Ph.D. Saso Koceski, organized by Prof. Ph.D. Biljana Zlatanovska, Prof. Ph.D. Marija Miteva and Prof. Ph.D. Limonka Koceva Lazarova.

The participants of this event were:

1. Prof. Ph.D. Aleksa Malcheski from the Faculty of Mechanical engineering at Ss.Cyril and Methodius University in Skopje;
2. Prof. Ph.D. Slagjana Brsakoska from the Faculty of Natural Sciences and Mathematics at Ss.Cyril and Methodius University in Skopje;
3. Prof. Ph.D. Natasa Koceska, Prof. Ph.D. Limonka Koceva Lazarova, Prof. Ph.D. Marija Miteva and Prof. Ph.D. Biljana Zlatanovska from the Faculty of Computer Sciences at Goce Delcev University in Stip;
4. Ass. Prof. Ph.D. Biljana Citkuseva Dimitrovska and Ass. M.Sc. Maja Kukuseva Panova from the Faculty of Electrical Engineering at Goce Delcev University in Stip.

Acknowledgments to Prof. Ph.D. Boro Piperevski, Prof. Ph.D. Borko Ilievski and Prof. Ph.D. Lazo Dimov for the wonderful idea and the successful realization of the event this year and in previous years.

Acknowledgments to the Dean of the Faculty of Computer Sciences, Prof. Ph.D. Saso Koceski for her overall support of the organization and implementation of the "Day of Differential Equations".

The papers that emerged from the "Day of Differential Equations" are in the appendix to this issue of BJAMI.

CAN CHATGPT BE USED FOR SOLVING ORDINARY DIFFERENTIAL EQUATIONS

SASO KOCESKI, NATASA KOCESKA, LIMONKA KOCEVA LAZAROVA, MARIJA MITEVA AND
BILJANA ZLATANOVSKA

Abstract. In this research we have conducted an empirical study to evaluate the capabilities of OpenAI's chat bot ChatGPT for automated software code generation and programming numerical methods for solving various types of differential equations. We have tested the ChatGPT for analytical as well as numerical solution of the first and second order ordinary differential equations. The obtained results suggest that ChatGPT is a promising AI tool for programming numerical algorithms and solving differential equations. However, there are still some limitations and challenges of using AI for numerical solution generation, related to the potential biases in the algorithm, as well as the need for a large amount of high-quality training data. However, as AI technology continues to develop, it is likely that the use of AI for programming will become more prevalent and effective in the future.

1. Introduction

A differential equation is an equation that contains one or more functions with its derivatives, which in turn define the rate of change of a function at a point. Differential equations are essential in cases where unevenly flowing processes are studied, that is, processes in which the change rate of the quantities varies over time or depends on the values of the quantities themselves. Since natural phenomena and physical processes are also based on certain changes, there are huge possibilities of applying differential equations for solving real-life problems. Differential equations are used to describe the motion of planets in the solar system, the motion of electrons in an atom, the electrical oscillations in a radio generator, the computation of a nuclear reactor, the wave distribution, the propagation of light and sound in the atmosphere and many other things. They can be used in various fields such as physics, engineering, biology, chemistry and so on [1-6].

Differential equations can be classified into several types based on the highest derivative (differential equations of first order, second order, third order, etc.), the power of derivatives (linear differential equations, quadratic equation, etc.) and the number of independent variables (ordinary differential equations and partial differential equations), [7, 8].

An ordinary differential equation of the n -th order is of the form:

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots, \frac{d^ny}{dx^n}\right) = 0 \quad (1.1)$$

Its general solution contains n arbitrary constants and is of the form:

$$\varphi(x, y, C_1, C_2, \dots, C_n) = 0 \quad (1.2)$$

Keywords. ChatGPT, Ordinary Differential Equations (ODE), Artificial Intelligence (AI), numerical solution.

To obtain its particular solution, conditions must be given so that the constants C_1, C_2, \dots, C_n can be determined. If these conditions are prescribed at one point only, then the differential equation together with the conditions constitute an initial value problem of the n -th order. If the conditions are prescribed at two or more points, then the problem is termed as a boundary value problem.

Although various analytical methods for determining general solutions of differential equations of different types exist, solving differential equations is still a challenging mathematical problem that requires significant human effort. Many types of differential equations cannot be solved with an analytical method but can be solved with numerical methods. Differential equations which can be solved analytically can also be solved numerically.

The following methods are the most popular for finding numerical solutions of ordinary differential equations from the first and second order [9]:

- Picard's method - is an iterative technique used to solve ordinary differential equations. It involves breaking down the original differential equation into a series of simpler equations and then approximating the solution using successive iterations until convergence is achieved.

- The Taylor series method - is a numerical approach that approximates the solution of a differential equation using the Taylor series expansion. It computes higher-order derivatives of the unknown function to achieve greater accuracy in the approximation.

- Euler's method - is a simple and widely used numerical technique for solving ordinary differential equations. It approximates the solution by taking small steps along the direction of the derivative at each point, making it easy to implement but less accurate for complex problems.

- The Runge-Kutta method - is a family of numerical techniques for solving both ordinary and partial differential equations. The most used is the fourth order Runge-Kutta method, which provides higher accuracy compared to the Euler's method by evaluating the derivative at multiple points within each step.

- Milne's method - is an explicit method used to solve ordinary differential equations. It is based on a predictor-corrector approach and uses three consecutive points to estimate the next value, making it more accurate than some lower-order methods.

- The Adams-Bashforth method - is a family of explicit numerical methods for solving ordinary differential equations. It uses previous function values to predict the next value, making it efficient for solving initial value problems. It is often combined with the Adams-Moulton method to form predictor-corrector pairs for higher accuracy.

Each of these numerical methods has its own advantages and limitations, and the choice of method used in a particular case depends on the specific characteristics of the differential equation and the level of accuracy required for the given problem.

Solving differential equations can be more complex than solving numerical equations, and specialized software is often used to tackle these problems efficiently. Some popular applications and programming languages used for solving differential equations are the following:

- **MATLAB** – is a versatile mathematical software widely used for solving differential equations. It provides built-in functions and solvers for ordinary differential equations (ODEs) and partial differential equations (PDEs). Engineers, scientists, and researchers commonly use MATLAB for simulations and numerical solutions involving differential equations [10].

- **Mathematica** - a powerful mathematical software that can handle differential equations symbolically and numerically. It has a wide range of built-in functions for solving various types of ODEs and PDEs and offers visualization capabilities for better understanding of solutions [11].

- **Wolfram Alpha** - an online computational knowledge engine that can solve a wide range of mathematical problems, including differential equations. It can handle various types of equations and provide step-by-step solutions [12].

- **COMSOL Multiphysics** - a powerful simulation software that allows engineers and scientists to solve complex coupled ODEs and PDEs. It is particularly useful for modeling physical systems with multiple interconnected phenomena [13].

- **Maple** - is a comprehensive mathematical software that can handle symbolic and numerical solutions of differential equations. It is commonly used in academia and industry for mathematical modeling and simulations [14].

- **FORTRAN** - is a general-purpose, compiled imperative programming language that is especially suited to numeric computation and scientific computing. ODEPACK is a collection of Fortran solvers for the initial value problem for ordinary differential equation systems [15]

- **Python** - a popular programming language for scientific computing. Several libraries are available for solving differential equations [16, 17], such as SciPy (a library that provides numerical methods for solving ODEs and PDEs. It is built on NumPy and offers several solvers and functions for mathematical operations), SymPy (symbolic mathematics library, which means it can manipulate equations symbolically and solve ODEs and PDEs analytically).

These software tools offer different features and capabilities, so the choice depends on the complexity of the differential equations and the specific needs of the user. Some tools are better suited for symbolic manipulation, while others are more focused on numerical solutions and simulations.

The calculations of differential equations are complex and computationally intensive, so high-end supercomputers are used to do the math. However, even when using supercomputers, the time for solving the equations is computationally high; that is why the Artificial Intelligence (AI) field has taken a particular interest in these equations. The latest development of Artificial Intelligence (AI) and models capable of mimicking human behavior have already shown their strength and capabilities in finding creative solutions to multiple problems in various fields [18], starting from tourism [19] through medicine [20, 21, 22], biology [23, 24], education [25, 26, 27, 28], robotics [29, 30, 31], and also economy [32].

ChatGPT (Chat Generative Pre-trained Transformer) [33] is a natural language processing model, developed by OpenAI [34], that can generate conversation-style responses to user input. It is the latest among a class of large language models (LLMs)

known as autoregressive language models [35], trained on a very large corpus of text data. At its core, ChatGPT uses deep learning techniques, specifically transformer neural networks, to process text and generate responses based on the patterns it learns from training data. ChatGPT is known for its ability to learn and adapt to user feedback. For instance, when users interact with the chatbot, they can rate the quality of the interaction. This rating is then used by the language model to refine itself and improve its performance over time.

The effectiveness of ChatGPT has been tested on various tasks and in various fields, like mathematics [36-40], medicine [41, 42], healthcare [43], physics [44, 45], education [46, 47] etc.

However, the use of ChatGPT for finding a numerical solution of differential equations is still at early stages [48, 49]. Therefore, we have decided to investigate the ChatGPT's capability for writing numerical algorithms for solving differential equations.

2. Methodology

In our research we were interested in discovering the frequency of correct answers and explanations of ChatGPT to the problems defined in our dataset. To investigate this, we have created a dataset composed of 50 differential equations (25 of them were first order and the other 25 second order equations). They were extracted from various academic books and textbooks available on the Internet, as well as our own written differential equations that could not be found in any of the previous sources.

After creating the database, we started with the experimental phase, which involved the conversation with ChatGPT. Questions were posted to ChatGPT in a single session. The answers generated by ChatGPT were graded by three independent domain experts using the 5 Likert scale (1-lowest grade, 5-highest grade). The final mark was obtained as an average of the experts' grades.

To investigate the efficiency of ChatGPT for solving differential equations we analyzed the following research question:

Q1: How frequent is ChatGPT providing correct methodological approach and correct result?

Q2: For which method does ChatGPT give the most accurate and reliable answer?

3. Evaluation

One way to get the best results from ChatGPT is to assign it a role. This is a fantastic method for getting more appropriate responses to your prompts. Asking ChatGPT a question will always produce a response of some sort, but its relevance might not be suited to your requirements.

Therefore, before starting our experimental interactive session, we assigned a role to ChatGPT and tried to test its initial theoretical knowledge about the specific subject. For this purpose, a free version of ChatGPT was used.

One such conversation is the following:

- » **S** Please act as mathematician and expert in numerical analysis. Can you please list me the methods for step-by-step methods for numerical solution of first and second order ordinary differential equations with a short explanation for each?

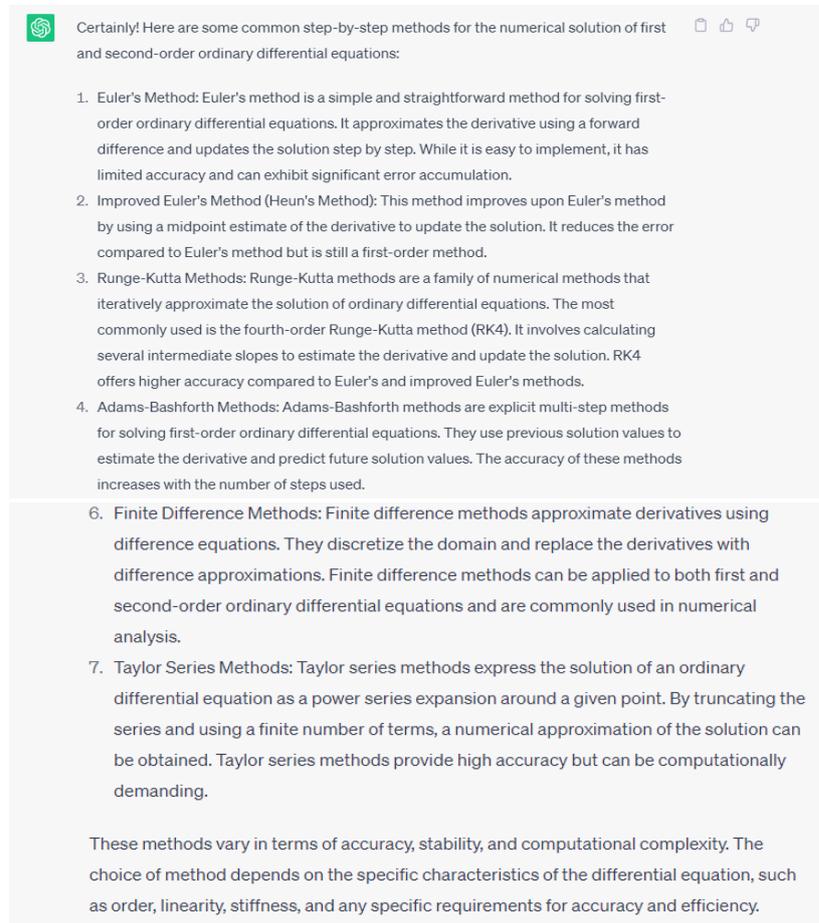
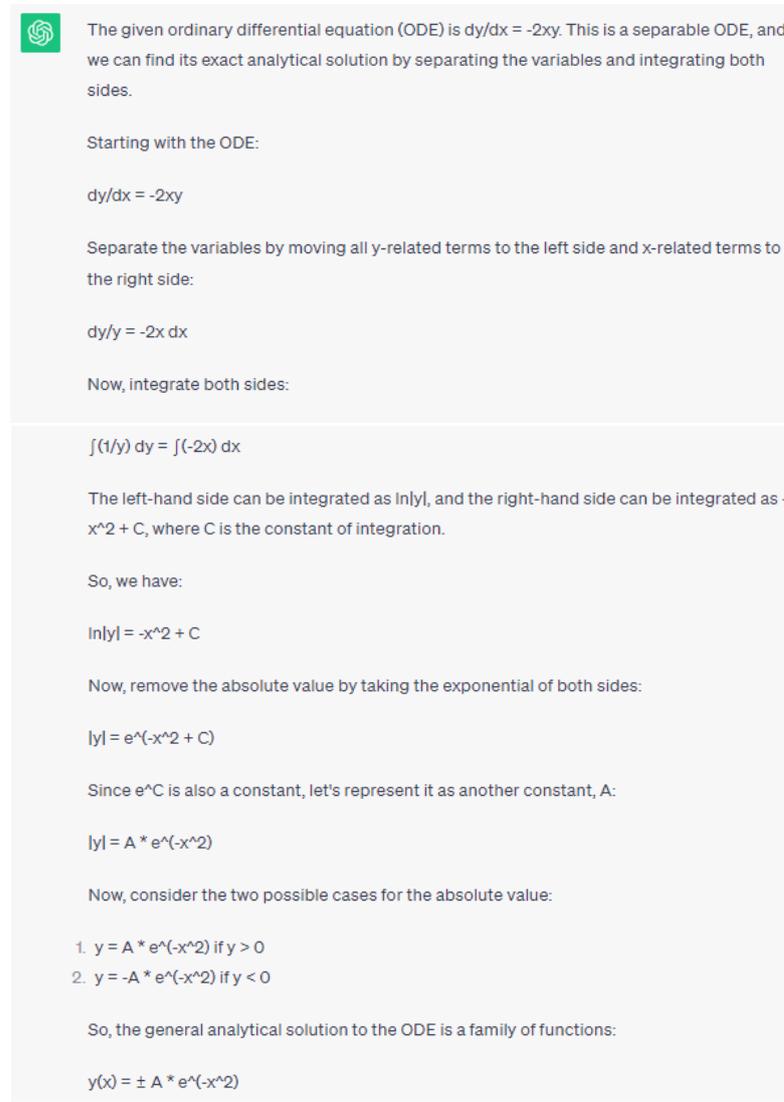


Figure 1. Conversation with ChatGPT when assigned a role

After that we proceeded with differential equations, which was the goal of our research. Each conversation with ChatGPT contained the following elements:

- Definition of the equation, definition of the initial conditions;
- Requirement to write the numerical solution using a particular method, plot the solution and find the value of y at given points.

We started with the first order ODE. The sample prompts and the received answer are given in the following:



The given ordinary differential equation (ODE) is $dy/dx = -2xy$. This is a separable ODE, and we can find its exact analytical solution by separating the variables and integrating both sides.

Starting with the ODE:

$$dy/dx = -2xy$$

Separate the variables by moving all y-related terms to the left side and x-related terms to the right side:

$$dy/y = -2x dx$$

Now, integrate both sides:

$$\int (1/y) dy = \int (-2x) dx$$

The left-hand side can be integrated as $\ln|y|$, and the right-hand side can be integrated as $-x^2 + C$, where C is the constant of integration.

So, we have:

$$\ln|y| = -x^2 + C$$

Now, remove the absolute value by taking the exponential of both sides:

$$|y| = e^{-x^2 + C}$$

Since e^C is also a constant, let's represent it as another constant, A:

$$|y| = A * e^{-x^2}$$

Now, consider the two possible cases for the absolute value:

1. $y = A * e^{-x^2}$ if $y > 0$
2. $y = -A * e^{-x^2}$ if $y < 0$

So, the general analytical solution to the ODE is a family of functions:

$$y(x) = \pm A * e^{-x^2}$$

Figure 2. Sample conversation showing ChatGPT capabilities for solving a first order ODE

Then we asked ChatGPT to generate a programming code in Python programming language for the same ODE. The generated code was manually transferred to Google Colaboratory, where it was analyzed, debugged, and executed.

S

Can you please solve the following ODE using Runge-Kutta 4th order method, $dy/dx = -2xy$, with initial condition $y(0) = 2$, on the interval from 0 to 3, with step of $h = .25$? Please write me a code in python that will draw the solution.



Certainly! To solve the ordinary differential equation (ODE) $dy/dx = -2xy$ with the initial condition $y(0) = 2$ on the interval from 0 to 3 using the fourth-order Runge-Kutta method with a step size of $h = 0.25$, you can use the following Python code:

```
python Copy code

import numpy as np
import matplotlib.pyplot as plt

# Define the ODE dy/dx = -2xy
def ode(x, y):
    return -2 * x * y

# Initial condition
x0 = 0.0
y0 = 2.0

# Time interval and step size
x_end = 3.0
h = 0.25

# Create arrays to store the values
x_values = np.arange(x0, x_end + h, h)
y_values = np.zeros(len(x_values))

# Set the initial value
y_values[0] = y0

# Perform the fourth-order Runge-Kutta integration
for i in range(0, len(x_values) - 1):
    x = x_values[i]
    y = y_values[i]

    k1 = h * ode(x, y)
    k2 = h * ode(x + h/2, y + k1/2)
    k3 = h * ode(x + h/2, y + k2/2)
    k4 = h * ode(x + h, y + k3)

    y_values[i+1] = y + (k1 + 2*k2 + 2*k3 + k4) / 6

# Plot the solution
plt.figure(figsize=(10, 6))
plt.plot(x_values, y_values, label='Numerical Solution', marker='o')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Solution of dy/dx = -2xy using Runge-Kutta (h=0.25)')
plt.grid(True)
plt.legend()
plt.show()
```

This code uses the fourth-order Runge-Kutta method to solve the ODE and then plots the solution over the specified interval. The result will show how y changes as x varies from 0 to 3 with a step size of 0.25.

Figure 3. Sample conversation showing ChatGPT code generation capabilities

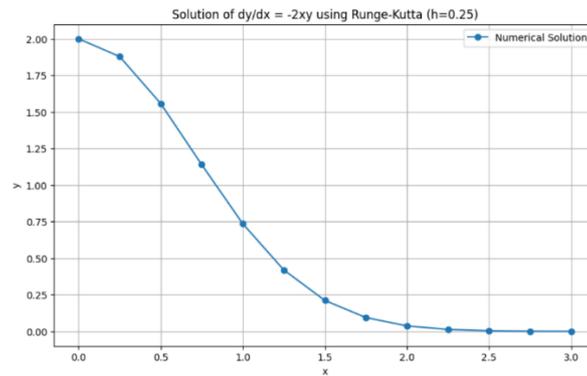


Figure 4. ChatGPT generated code in Google Colaboratory environment

We continued with the second order ODE. The sample prompts and the received answers are given in the following:

S Can you please solve the following second order ODE $y'' - 6y' + 5y = 0$

G Certainly! To solve the second-order ordinary differential equation (ODE) $y'' - 6y' + 5y = 0$, we can start by finding the characteristic equation and then use it to find the general solution. The characteristic equation is obtained by assuming a solution of the form $y(x) = e^{rx}$ and substituting it into the ODE. This leads to a quadratic equation in terms of r .

The characteristic equation for the given ODE is:

$$r^2 - 6r + 5 = 0$$

Now, we can solve this quadratic equation for r . Factoring it, we get:

$$(r - 5)(r - 1) = 0$$

Setting each factor equal to zero gives us two possible values for r :

1. $r = 5$
2. $r = 1$

These values of r correspond to the roots of the characteristic equation.

Now, we'll use these roots to write down the general solution of the ODE:

1. For $r = 5$:
The solution corresponding to this root is $y_1(x) = e^{5x}$.
2. For $r = 1$:
The solution corresponding to this root is $y_2(x) = e^x$.

The general solution of the ODE is a linear combination of these solutions:

$$y(x) = C_1 * e^{5x} + C_2 * e^x$$

Here, C_1 and C_2 are arbitrary constants that depend on the initial conditions of the problem.

If you have specific initial conditions (e.g., $y(0)$ and $y'(0)$), you can use them to determine the values of C_1 and C_2 and obtain the particular solution that satisfies those conditions.

Figure 5. Sample conversation showing ChatGPT capabilities for solving a second order ODE

4. Results and discussion

ChatGPT demonstrates proficiency in providing precise definitions and explanations of mathematical concepts. It performs the role of an expert/mathematician very well and gives detailed answers to the mathematical questions asked. Therefore, we can state that ChatGPT can be used as an additional learning resource. However, it should never supersede human expertise and knowledge.

Regarding the research question (Q1), the experiments revealed that ChatGPT is capable of providing correct methodological approach and correct result with an accuracy of about 72% and 64%, for first and second order ordinary differential equations, respectively (Table 1). There was no significant difference in the application's capability to solve both types of ODEs. Most of the results differ by a few decimals than the results obtained manually. This does not mean that ChatGPT made a mistake because calculating manually we work with less decimals from the beginning and there we also lose exactness.

Table 1. *ChatGPT responses in terms of solutions and methodological approach*

Type of equation (Solutions in a single thread)	Correct solution and methodological approach	Partially correct solution or methodological approach	Wrong solution and wrong methodological approach
First order ODE	18 (72%)	5 (20%)	2 (8%)
Second order ODE	16 (64%)	6 (24%)	3 (12%)

Regarding the second research question (Q2), the opinion of the experts is the most consistent about ChatGPT answers for the solution of ODEs obtained using the Runge-Kutta method. This conclusion is relevant for both first and second order ODEs, as shown in Table 2. The obtained results also suggest that ChatGPT is a promising AI tool for programming numerical algorithms and solving differential equations. It can generate a fully usable code in Python that could be compiled with no or minimal human intervention.

Table 2. *Average grades for the solutions obtained by ChatGPT of first and second order ODEs, with respect to the method used.*

	First order ODE	Second order ODE
Picard's method	3.97	3.92
Taylor's series method	3.95	3.88
Euler's method	4.11	4.03
Runge-Kutta method	4.21	4.3
Milne's method	3.6	3.69
Adams-Bashforth method	3.75	

As a conclusion we can say that although ChatGPT has demonstrated a strong conceptual understanding of the subject and ability to provide numerical and analytical solution to first and second order ODEs, it should be used with caution, and only under the supervision of experts and educators capable of discerning its benefits and effectively navigating any potential limitations.

References

- [1]. *I. Abed, R. K. Khare* (2013) Differential equations and their applications, *International Journal of Scientific Engineering and Technology Research*, 2 (19), 2208-2217.
- [2]. *H. Mishi, A. I. Sabari, D. A. Amos, C. F. Egbogu, C. A. Kuje, J. O. Ojosipe* (2020), Application of differential equations in physics, *Global Scientific Journal*, 8 (9).
- [3]. *N. Goyal, P. Kulczycki, M. Ram* (2022) *Differential Equations in Engineering: Research and Applications*, CRC/Taylor & Francis.
- [4]. *Mihai, M Turnea, M. Rotariu,* (2012) *Ordinary differential equations with applications in molecular biology*, PubMed.
- [5]. *L. K. Lazarova, N. Stojković, M. Miteva, A. Stojanova* (2021) Application of differential equations in epidemiological models, *Balkan Journal of Applied Mathematics and Informatics*, 4(2), 91-102.
- [6]. *E. A. Loginova* (2020) Application of ordinary differential equations in chemistry, DOI: 10.18411/lj-07-2020-25.
- [7]. *E. Kreyszik* (2006) *Advanced Engineering Mathematics*, John Wiley and Sons.
- [8]. *P. Dawkins* (2007) *Differential Equations*
- [9]. *K. Atkinson, W. Han, D. Stewart* (2009) *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, Inc.
- [10]. *L.F. Shampine, S. Thompson* (2001) Solving DDEs in Matlab, *Applied Numerical Mathematics*, 37 (4), 441-458
- [11]. *U. Goktas, D. Kapadia* (2011) Methods in Mathematica for solving ordinary differential equations, *Mathematical and Computational Applications*, 16 (4), 784-796.
- [12]. *A. Hvatov and T. Tikhonova* (2022) Automated differential equation solver based on the parametric approximation optimization.
- [13]. *P. Pirnia, F. Duhaime, Y. Ethier, J-S Dubé* (2019) ICY: An interface between COMSOL and discrete element code YADE for the modelling of porous media, *Computers & Geosciences*, 123, 38-46.
- [14]. *Meade, D.B.* (2020) How Maple Has Improved Student Understanding in Differential Equations. In: Gerhard, J., Kotsireas, I. (eds) *Maple in Mathematics Education and Research*. MC 2019. Communications in Computer and Information Science, vol 1125. Springer, Cham.
- [15]. *ODEPACK: Fortran ODE Solvers* <https://computing.llnl.gov/projects/odepack>
- [16]. *A. Koryagin, R. Khudorozkov, S. Tsinfer* (2019) PyDEns: a Python framework for solving differential equations with neural networks, *Computer Science, Machine Learning*.
- [17]. *J. Sundnes* (2023) *Solving Ordinary Differential Equations in Python*, Simula Research Laboratory.
- [18]. *Loshkovska, S., Koceski. S.* (2015) *ICT innovations: Emerging technologies for better living*. Springer, 2015.
- [19]. *Koceski, S., Petrevska B.* (2012) Empirical evidence of contribution to e-tourism by application of personalized tourism recommendation system. *Annals of the Alexandru Ioan Cuza University-Economics*, vol. 59, No 1, 363-374.
- [20]. *Kocev, I., J. Achkoski, D. Bogatinov, S. Koceski, V. Trajkovic, G. Stevanoski, B. Temelkovski* (2018). Novel approach for automating medical emergency protocol in military environment. – *Technology and Health Care*, Vol. 26, No 2, 249-261.
- [21]. *Trajkovic, V., E. V. Gjorgievska, S. Koceski, I. Kulev* (2014) General assisted living system architecture model. – *International Conference on Mobile Networks and Management*, Springer, Cham, 329-343.

- [22]. *Stojanov, D., Koceski S.* (2014) Topological MRI prostate segmentation method.– Computer Science and Information Systems (FedCSIS), 2014 Federated Conference, IEEE 2014, 219-225.
- [23]. *Stojanov, D., A. Mileva, Koceski S.* (2012) A new, space-efficient local pairwise alignment methodology. – Advanced Studies in Biology, Vol. 4, No 2, 85-93.
- [24]. *Terziyska, M., Todorov, Y., Miteva, D., Doneva, M., Dyankova, S., Metodieva, P., & Nacheva, I* (2020) Bayesian Regularized Neural Network for Prediction of the Dose in Gamma Irradiated Milk Products. Cybernetics and Information Technologies, Vol. 20, No 2, 141-151.
- [25]. *Choi, J. H., K. E. Hickman, A. Monahan, D. Schwarcz* (2023) Chatgpt goes to law school – Available at SSRN.
- [26]. *Cotton, D. R., P. A. Cotton, J. R. Shipway* (2023) Ensuring academic integrity in the era of ChatGPT. – Innovations in Education and Teaching International, 1-2.
- [27]. *Kumar, V., D. Boulanger* (2020) Explainable automated essay scoring: Deep learning really has pedagogical value. – InFrontiers in education, Vol. 5.
- [28]. *Tapalova, O., N. Zhiyenbayeva* (2022) Artificial Intelligence in Education AIED for Personalised Learning Pathways.–Electronic Journal of e-Learning, Vol. 20, No 5, 639-653.
- [29]. *Koceska, N., S. Koceski, P. Z. Beomonte, V. Trajkovik, N. Garcia* (2019) A telemedicine robot system for assisted and independent living. – Sensors, Vol. 19, No 4, 834.
- [30]. *Koceski, S., Koceska N.* (2016) Evaluation of an assistive telepresence robot for elderly healthcare. – Journal of medical systems, Vol. 40, No 5, 121.
- [31]. *Koceski, S., S. Panov, N. Koceska, P. Z. Beomonte, F. Durante* (2014) A novel quad harmony search algorithm for grid-based path finding. – International Journal of Advanced Robotic Systems, Vol. 11, No 9, 144.
- [32]. *Koceska, N., Koceski S.* (2014) Financial-Economic Time Series Modelling and Prediction Techniques Review. – Journal of Applied Economics and Business, Vol. 2, No 4, 28-33.
- [33]. *ChatGPT* website. <https://chat.openai.com/>
- [34]. *OpenAi* website. <https://openai.com/chatgpt>
- [35]. *Brown, T., Mann, B., Ryder, N., Subbiah, M., Kaplan, J. D., Dhariwal, P., ... & Amodei, D.* (2020). Language models are few-shot learners. Advances in neural information processing systems, 33, 1877-1901.
- [36]. *Frieder, S., Pinchetti, L., Griffiths, R. R., Salvatori, T., Lukasiewicz, T., Petersen, P. C., ... & Berner, J.* (2023). Mathematical capabilities of chatgpt. arXiv preprint arXiv:2301.13867.
- [37]. *Pardos, Z. A., S. Bhandary* (2023) Learning gain differences between ChatGPT and human tutor generated algebra hints. – arXiv preprint arXiv:2302.06871, Feb 14, 2023.
- [38]. *Shakarian P., A.Koyalamudi, N. Ngu, L. Mareedu* (2023) An Independent Evaluation of ChatGPT on Mathematical Word Problems (MWP).– arXiv:2302.13814, Feb 23, 2023
- [39]. *Dao, X. Q., N. B. Le.* (2023) Investigating the Effectiveness of ChatGPT in Mathematical Reasoning and Problem Solving: Evidence from the Vietnamese National High School Graduation Examination. – arXiv preprint arXiv:2306.06331, Jun 10, 2023.
- [40]. *Borji, A.* (2023) A categorical archive of chatgpt failures–arXiv preprint arXiv:2302.03494., Feb 6, 2023
- [41]. *Wang, J., Liang, Y., Meng, F., Shi, H., Li, Z., Xu, J., ... & Zhou, J.* (2023). Is chatgpt a good nlg evaluator? a preliminary study. arXiv preprint arXiv:2303.04048.
- [42]. *Lederman, A., R. Lederman, K. Verspoor.* (2022) Tasks as needs: reframing the paradigm of clinical natural language processing research for real-world decision support. – Journal of the American Medical Informatics Association, Vol. 29, 2022, No 10, 1810-1817.
- [43]. *Nov, O., Singh, N., & Mann, D. M.* (2023). Putting ChatGPT's medical advice to the (Turing) test. medRxiv, 2023-01.
- [44]. *Lehnert, K.* (2023) AI insights into theoretical physics and the Swampland Program: A journey through the cosmos with ChatGPT – arXiv preprint arXiv:2301.08155, Jan 10, 2023.
- [45]. *Kortemeyer, G.* (2023). Could an artificial-intelligence agent pass an introductory physics course?. Physical Review Physics Education Research, 19(1), 010132.
- [46]. *Choi, J. H., K. E. Hickman, A. Monahan, D. Schwarcz* (2023) Chatgpt goes to law school – Available at SSRN. 2023.
- [47]. *Cotton, D. R., P. A. Cotton, J. R. Shipway* (2023) Ensuring academic integrity in the era of ChatGPT. – Innovations in Education and Teaching International, March 19, 2023, 1-2.

- [48]. *Kashefi A, Mukerji T.* (2023) Chatgpt for programming numerical methods. *Journal of Machine Learning for Modeling and Computing.* 2023;4(2).
- [49]. *Orlando, G.* (2023) Assessing ChatGPT for coding finite element methods. *Journal of Machine Learning for Modeling and Computing,* 4(2).

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