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REICH-TYPE CONTRACTIVE MAPPING INTO A COMPLETE METRIC SPACE AND CONTINUOUS, INJECTIVE AND SUBSEQUENTIALLY CONVERGENT MAPPING

SAMOIL MALCHESKI

Abstract. In this paper, a generalization of the fixed point theorem of the Reich-type mapping on a complete metric space (X, d) is given. Continuous, injective and sub-sequentially convergent mapping T was used, as well as it is taken that function f is from the class \mathcal{O} continuous monotonically nondecreasing functions $f : [0, +\infty) \rightarrow [0, +\infty)$ such that $f^{-1}(0) = \{0\}$, where it is additionally assumed that it is subadditive, i.e. $f(x+y) \leq f(x) + f(y)$, for each $x, y \in [0, +\infty)$.

1. Introduction

Let (X, d) be a metric space. The mapping $S : X \rightarrow X$ is called a contraction if it exists $\lambda \in (0, 1)$ such that for each $x, y \in X$ holds true

$$d(Sx, Sy) \leq \lambda d(x, y). \quad (1.1)$$

If the metric space (X, d) is complete, then the mapping T for which condition (1.1) is satisfied has a unique fixed point. This result is known in the literature as Banach's fixed point principle ([4]). Later, in 1968, it was generalized by R. Kannan ([3]), and in 1972 its generalization was given by S. K. Chatterjea ([5]).

In 1971, S. Reich ([10]) gave a new generalization of the Banach's fixed point principle as follows:

Theorem 1.1. *If the mapping $S : X \rightarrow X$ where (X, d) is a complete metric space, it satisfies the inequality*

$$d(Sx, Sy) \leq ad(x, Sx) + bd(y, Sy) + cd(x, y), \quad (1.2)$$

where $a > 0, b > 0$ and $c > 0$ are such that $a + b + c < 1$ and $x, y \in X$, then S has a unique fixed point.

If S satisfies the condition (1.2), then for S we say that it is a Reich-type mapping.

Furthermore, in [9] S. Moradi and D. Alimohammadi generalize R. Kannan's result, using the sequentially convergent mappings, and in [1] several generalizations of Kannan and Chatterjea's theorems are proved, using the sequentially convergent mappings, which are defined as follows.

Keywords. Reich-type contractive mapping, complete metric space, sequentially convergent, subsequentially convergent mapping.

Definition 1.1. ([8]). Let (X, d) be a metric space. A mapping $T : X \rightarrow X$ is said to be sequentially convergent if we have, for every sequence $\{y_n\}$, if $\{Ty_n\}$ is convergence then $\{y_n\}$ is also convergence. A mapping T is said to be sub-sequentially convergent if we have, for every sequence $\{y_n\}$, if $\{Ty_n\}$ is convergence then $\{y_n\}$ has a convergent subsequence.

In [8] S. Moradi and A. Beiranvand introduce the concept of T_f contractive mapping, whereby they use the class Θ of continuous monotonically nondecreasing functions $f : [0, +\infty) \rightarrow [0, +\infty)$ such that $f^{-1}(0) = \{0\}$, which is defined as follows.

Definition 2 ([8]). Let (X, d) be a metric space, $S, T : X \rightarrow X$ and $f \in \Theta$. A mapping S is said to be T_f -contraction if there exist $\lambda \in (0, 1)$ such that

$$f(d(TSx, TSy)) \leq \lambda f(d(Tx, Ty)), \quad (1.3)$$

for all $x, y \in X$.

Let us note here that, if $f \in \Theta$, then from $f^{-1}(0) = \{0\}$ it follows that $f(t) > 0$, for each $t > 0$. S. Moradi and A. Beiranvand prove that if S is T_f contractive mapping, then S has a unique fixed point. Then, in [2], M. Kir and H. Kiziltunc generalize the result of S. Moradi and A. Beiranvand for Kannan and Chatterjea type mappings. In [7] the results of Kir and Kiziltunc are generalized and their application is given, while in [6] a generalization of the Reich-type mapping using sequentially convergent mappings is given. In the following considerations we will give a generalization of the Reich-type mapping using sub-sequentially convergent mappings, which were also introduced by S. Moradi and A. Beiranvand ([8]).

2. Mains results

Theorem 2.1. Let (X, d) be a complete metric space $S : X \rightarrow X$, $f \in \Theta$ is such that $f(p+q) \leq f(p) + f(q)$, for each $p, q \in [0, +\infty)$ and mapping $T : X \rightarrow X$ is continuous, injective and sub-sequentially convergent. If exist $a > 0, b > 0$ and $c > 0$ such that $a + b + c < 1$ and

$$f(d(TSx, TSy)) \leq af(d(Tx, TSx)) + bf(d(Ty, TSy)) + cf(d(Tx, Ty)) \quad (2.4)$$

for each $x, y \in X$, then S has a single fixed point and for each $x_0 \in X$ the sequence $\{S^n x_0\}$ converges to the fixed point.

Proof. Let x_0 be an arbitrary point from X and let the sequence $\{x_n\}$ be determined by $x_{n+1} = Sx_n$, $n = 0, 1, 2, 3, \dots$. From inequality (4) it follows that

$$\begin{aligned} f(d(Tx_{n+1}, Tx_n)) &= f(d(TSx_n, TSx_{n-1})) \\ &\leq af(d(Tx_n, TSx_n)) + bf(d(Tx_{n-1}, TSx_{n-1})) + cf(d(Tx_n, Tx_{n-1})) \\ &= af(d(Tx_n, Tx_{n+1})) + (b+c)f(d(Tx_{n-1}, Tx_n)), \end{aligned}$$

i.e.

$$f(d(Tx_{n+1}, Tx_n)) \leq \frac{b+c}{1-a} f(d(Tx_n, Tx_{n-1})) .$$

According to that, for $\lambda = \frac{b+c}{1-a} < 1$ holds true

$$f(d(Tx_{n+1}, Tx_n)) \leq \lambda f(d(Tx_n, Tx_{n-1})) , \quad (2.5)$$

for each $n = 1, 2, 3, \dots$. From inequality (2.5) it follows that

$$f(d(Tx_{n+1}, Tx_n)) \leq \lambda^n f(d(Tx_1, Tx_0)) , \quad (2.6)$$

for each $n = 1, 2, 3, \dots$. Now, from inequality (2.6), the properties of the metric and the monotonicity and subadditivity of the function f , it follows that for each $m, n \in \mathbb{R} \quad n > m$ holds true

$$\begin{aligned} f(d(Tx_n, Tx_m)) &\leq f\left(\sum_{k=m}^{n-1} d(Tx_{k+1}, Tx_k)\right) \\ &\leq \sum_{k=m}^{n-1} f(d(Tx_{k+1}, Tx_k)) \\ &\leq \sum_{k=m}^{n-1} \lambda^k f(d(Tx_1, Tx_0)) \\ &< \frac{\lambda^m}{1-\lambda} f(d(Tx_1, Tx_0)). \end{aligned}$$

From the last inequality, it follows

$$\lim_{m, n \rightarrow \infty} f(d(Tx_n, Tx_m)) = 0 ,$$

and because $f \in \mathcal{O}$ we have $\lim_{m, n \rightarrow \infty} d(Tx_n, Tx_m) = 0$. According to that, $\{Tx_n\}$ is a Cauchy sequence. But X is a complete metric space, so therefore the sequence $\{Tx_n\}$ is convergent. Further, the mapping $T : X \rightarrow X$ is sub-sequentially convergent, therefore the sequence $\{x_n\}$ contains a convergent subsequence $\{x_{n(k)}\}$, i.e., there exists $u \in X$ such that $\lim_{n \rightarrow \infty} x_{n(k)} = u$. From the continuity of T follows $\lim_{n \rightarrow \infty} Tx_{n(k)} = Tu$. Now, the sequence $\{Tx_{n(k)}\}$ is a subsequence of the convergent sequence $\{Tx_n\}$, so the equation holds true

$$\lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} Tx_{n(k)} = Tu .$$

We will prove that $u \in X$ is a fixed point for the mapping S . We have

$$\begin{aligned} f(d(TSu, Tx_{n+1})) &= f(d(TSu, TSx_n)) \\ &\leq af(d(TSu, Tu)) + bf(d(TSx_n, Tx_n)) + cf(d(Tu, Tx_n)) \\ &= af(d(TSu, Tu)) + bf(d(Tx_{n+1}, Tx_n)) + cf(d(Tu, Tx_n)). \end{aligned}$$

If in the last inequality we take $n \rightarrow \infty$, then from $\lim_{n \rightarrow \infty} Tx_n = Tu$ and the continuity of metric and function f follows the inequality

$$f(d(TSu, Tu)) \leq \frac{b+c}{1-a} f(0) .$$

But, $0 < \frac{b+c}{1-a} < 1$ and $f^{-1}(0) = \{0\}$, so from the last inequality it follows that $d(TSu, Tu) = 0$, i.e., $TSu = Tu$. Finally, T is injection and therefore $Su = u$, i.e., the mapping S has a fixed point.

Let $u, v \in X$ be two fixed points for S , i.e., $Su = u$ и $Sv = v$. From the inequality, (1.4) it follows that

$$\begin{aligned} f(d(Tu, Tv)) &= f(d(TSu, TSv)) \\ &\leq af(d(Tu, TSu)) + bf(d(Tv, TSv)) + cf(d(Tu, Tv)) \end{aligned}$$

i.e.

$$f(d(Tu, Tv)) \leq \frac{a+b}{1-c} f(0),$$

So, similarly as above, we conclude that $d(Tu, Tv) = 0$. Therefore, $Tu = Tv$. But T is an injection, and therefore $u = v$, i.e., S has a unique fixed point.

Finally, from the arbitrariness of the point x_0 , it follows that for each $x_0 \in X$ the sequence $\{S^n x_0\}$ converges to the fixed point. \square

References

- [1] A. Malčeski, S. Malcheski, K. Anevska, R. Malčeski, *New extension of Kannan and Chatterjea fixed point theorems on complete metric spaces*. British Journal of Mathematics & Computer Science. Vol. 17 No. 1 (2016), 1-10.
- [2] M. Kir, H. Kiziltunc, *T_F type contractive conditions for Kannan and Chatterjea fixed point theorems*, Adv. Fixed Point Theory, Vol. 4, No. 1 (2014), pp. 140-148
- [3] R. Kannan, *Some results on fixed points*, Bull. Calc. Math. Soc. Vol. 60 No. 1, (1968), 71-77
- [4] S. Banach, *Sur les operations dans les ensembles abstraits et leur application aux equations intégrales*, Fund. Math. 2 (1922), 133-181
- [5] S. K. Chatterjea, *Fixed point theorems*, C. R. Acad. Bulgare Sci., Vol. 25 No. 6 (1972), 727-730
- [6] S. Malcheski, R. Malcheski, *Generalization of a Reich-type contractive mapping in a complete metric space*, Proceedings of the CODEMA2022, Armaganka, Skopje, 2023
- [7] S. Malcheski, R. Malcheski, *Three Theorems about Fixed Points for T_f Contraction in a Complete Metric Space*, Proceedings of the CODEMA2020, 13-21
- [8] S. Moradi, A. Beiranvand, *Fixed Point of T_F -contractive Single-valued Mappings*, Iranian Journal of Mathematical Sciences and Informatics, Vol. 5, No. 2 (2010), pp 25-32
- [9] S. Moradi, D. Alimohammadi, *New extensions of kannan fixed theorem on complete metric and generalized metric spaces*. Int. Journal of Math. Analysis. 2011;5(47):2313-2320.
- [10] S. Reich, *Some remarks concerning contraction mappings*. Canad. Math. Bull. Vol. 14 (1), 1971:121-124.

Samoil Malcheski

International Slavic University, Sv. Nikole, R. North Macedonia

E-mail address: samoil.malcheski@gmail.com