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CONTENT

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APPLICATION OF CENTER OF GRAVITY METHOD FOR LOCATIONS OF FACILITIES

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Abstract. The facility location decision is a critical aspect of operations management, with transportation costs often serving as a primary consideration. The Center of Gravity (COG) method has emerged as a practical tool for its simplicity in estimating optimal facility locations. This paper provides a comprehensive review of the COG method, acknowledging its strengths as a quick approximation tool but also highlighting its limitations. We delve into the method's assumptions, notably the uniformity of transportation costs, and discuss its application in dynamic and complex environments. Furthermore, we advocate for a holistic approach to facility location decisions, incorporating factors such as market competition, legal constraints, and environmental sustainability. To address these complexities, we propose the integration of advanced tools and techniques, including optimization algorithms, geographical information systems (GIS), and machine learning. By embracing a more sophisticated analytical framework, organizations can make informed decisions that align with broader business objectives, ensuring that facility locations are strategically chosen beyond the narrow focus on transportation costs. This paper aims to guide practitioners and researchers in navigating the intricacies of facility location planning, emphasizing the importance of a nuanced and advanced approach.

1. Introduction

In the dynamic landscape of modern business and logistics, the strategic placement of facilities plays a pivotal role in optimizing operational efficiency and minimizing costs. The process of determining the optimal locations for facilities is a complex task that involves consideration of various factors such as transportation costs, market demands, and geographic constraints. One of the widely recognized methodologies employed for this purpose is the Center of Gravity Method.[1]

The Center of Gravity Method, rooted in spatial analysis and mathematical modeling, provides a systematic approach to facility location by identifying a central point that minimizes the overall transportation costs. Originally developed for military logistics during World War II, this method has since found widespread application in diverse industries ranging from manufacturing and distribution to service-oriented sectors.

This paper explores the principles and application of the Center of Gravity Method in the context of facility location planning. Through a comprehensive review of the method's theoretical foundations and practical considerations, we aim to shed light on its efficacy in addressing the challenges posed by the intricate network of supply chains and distribution channels.

As global markets become increasingly interconnected and customer expectations continue to evolve, businesses are compelled to reassess their facility location strategies. The Center of Gravity Method, with its ability to integrate spatial analysis with cost optimization, emerges as a valuable tool in this endeavor. By leveraging geographic information and quantitative analysis, organizations can make informed decisions that contribute not only to cost reduction but also to enhanced customer satisfaction and overall competitiveness.

This paper unfolds by delving into the historical evolution of the Center of Gravity Method, elucidating its underlying principles, and subsequently examining its applications across diverse industries. Through real-world case studies and comparative analyses, we aim to highlight the method's strengths, limitations, and potential areas for refinement. In doing so, this paper aims to provide a valuable resource for practitioners, researchers, and decision-makers involved in the strategic planning of facility locations.

Key words: *COG method, Location, Relationship*

In the subsequent sections, we will explore the theoretical foundations of the Center of Gravity Method, discuss its application in various industries, and critically assess its suitability in the contemporary business environment. Through a synthesis of academic research and practical insights, this paper seeks to contribute to the ongoing discourse on effective facility location strategies in an ever-evolving global marketplace.

2. Material and methods

Several mathematical methods are employed in object location and tracking applications, ranging from classical mathematical models to advanced algorithms. Here are some notable mathematical methods [2,3]:

— Trilateration involves determining an object's location by measuring its distance from three known points.

— Multilateration extends this concept to more than three reference points, providing increased accuracy. These methods are commonly used in GPS and wireless positioning systems.

— Least Squares is a statistical method used to minimize the sum of the squared differences between observed and calculated values. In object location, it can be employed to refine location estimates, especially when dealing with noisy or imprecise data.

— The Kalman filter is an algorithm that estimates the state of a dynamic system by continuously predicting and updating the system's state based on noisy measurements. It is widely used in navigation systems, robotics, and object tracking.

— Monte Carlo methods involve random sampling to obtain numerical results. In object location, Particle Filtering is a Monte Carlo-based approach used for recursive Bayesian filtering, particularly in scenarios where the system's state is uncertain.

— Voronoi diagrams partition a space into regions based on the distance to a given set of points. In object location, Voronoi diagrams are applied to spatially divide an area, aiding in proximity-based analysis and location-based services.

— Cubic spline interpolation is used to estimate the trajectory of an object based on a set of known points. This method is common in computer graphics, animation, and path planning for moving objects.

— Graph theory models spatial relationships between objects as a set of nodes and edges. It is applied in network-based approaches for route optimization, logistics, and transportation planning.

— Nonlinear optimization methods, such as gradient descent or genetic algorithms, are employed to optimize objective functions related to object location. These techniques are useful when dealing with complex and nonlinear relationships.

— Delaunay triangulation is a method for dividing a set of points into non-overlapping triangles, providing a way to understand the spatial relationships between points. It finds applications in spatial analysis and mesh generation.

— Markov models are used to represent systems where the future state depends only on the present state. In object location, Markov models are employed for predicting and analyzing the movement patterns of objects.

Regression analysis models the relationship between variables. In object location, it can be used to predict an object's location based on historical data and other relevant factors.

These mathematical methods offer diverse approaches to address the challenges associated with object location, providing tools for accurate estimation, prediction, and optimization in various applications. The choice of method often depends on the specific characteristics of the problem at hand and the nature of the available data.

3. Center of gravity method

The Center of Gravity (COG) method is a technique used in operations management and facility location planning to determine the best location for a facility or distribution center. The goal is to minimize transportation costs, taking into account the location of demand points and the volume of goods to be transported.

It's important to note that the Center of Gravity Method is a simplification and has limitations. It assumes uniform transportation costs, which may not be accurate in real-world scenarios. Additionally, it does not consider factors such as competition, labor availability or local regulations, which are crucial in actual site selection decisions.

Despite its limitations, the Center of Gravity Method provides a quick and useful initial approximation for facility location decisions, especially when transportation costs are a significant factor. Advanced optimization techniques may be employed for more accurate and comprehensive analyses in complex situations.

The Center of Gravity Method involves mathematical calculations to determine the optimal location for facilities based on minimizing transportation costs. Here is a simplified explanation of the mathematical steps involved:

Let us consider a scenario where you have potential facility locations (warehouses, distribution centers, etc.) and demand points (customers, retail outlets, etc.) with associated quantities and transportation costs.

1. Coordinate System: Assign coordinates to each facility location and demand point on a coordinate system (e.g., x, y coordinates on a map).
2. Calculate Weighted Coordinates: For each facility, multiply the quantity or demand at each demand point by the coordinates of that point. Sum these values for all demand points. This is done separately for the x and y coordinates.

Similarly, sum the quantities for all demand points.[4]

$$D_x = \frac{\sum D_{ix} V_i}{\sum V_i} \text{ and } D_y = \frac{\sum D_{iy} V_i}{\sum V_i} \tag{1}$$

3. Center of Gravity Coordinates: The calculated weighted coordinates represent the center of gravity, and these coordinates give the optimal location for the facility that minimizes transportation costs.

The center of gravity coordinates provides a location that minimizes the overall transportation cost based on the weighted distances from the demand points.

It is important to note that this is a simplified explanation, and in practice, additional considerations, such as the type of transportation network, capacity constraints, and the dynamic nature of demand, may require more complex mathematical modeling and optimization techniques. Advanced mathematical methods, including linear programming or network optimization algorithms, can be incorporated for a more accurate representation of real-world logistics scenarios. [5..7]

4. Results and discussion

Using the Center of Gravity Method and the data shown in Figure 1 and Table 1, find the best location for the warehouse (the reference point). Adopt a linear relationship between shipped volume and shipping costs.

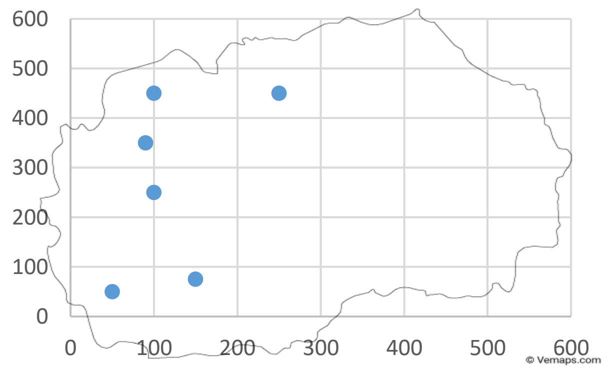

Figure 1. *Location of store*

Table 1. *Quantity of compressors requested by each supplier*

Store	Location of the x and y axis	Monthly storage of items
A:Skopje	250 x 450	18.000
B:Tetovo	100 X 450	12.000
C:Gostivar	90 x 350	8.200
D: Kicevo	100 x 250	7.000
E:Ohrid	50 x 50	13.000
G:Prilep	150 x 75	6.500

Also predict how many days' management decided to deliver 5,000 items from store A to store E. Check if management's decision would change the location of warehouse D.

$$D_x = \frac{(250*13000)+(50*18000)+(90*8200)+(100*7000)+(100*12000)+(150*6500)}{18000+12000+8200+7000+13000+6500} = 137.39$$

$$D_y = \frac{(450*13000)+(50*18000)+(350*8200)+(250*7000)+(450*12000)+(75*6500)}{18000+12000+8200+7000+13000+6500} = 305.44$$

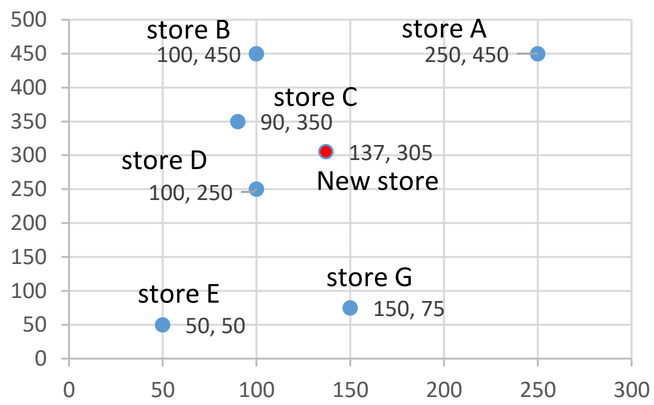

Figure 2. *Calculation result for the reference point*

An example of how you can implement the Center of Gravity (COG) method in MATLAB is shown below. This code allows you to enter the coordinates and weights of the demand points yourself.

Algorithm 1 Potential locations and the facility

```
% Number of potential locations
numLocations = 6;

% Random weights for each criterion
for i = 1:numLocations
   weights(i) = input(['Enter weights ', num2str(i), ': ']);
end

% Random distances for each criterion and location
for i = 1:numLocations
   xCoordinates(i) = input(['Enter X coordinate for facility ', num2str(i), ': ']);
   yCoordinates(i) = input(['Enter Y coordinate for facility ', num2str(i), ': ']);
end

% Calculate the reference point for locationS
CoordinatesDx=xCoordinates.*weights/weights
CoordinatesDy=yCoordinates.*weights/weights

% Display the results
disp(CoordinatesDx);
disp(CoordinatesDy);

% Plot the locations and the facility
figure;
scatter(xCoordinates, yCoordinates, 100, 'filled', 'DisplayName', 'Original Coordinates');
hold on;
scatter(CoordinatesDx, CoordinatesDy, 100, 'r', 'filled', 'DisplayName', 'New Location');
text(CoordinatesDx, CoordinatesDy, 'New Location', 'VerticalAlignment', 'bottom', 'HorizontalAlignment',
'right');
title('Potential Locations and Facility');
xlabel('X Coordinate');
ylabel('Y Coordinate');
legend('show');
```

If you are referring to the "reference point" as a point of reference or a point used as a base for comparison, then in the provided MATLAB code, the facility coordinates (CoordinatesDx and CoordinatesDy) can be considered as the reference point. This is the point from which distances to potential locations are calculated.

In location analysis, the reference point is often the location of the facility or a central point that serves as a base for evaluating the distances to other points. In the code example, the facility coordinates represent the point of reference, and distances are calculated from this point to each potential location.

If you have a specific concept of "reference point" in mind or if you are asking about a different aspect, please provide more details so that we can assist you more accurately

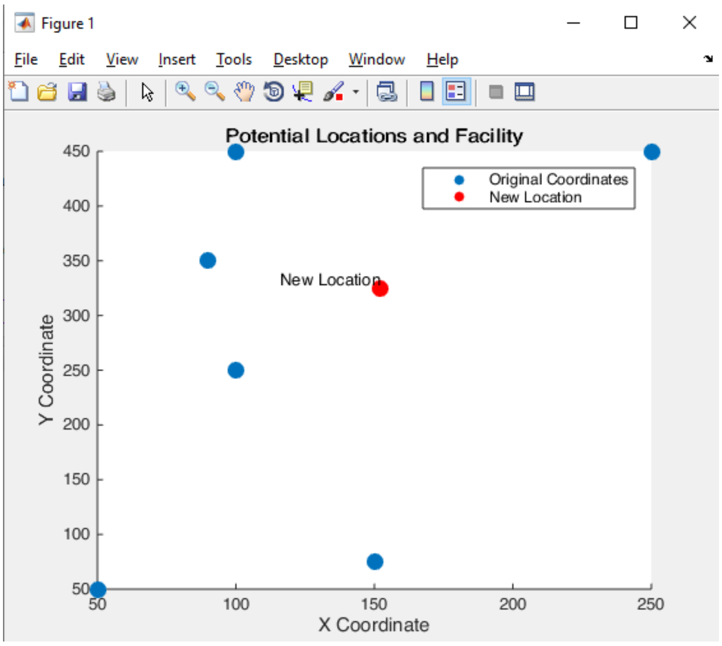

Figure 3. *Calculation result from MATlab*

5. **Conclusion**

The Center of Gravity (COG) method provides a straightforward and intuitive approach to preliminary facility location planning, particularly when minimizing transportation costs is a primary concern. The method offers a quick estimation of the optimal location by considering the weighted average of demand point coordinates.

This code calculates the weighted coordinates, then computes the center of gravity by dividing the sum of weighted coordinates by the sum of weights. Finally, it plots the demand points and the center of gravity for visualization.

Keep in mind that this is a basic implementation, and for a real-world scenario, you might need to consider additional factors and constraints, as discussed in the advanced explanation.

In practice, while the COG method serves as a useful initial step, organizations should complement it with advanced analyses and tools to make informed and strategic decisions in facility location planning. Taking into account the complexity and variability of real-world scenarios ensures that the chosen location aligns with broader business objectives and considerations beyond transportation costs.

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