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ADVANCED TOOLPATH VERIFICATION IN CNC DRILLING: APPLYING NEWTON'S INTERPOLATION THROUGH MATLAB

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Abstract. This paper presents an advanced methodology for toolpath verification in CNC drilling, utilizing Newton's interpolation method along with Matlab. The primary objective is to develop a highly accurate model that precisely captures the dynamic behavior of CNC drilling machines, allowing for meticulous control over the drill bit's trajectory. Newton's interpolation is crucial for accurately modelling the toolpath, which is essential for ensuring precision in drilling operations. Matlab is employed as the computational platform for implementing the interpolation method, enabling the generation of detailed visualizations of the CNC drilling process. This approach enhances drilling accuracy and efficiency, reduces tool wear, and effectively predicts the dynamic motion of the drill bit, offering significant benefits for both professionals and researchers in the field of precision machining.

1. **Introduction**

In contemporary manufacturing, computer numerically controlled (CNC) drilling has emerged as a fundamental technique for precision machining, with toolpath verification playing a crucial role in this process. Toolpath verification is essential for ensuring that the drilling tool follows the specified trajectory detailed in the design, which directly influences the accuracy, quality, and functionality of the final product. This verification involves simulating and analyzing the toolpath to identify potential deviations or errors that may occur during actual drilling processes. By validating the toolpath prior to physical machining, manufacturers can proactively address issues that could otherwise lead to costly errors, inefficient production, and compromised product quality [1].

The significance of toolpath verification is amplified by the increasing complexity of the CNC drilling processes. As technological advancements progress, the designs and toolpaths utilized in drilling become more intricate, necessitating more sophisticated verification methods. Traditional approaches, which often depend on basic simulations and manual inspections, may no longer be adequate to manage the intricate dynamics and variations inherent in modern CNC drilling. These conventional methods can struggle to account for factors such as cutting tool wear, machine precision, and material inconsistencies, potentially resulting in inaccuracies and inefficiencies in the drilling process. Consequently, there is an escalating need for advanced verification techniques that offer a higher degree of precision and reliability [2].

Material removal processes are central to CNC drilling, involving the systematic extraction of material from a workpiece to achieve the desired shape and dimensions. These processes encompass various techniques, including turning, milling, and drilling,

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each tailored to specific applications and materials. In CNC drilling, material removal is accomplished through the controlled rotation of a cutting tool, which penetrates the workpiece to create holes or cavities with high precision. The efficiency and effectiveness of material removal depend on several factors, including the type of material being machined, the properties of the cutting tools, and operational parameters such as feed rate and cutting speed. As materials become more diverse and complex, optimizing material removal processes to enhance productivity and precision remains a significant focus in manufacturing research and development.

Metal cutting processes, a subset of material removal techniques, are particularly important in CNC drilling due to their role in shaping and refining metal components. Metal cutting involves the use of specialized cutting tools and techniques to achieve precise cuts, holes, and contours in various metal alloys. This process is characterized by its ability to produce high-quality surfaces and tight tolerances, which are critical in fields such as aerospace, automotive, and precision engineering. Effective metal cutting requires a thorough understanding of material properties, cutting tool dynamics, and cutting conditions to minimize cutting tool wear and ensure optimal performance. Advances in metal cutting techniques, including the development of high-speed machining tools and advanced cooling methods, continue to drive improvements in efficiency, accuracy, and surface finish in CNC drilling and other machining processes.

Drilling, as a fundamental machining process, involves creating holes in materials by rotating a drill bit to cut through and produce a cylindrical hole (Fig. 1). This process is widely applied in various industrial sectors due to its simplicity, efficiency, and capability to produce holes of different diameters and depths in a range of materials. The quality of the drilled hole, including its dimensional accuracy and surface finish, depends on several factors, such as the type of drill bit, cutting parameters, material properties, and machine tool characteristics. Proper selection and control of these factors are essential to ensure the desired precision and performance of the final product [3].

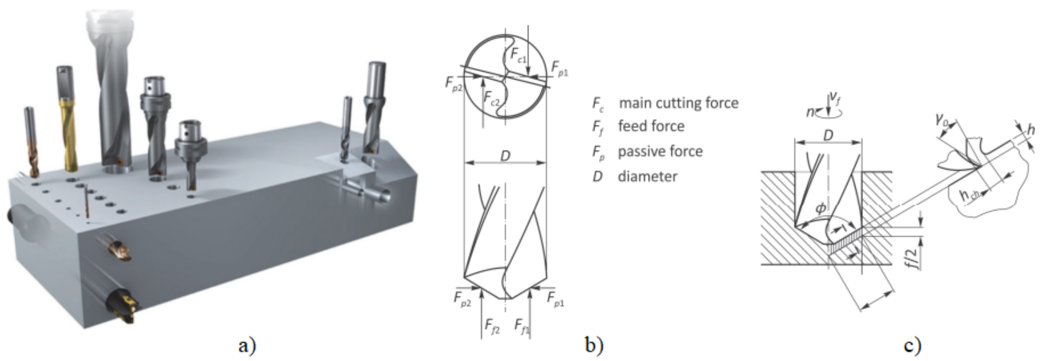


Figure 1. *a) Drilling process, b) Geometry of the drill, c) Chip formation [4]*

In the context of toolpath verification, CNC drilling machines exhibit remarkable versatility, allowing them to be programmed to perform a wide array of operations with exceptional precision and repeatability. Toolpath verification is a critical process that involves a thorough examination and simulation of the drill bit's trajectory to identify and correct any deviations or errors that could compromise the final product. This verification ensures that drilling operations adhere to the stringent specifications and standards necessary for high-quality manufacturing.

As the complexity of toolpaths in contemporary CNC drilling increases, traditional verification methods, often dependent on basic simulations and manual inspections, are becoming increasingly inadequate. These conventional approaches struggle to address the intricate dynamics and challenges posed by advanced drilling processes, necessitating more sophisticated solutions for accurate toolpath verification.

The key element of the modern verification process is the CNC interpolator, a function within the CNC system responsible for generating intermediate points along the toolpath based on programmed commands. The CNC interpolator plays a crucial role in ensuring smooth and continuous cutting tool movement between specified points and maintaining desired feed rates and motion dynamics. Accurate interpolation is vital to avoid abrupt changes in direction or speed, which could adversely affect the cutting tool's performance and the quality of the machined surface. In advanced CNC drilling, where complex geometries and tight tolerances are common, the effectiveness of the CNC interpolator directly influences the success of the toolpath verification process [5].

Interpolation in CNC machining refers to the method used to create a smooth and continuous toolpath between specified points. The two most common types of interpolation in CNC systems are linear and circular interpolation. Linear interpolation (Figs. 2a and 2b) generates a straight-line path between two points, ensuring the cutting tool moves in a straight trajectory with constant velocity.

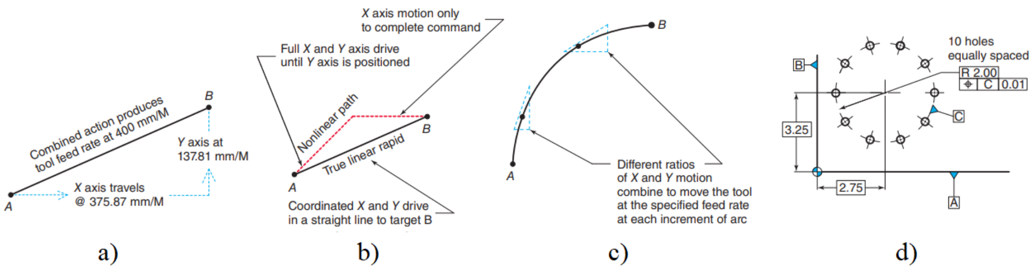


Figure 2. *a) and b) Linear interpolation, c) and d) Circular interpolation [6]*

This method is particularly useful for machining flat surfaces, sharp edges, or when transitioning between different geometrical features that require straight cuts. In contrast, circular interpolation (Figs. 2c and 2d) is used to create arcs or complete circles by defining a curved path between two points, which is essential for machining components with rounded features such as holes, arcs, or complex curved surfaces. Both types of interpolation are fundamental to achieving precise and smooth tool movements, ensuring that the finished product matches the design intent.

However, interpolation in CNC machining is not limited to just linear and circular paths. Advanced processes often require more complex forms of interpolation, such as parabolic and helical interpolation. Parabolic interpolation is employed to create smooth, curved toolpaths following a parabolic trajectory, ideal for machining components with gentle, flowing curves or parabolic shapes. This type of interpolation is particularly useful in applications where a smooth transition between varying slopes is required, ensuring a seamless and accurate toolpath. Helical interpolation, on the other hand, is used to create spiral or helical paths, essential for machining threaded holes or complex 3D surfaces. This method allows the cutting tool to move along a helical trajectory, combining both circular and linear movements.

Equally important in the verification process is the CNC simulator, a software tool that replicates the behavior of the CNC machine in a virtual environment. The CNC simulator enables engineers to visualize and analyze the toolpath before actual machining occurs, providing a detailed preview of the drilling process. By simulating the drill bit's movements, the CNC simulator can identify potential collisions, errors, or inefficiencies in the toolpath, allowing for adjustments to be made before any physical material is processed. This preemptive analysis is crucial for optimizing the toolpath and ensuring that the drilling process adheres to design specifications without wasting resources or time. In conjunction with the CNC interpolator, the CNC simulator enhances the accuracy and reliability of toolpath verification, making it an indispensable tool in modern precision machining [7].

In this context, the application of Newton's interpolation method through Matlab offers a promising advancement. Newton's interpolation, a numerical analysis technique, provides a robust framework for refining toolpath verification by enabling precise and efficient interpolation of data points along the toolpath. In contrast to basic linear or circular interpolation, Newton's interpolation can also handle complex, non-linear toolpaths by constructing polynomials that fit the toolpath data more accurately. Matlab, with its powerful computational and visualization capabilities, facilitates the implementation of this interpolation method, enhancing the accuracy and reliability of toolpath simulations [8].

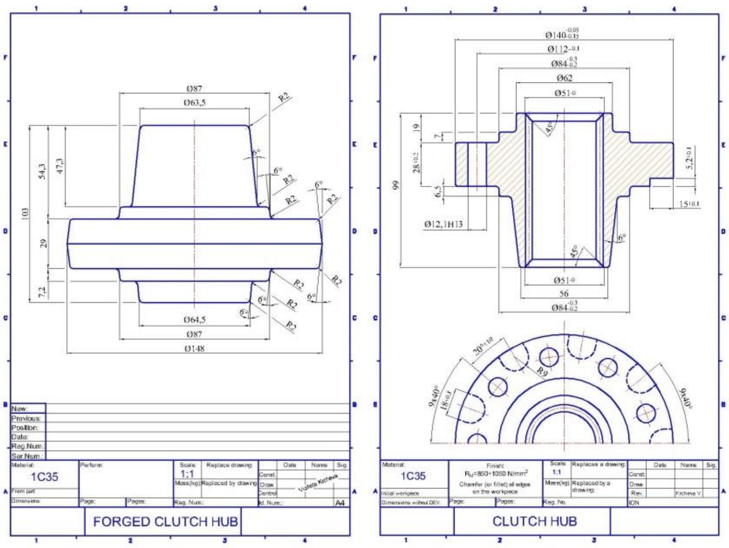
By utilizing Newton's interpolation through Matlab, engineers can overcome the limitations of traditional verification methods, ensuring that CNC machining processes meet the increasingly demanding standards of precision and performance. This approach not only improves the accuracy of toolpath simulations but also enhances the overall efficiency of the verification process, paving the way for advancements in CNC machines and technological innovation. Accordingly, the structure of this paper comprises four main sections: an introduction, a description of the research methodology, a demonstration and discussion of the results, and a conclusion with potential applications.

2. **Research**

This research examines a particular facet of machining a clutch hub, with a specific emphasis on the CNC drilling process utilized. The clutch hub constitutes a critical element within a vehicle's clutch system, being integral to the clutch disc assembly necessary for manual transmission vehicles. Its significance is underscored by its impact on the performance and dependability of the transmission system, thereby highlighting the necessity for meticulous manufacturing practices.

To provide a thorough and precise analysis of the drilling process, detailed two-dimensional technical drawings were developed using AutoCAD software, a sophisticated CAD (computer-aided design) tool. These drawings serve not merely as illustrations but also as exact representations of the machining process.

The technical drawings document the transformation of the clutch hub from its initial forged condition (Fig. 3a) to its completed, fully machined form (Fig. 3b). This visual documentation is essential for comprehending the intricacies and precision required in the CNC drilling process, contributing to the attainment of the high accuracy and quality standards demanded in automotive manufacturing.



a) b)

Figure 3. *a) Forged clutch hub; b) Machined clutch hub [9]*

The CNC drilling process is thoroughly detailed in the operational sheet depicted in Figure 4. This technical drawing provides an in-depth overview of the drilling requirements, including precise dimensions and the cutting tool specifications. Designed for clarity and accuracy, the sheet offers a comprehensive depiction of the geometry and dimensions that must be achieved during the drilling process on a modern CNC drilling machine, ensuring that every phase of the process is executed correctly.

The process begins with the precise alignment and secure clamping of the workpiece within the CNC drilling machine, which is essential for maintaining stability and ensuring accuracy. Subsequently, the machine employs advanced technology to control the cutting tool along a predetermined trajectory, systematically removing excess material to achieve the expected geometry and dimensions of the specified holes.

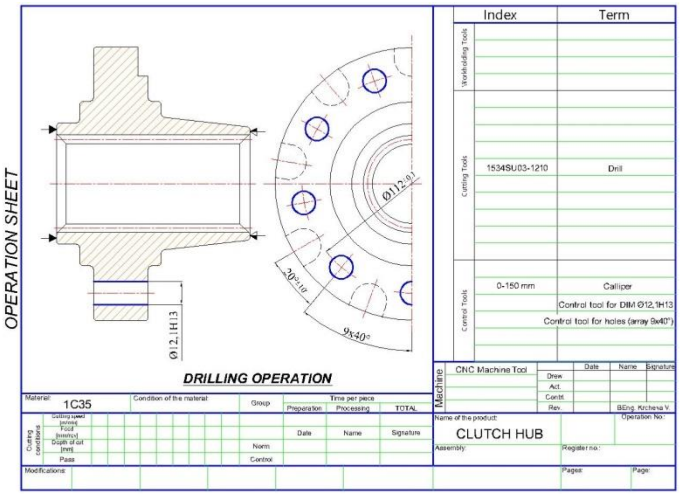


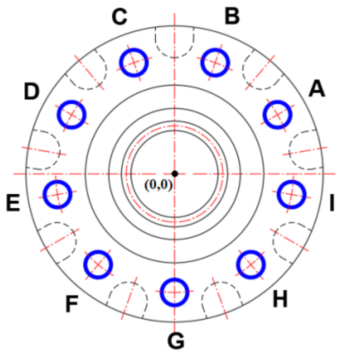
Figure 4. *CNC drilling*

Once the workpiece is securely clamped, selecting an appropriate cutting tool becomes crucial for obtaining the desired result. For this operation, the chosen cutting tool is the 1534SU03-1210 drill bit (Fig. 5), manufactured by ZCC-CT. This cutting tool meets the required geometry and dimensions for the workpiece holes, ensuring that the final shape of the workpiece matches the exact specifications with high precision.

Figure 5. *The drill bit [10]*

To outline the toolpath for the designated cutting tool, Figure 6 provides a comprehensive two-dimensional visualization of the circular arrangement of holes on the clutch hub. The figure features a systematic array of coordinates, marked as points A through I, which act as reference points to depict the necessary toolpath trajectory. By detailing these positions, the figure clarifies the geometric relationship between the cutting tool and the workpiece, enabling accurate control of the drilling process.

These coordinates define the cutting toolpath using linear interpolation, where the drill bit moves along straight lines connecting each designated point, ensuring smooth transitions from point A to point I. This approach creates a series of linear segments linking the points, with the drill bit adjusting its direction at each point to maintain a straight trajectory. It offers precision and control, which are essential for achieving the desired shape of the workpiece.



a)

Coordinates (x;y)
A (48.5;28)
B (19.2;52.6)
C (-19.2;52.6)
D (-48.5;28)
E (-55.1;-9.7)
F (-36;-42.9)
G (0;-56)
H (36;-42.9)
I (55.2;-9.7)

b)

Figure 6. *a) Circular array of specified holes, b) Coordinates of the holes*

When analyzing linear motion, Newton's interpolation formula estimates function values at specific points within a set of known data. This approach, a type of polynomial interpolation using a linear polynomial, presumes that values between consecutive data points can be approximated by a straight line. Newton's formula constructs this line, enabling the estimation of intermediate values. The formula used to compute the interpolating polynomial within these linear segments is:

$$P_i(x) = y_i + (x - x_i) \cdot \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \qquad (2.1)$$

In this context, P(x) denotes the predicted value of the function at a specific point x; x_i and x_{i+1} are the x-values of the given points, while y_i and y_{i+1} are their corresponding y-values. Newton's linear interpolation formula assumes that x falls within the interval $[x_i;x_{i+1}]$, which is the range over which the function is approximated. This means the interpolation is expected to be accurate only for x values within this interval. Outside this range, the linear interpolation might not provide reliable results.

3. **Results and discussion**

This section presents the results derived from the application of Newton's linear interpolation formula to a set of specific data points. The methodology entailed selecting the data points, identifying the relevant intervals, and employing the interpolation formula to estimate the unknown values.

The results are systematically summarized in Table 1, which outlines the interpolation procedure in detail. The table includes columns representing the initial and final coordinates of the data points, the interval points, and the associated interpolating polynomial for each segment.

The interpolation procedure commenced with the coordinates (x_i, y_i) and (x_{i+1}, y_{i+1}), which served as the foundational basis for the process. Following this, the intervals between successive data points were computed to structure the interpolation.

Newton's linear interpolation formula was subsequently utilized to estimate values within these intervals, with equation (2.1) being employed to determine the polynomial P(x) for any x within the interval [x_i; x_{i+1}]. The interpolated values are presented in Table 1.

Table 1. *Results obtained through the interpolation procedure*

Intvl. No.	**Initial Coordinates**		**Final Coordinates**		**Intvl.**	**Interpolating Polynomial $P_i(x)$**
	x_i	y_i	x_{i+1}	y_{i+1}		
1	48.5	28	19.2	52.6	[A;B]	$P_0(x)=68.7-0.8x$
2	19.2	52.6	−19.2	52.6	[B;C]	$P_1(x)=52.6$
3	−19.2	52.6	−48.5	28	[C;D]	$P_2(x)=68.7+0.8x$
4	−48.5	28	−55.1	−9.7	[D;E]	$P_3(x)=305+5.7x$
5	−55.1	−9.7	−36	−42.9	[E;F]	$P_4(x)=-105.5-1.7x$
6	−36	−42.9	0	−56	[F;G]	$P_5(x)=-56-0.4x$
7	0	−56	36	−42.9	[G;H]	$P_6(x)=-56+0.4x$
8	36	−42.9	55.2	−9.7	[H;I]	$P_7(x)=-105+1.7x$

Furthermore, the coordinates of the given points were systematically incorporated into a Matlab script to develop a mathematical model using Newton's interpolation formula for linear interpolation. This process included writing and executing Matlab script to implement the interpolation formula. The script accepts the x and y coordinates of the data points, applies Newton's interpolation formula to estimate values at intermediate points, and generates a plot to visually display the interpolation results. The Matlab script used is provided below.

Matlab script

```
% Coordinates of the given points
x = [48.5, 19.2, -19.2, -48.5, -55.1, -36, 0, 36, 55.2];
y = [28, 52.6, 52.6, 28, -9.7, -42.9, -56, -42.9, -9.7];

% Newton's linear interpolation
n = length(x);
coefficients = zeros(n, n);

% Determine the divided differences
coefficients(:, 1) = y';
```

```
for j = 2:n
    for i = 1:n-j+1
        coefficients(i, j) = (coefficients(i+1, j-1) - coefficients(i, j-1)) / (x(i+j-1) - x(i));
    end
end

% Generate interpolated values
x_interpolated = linspace(min(x), max(x), 1000);
y_interpolated = zeros(size(x_interpolated));

for i = 1:length(x_interpolated)
    y_interpolated(i) = coefficients(1, 1);
    for j = 2:n
        y_interpolated(i) = y_interpolated(i) + coefficients(1, j) * prod(x_interpolated(i) -
x(1:j-1));
    end
end

% Plot a diagram
figure;
plot(x, y, 'o', 'MarkerFaceColor', 'b', 'MarkerSize', 8, 'DisplayName', 'Data Points');
hold on;
plot(x_interpolated, y_interpolated, 'r-', 'LineWidth', 2, 'DisplayName', 'Linear
Interpolation');
xlabel('x');
ylabel('y');
title('Newton''s Linear Interpolation');
legend('show');
grid on
```

The Matlab programming environment, as depicted in Figure 7, provides a comprehensive setup where the script for Newton's linear interpolation has been implemented. This environment comprises a command window for interactive programming, an editor designed for developing scripts and functions, and a workspace dedicated to monitoring variables.

Following the implementation of the developed script, Matlab was utilized to produce a comprehensive graphical representation of the results, as shown in Figure 8. This figure illustrates the use of linear interpolation in the CNC drilling process, utilizing the coordinates of the holes provided in Figure 6b. It offers a clear visual understanding of how linear interpolation contributes to the CNC drilling process, emphasizing its significance in achieving precise enhancements and modifications throughout the process.

Specifically, the interpolation model employs the coordinates from Figure 6b to convert discrete data points into a smooth, continuous curve, thereby effectively representing the verification of the cutting toolpath.

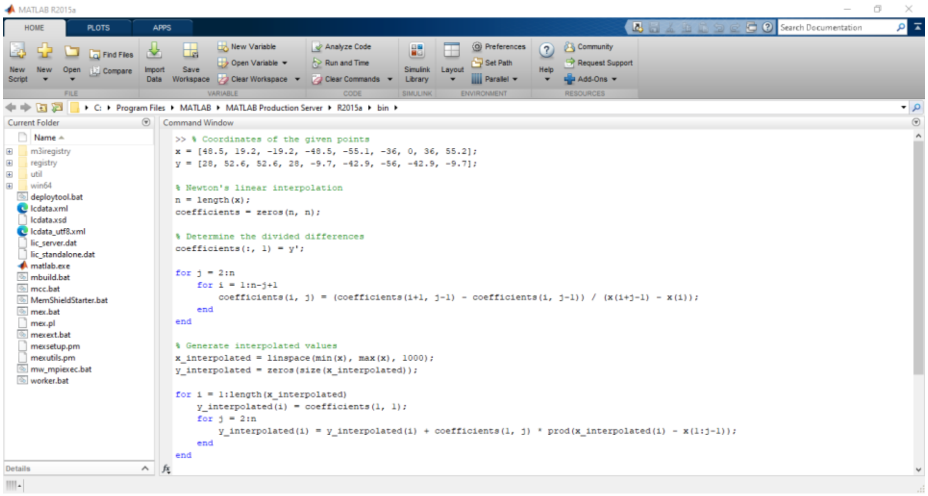


Figure 7. *The Matlab workspace visualization*

This graphical plot fulfils multiple critical roles within the framework of CNC drilling processes. Initially, it confirms the proper application of the interpolation model by illustrating a seamless transition between data points that precisely corresponds with the intended toolpath. This verification guarantees that the model operates as expected and conforms to the specified trajectory.

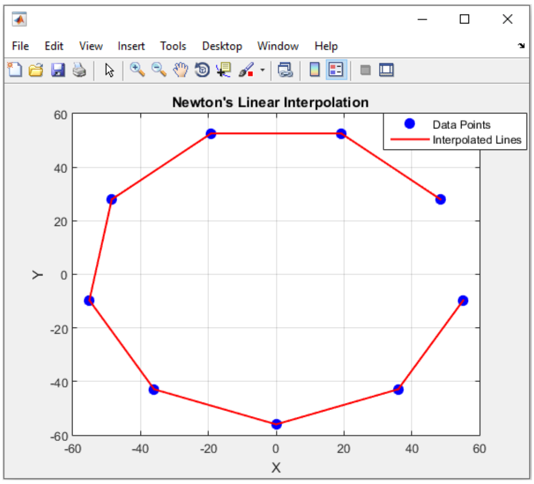


Figure 8. *The graphical plot*

Furthermore, the visual representation facilitates deeper analysis by offering engineers and technicians an in-depth perspective on the interpolation model's performance in real-world contexts. This allows for a comprehensive evaluation of the linear interpolation's alignment with the designated CNC drilling strategy, thereby enabling the identification of areas needing improvement and the application of modifications to optimize performance.

Moreover, the graphical representation is vital for the verification of the toolpath. It permits a meticulous evaluation of the toolpath by demonstrating the extent to which the CNC machine adheres to the intended trajectory. By comparing the actual toolpath with the planned trajectory, engineers can identify deviations, verify that the toolpath conforms to the designated design parameters, and ensure that any inconsistencies are rectified. This procedure is fundamental for preserving machining precision and ensuring that the final components conform to the specified requirements.

Specifically, the graphical representation enhances communication by converting complex data into a more understandable format. This improvement in clarity promotes greater comprehension and supports more effective coordination among mechanical engineers, machinists, project managers, and operators.

In summary, the graphical representation produced by Matlab is crucial for both the verification and analysis of toolpaths in CNC drilling operations. It not only validates the successful application of linear interpolation but also establishes a solid basis for ensuring that the toolpath conforms to the design and operational standards of contemporary CNC drilling machines.

4. **Conclusion**

In this paper, a mathematical model for linear interpolation motion in CNC drilling, utilizing Newton's linear interpolation formula, is developed and examined. This model, implemented using Matlab, offers significant insights into the dynamic characteristics of CNC drilling machines, thereby enhancing both theoretical comprehension and practical application in CNC drilling processes.

Newton's interpolation formula has been demonstrated to be effective in estimating and forecasting the CNC drilling machine motion, providing a precise representation of linear interpolation motion that is essential for enhancing accuracy in CNC drilling processes. The Matlab implementation enabled graphical representation and visualization, thereby improving the understanding of the model's dynamics and its practical applications.

The verification of toolpath dynamics in CNC drilling through the application of Newton's interpolation formula signifies a notable advancement in CNC machining technology. This formula can be employed to verify toolpath trajectories by systematically predicting and adjusting intermediate points between designated coordinates. Such verification improves the smoothness and precision of cutting tool movements, reducing errors and enhancing overall machining performance.

The study underscores the essential role of mathematical modelling in CNC machining, particularly in the verification and understanding of the dynamic performance of CNC drilling machines. The insights derived from this model hold significant implications for advancing CNC technology, refining drilling precision, and progressing automated machining processes. The integration with Matlab provides a robust framework for ongoing research, supporting further development and exploration of the model.

In conclusion, this linear interpolation methodology makes a significant contribution to the field of CNC machining, providing a robust foundation for both theoretical analysis and practical application. Future research could delve into more advanced interpolation techniques, integrate additional influencing factors, and further advance CNC machining processes.

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