GOCE DELCEV UNIVERSITY, SHTIP, NORTH MACEDONIA
FACULTY OF ELECTRICAL ENGINEERING

ETIMA 2021

FIRST INTERNATIONAL CONFERENCE

19-21 OCTOBER, 2021

TECHNICAL SCIENCES APPLIED IN ECONOMY, EDUCATION AND INDUSTRY

УНИВЕРЗИТЕТ „ГОЦЕ ДЕЛЧЕВ” - ШТИП
ЕЛЕКТРОТЕХНИЧКИ ФАКУЛТЕТ

UNIVERSITY „GOCE DELCHEV” - SHTIP
FACULTY OF ELECTRICAL ENGINEERING

ПРВА МЕЃУНАРОДНА КОНФЕРЕНЦИЈА
FIRST INTERNATIONAL CONFERENCE

ЕТИМА / ETIMA 2021

ЗБОРНИК НА ТРУДОВИ
CONFERENCE PROCEEDINGS

19-21 Октомври 2021| 19-21 October 2021

Главен и одговорен уредник / Editor in Chief
Проф.д-р Сашо Гелев
Prof.d-r Saso Gelev

Јазично уредување / Language Editor
Весна Ристова (Македонски) / Vesna Ristova (Macedonian)

Техничко уредување / Technical Editing
Доц.д-р Далибор Серафимовски / d-r Dalibor Serafimovski

Издавач / Publisher
Универзитет „Гоце Делчев” - Штип / University Goce Delchev - Stip
Електротехнички факултет / Faculty of Electrical Engineering

Адреса на организационен комитет / Adress of the organizational committee
Универзитет „Гоце Делчев” – Штип / University Goce Delchev - Stip
Електротехнички факултет / Faculty of Electrical Engineering
Адреса: ул. „Крсте Мисирков“ бр. 10-А / Adress: Krste Misirkov, 10 - A
Пош. фах 201, Штип - 2000, С.Македонија / PO BOX 201, Stip 2000, North Macedonia
E-mail: conf.etf@ugd.edu.mk

CIP - Каталогизација во публикација
Национална и универзитетска библиотека "Св. Климент Охридски", Скопје

62-049.8(062)
004-049.8(062)

МЕЃУНАРОДНА конференција ЕТИМА (1 ; 2021)
Зборник на трудови [Електронски извор] / Прва меѓународна
конференција ЕТИМА 2021, 19-21 Октомври 2021 = Conference proceedings /
First international conferece ETIMA 2021, 19-21 October 2021 ; [главен и
одговорен уредник Сашо Гелев]. - Штип: Универзитет "Гоце Делчев",
Електротехнички факултет = Shtip: University "Goce Delchev", Faculty of
Electrical Engineering, 2021

Начин на пристапување (URL): https://js.ugd.edu.mk/index.php/etima. -
Текст во PDF формат, содржи 358 стр.илустр. - Наслов преземен од
екранот. - Опис на изворот на ден 15.10.2021. - Трудови на мак. и англ.
јазик. - Библиографија кон трудовите

ISBN 978-608-244-823-7

1. Напор. ств. насл.
а) Електротехника -- Примена -- Собири б) Машинство -- Примена -- Собири
в) Автоматика -- Примена -- Собири г) Информатика -- Примена -- Собири

COBISS.MK-ID 55209989

Прва меѓународна конференција ЕТИМА
19-21 Октомври 2021
First International Conference ETIMA
19-21 October 2021

ОРГАНИЗАЦИОНЕН ОДБОР
ORGANIZING COMMITTEE

Василија Шарац / Vasilija Sarac
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Сашо Гелев / Saso Gelev
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Тодор Чекеровски / Todor Cekerovski
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Далибор Серафимовски / Dalibor Serafimovski
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Маја Кукушева Панева / Maja Kukuseva Paneva
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Билјана Читкушева Димитровска / Biljana Citkuseva Dimitrovska
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Весна Конзулова / Vesna Konzulova
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

ПРОГРАМСКИ И НАУЧЕН ОДБОР
SCIENTIFIC COMMITTEE

Со Ногучи / So Noguchi
Висока школа за информатички науки и технологии
Универзитет Хокаидо, Јапонија
Graduate School of Information Science and Technology
Hokkaido University, Japan

Диониз Гашпаровски / Dionýz Gašparovský
Факултет за електротехника и информациони технологии,
Словачки Технички Универзитет во Братислава, Словачка
Faculty of Electrical Engineering and Information Technology
Slovak Technical University in Bratislava, Slovakia

Антон Белан / Anton Beláň
Факултет за електротехника и информациони технологии
Словачки Технички Универзитет во Братислава, Словачка
Faculty of Electrical Engineering and Information Technology
Slovak Technical University in Bratislava, Slovakia

Георги Иванов Георгиев / Georgi Ivanov Georgiev,
Технички Универзитет во Габрово, Бугарија
Technical University in Gabrovo, Bulgaria

Ивелина Стефанова Балабанова / Ivelina Stefanova Balabanova,
Технички Универзитет во Габрово, Бугарија
Technical University in Gabrovo, Bulgaria

Бојан Димитров Карапенев / Boyan Dimitrov Karapenev
Технички Универзитет во Габрово, Бугарија
Technical University in Gabrovo, Bulgaria

Сашо Гелев / Saso Gelev
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Влатко Чингоски / Vlatko Cingoski
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Божо Крстајиќ / Bozo Krstajic
Електротехнички факултет
Универзитет во Црна Гора, Црна Гора
Faculty of Electrical Engineering,
University in Montenegro, Montenegro

Милован Радуловиќ / Milovan Radulovic
Електротехнички факултет
Универзитет во Црна Гора, Црна Гора
Faculty of Electrical Engineering,
University in Montenegro, Montenegro

Гоце Стефанов / Goce Stefanov
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Мирјана Периќ / Mirjana Peric
Електронски факултет
Универзитет во Ниш, Србија
Faculty of Electronic Engineerig,
University of Nis, Serbia

Ана Вучковиќ / Ana Vuckovic
Електронски факултет
Универзитет во Ниш, Србија
Faculty of Electronic Engineerig,
University of Nis, Serbia

Тодор Чекеровски / Todor Cekerovski
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Далибор Серафимовски / Dalibor Serafimovski
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Мирослава Фаркаш Смиткова / Miroslava Farkas Smitková
Факултет за електротехника и информациони технологии
Словачки Технички Универзитет во Братислава, Словачка
Faculty of Electrical Engineering and Information Technology
Slovak Technical University in Bratislava, Slovakia

Петер Јанига / Peter Janiga
Факултет за електротехника и информациони технологии
Словачки Технички Универзитет во Братислава, Словачка
Faculty of Electrical Engineering and Information Technology
Slovak Technical University in Bratislava, Slovakia

Јана Радичова / Jana Raditschová,
Факултет за електротехника и информациони технологии
Словачки Технички Универзитет во Братислава, Словачка
Faculty of Electrical Engineering and Information Technology
Slovak Technical University in Bratislava, Slovakia

Драган Миновски / Dragan Minovski
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Василија Шарац / Vasilija Sarac
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Александар Туџаров / Aleksandar Tudzarov
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

Владимир Талевски / Vladimir Talevski
Електротехнички факултет,
Универзитет „Гоце Делчев” - Штип, Северна Македонија
Faculty of Electrical Engineering,
Goce Delchev University - Stip, North Macedonia

PREFACE

The Faculty of Electrical Engineering at University Goce Delcev (UGD), has organized the International Conference ***Electrical Engineering, Informatics, Machinery and Automation - Technical Sciences applied in Economy, Education and Industry-ETIMA.***

ETIMA has a goal to gather the scientists, professors, experts and professionals from the field of technical sciences in one place as a forum for exchange of ideas, to strengthen the multidisciplinary research and cooperation and to promote the achievements of technology and its impact on every aspect of living. We hope that this conference will continue to be a venue for presenting the latest research results and developments on the field of technology.

Conference ETIMA was held as online conference where contributed more than sixty colleagues, from six different countries with forty papers.

We would like to express our gratitude to all the colleagues, who contributed to the success of ETIMA'21 by presenting the results of their current research activities and by launching the new ideas through many fruitful discussions.

We invite you and your colleagues also to attend ETIMA Conference in the future. One should believe that next time we will have opportunity to meet each other and exchange ideas, scientific knowledge and useful information in direct contact, as well as to enjoy the social events together.

The Organizing Committee of the Conference

ПРЕДГОВОР

Меѓународната конференција ***Електротехника, Технологија, Информатика, Машинство и Автоматика-технички науки во служба на економија, образование и индустрија-ЕТИМА*** е организирана од страна на Електротехничкиот факултет при Универзитетот Гоце Делчев.

ЕТИМА има за цел да ги собере на едно место научниците, професорите, експертите и професионалците од полето на техничките науки и да претставува форум за размена на идеи, да го зајканува мултидисциплинарното истражување и соработка и да ги промовира технолошките достигнувања и нивното влијание врз секој аспект од живееењето. Се надеваме дека оваа конференција ќе продолжи да биде настан на кој ќе се презентираат најновите резултати од истражувањата и развојот на полето на технологијата.

Конференцијата ЕТИМА се одржа online и на неа дадоа свој допринос повеќе од шеесет автори од шест различни земји со четириесет труда.

Сакаме да ја искажеме нашата благодарност до сите колеги кои допринесоа за успехот на ЕТИМА'21 со презентирање на резултати од нивните тековни истражувања и со лансирање на нови идеи преку многу плодни дискусии.

Ве покануваме Вие и Вашите колеги да земете учество на ЕТИМА и во иднина. Веруваме дека следниот пат ќе имаме можност да се сретнеме, да размениме идеи, знаење и корисни информации во директен контакт, но исто така да уживаме заедно и во друштвените настани.

Организационен одбор на конференцијата

Содржина / Table of Contents

UDC: 334.72.012.63:519.862(497.7)

ON APPLICABILITY OF BLACK-SCHOLES MODEL TO MSE

Toni Stojanovski
Vodno Pty Ltd, Melbourne Australia, toni_stojanovski@ieee.org

Abstract *In this paper we test the accuracy of assumptions of Brownian motion model for stock pricing process for the Macedonian Stock Exchange (MSE)), which serves as an example of an emerging incomplete market. For this purpose we use historical prices data for three traded companies on the MSE with the highest market capitalisation: Alkaloid (ALK), Makpetrol (MPT) and Komercijalna banka (KMB). We also evaluate the accuracy of Black-Scholes (BS) options pricing model for stocks traded on MSE. For this purpose we define a hypothetical trader whose investment strategy is to buy a fixed number of options every day. Our analysis proves that BS model is not suitable for evaluation of out-of-money options on incomplete markets since the assumptions of its underlying stock pricing process model are not satisfied. The BS model has limited value only for in-the-money options whose value is not significantly overestimated by leptokurtic nature of the distribution of daily returns.*

Key words *Financial Derivatives, Options, Black-Scholes Formula*

Introduction

At the turn of the 20th century Louis Bachelier [1] proposed using random walk as a model for stock pricing processes - all subsequent price changes represent random departures from previous prices. This model for stochastic process, now called Brownian motion, has since become the dominant model for stock pricing processes and is at the core of pillars of modern finance: Markowitz portfolio theory [2], capital asset pricing model [3], and option pricing theory [4]. Efficient market hypothesis is dominant theory in financial economics associated with the idea of random walk. The logic of the random walk idea is that when the flow of information is unconstrained and information is immediately built in stock prices, then tomorrow’s price change will reflect only tomorrow’s news and will be independent of the former price changes. News is by definition unpredictable, hence, resulting price changes must be unpredictable and random [5].

Criticisms of random walk model are numerous. Benoit Mandelbrot [6] elaborates that random walk and Gaussian daily returns simply do not correspond to reality, and grossly underestimate the risk of huge market swings. Historical data from many markets indicate that the daily changes in stock prices do not follow Gaussian distribution. Huge changes in market prices occur much more frequently than predicted by the Gaussian distribution, which is popularly known as “heavy tails”.

In this paper we test the accuracy of assumptions of the random-walk model, and of one of its most popular derivations – Black-Scholes (BS) model. Accuracy of BS model for option pricing has already been subject of several empirical investigations, indicating that BS model is biased in its estimation. Duan [7] suggests the tail properties of the underlying lognormal distribution are too small, and thus the assumption of an underlying lognormal distribution does not hold. In [8], authors analyse error in BS model for valuation of European call options derived from the CAC-40 money-market index. Dependence of estimation error on moneyness and due-term of options is analysed, discovering error for both out-of-the-money (OTM) and in-the-money (ITM) options. McKenzie *et all*. in [9] evaluate the probability of an exchange traded European call option being exercised on the ASX200 Options Index. They conclude that the BS model is relatively accurate. More precisely, BS model is significant at the 1% level in estimating the probability of an option being exercised.

This paper extends previous results in the following direction. Statistical analysis of historical data on stock prices is given for the Macedonian Stock Exchange (MSE) as an emerging stock market with low liquidity. The aim is to determine the parameters for the model for the stock pricing process. Then, we address the following research questions: How accurate can we expect the BS option pricing models to be for MSE? This question is especially important for the process of introduction of options trading on MSE and other emerging markets. Traders need a working model for option valuation which they can reliably use to estimate the value of options. While we draw our conclusions from the historical data on MSE only, we argue that our results are valid for other emerging stock markets too.

The paper is organised as follows. In Section 0 we give summary of concepts used in the remainder of the paper: introductions to financial derivatives, option valuation models, and derivations of stochastic parameters make the paper self-contained and define the concepts and notation. Section 0 gives the results on the derivation of stochastic parameters from the analysis of historical data from MSE. Section 0 analyses the accuracy of BS formulae for MSE stocks. Section 0 gives conclusions and possible directions for future research.

Background

A. Financial derivatives

A financial derivative [10] is a financial instrument whose price depends on the price of the underlying asset such as oil, gold, stocks, interest rates, or currencies. Traders are using forwards, futures, options and swaps, and various combinations of these fundamental derivative instruments both (i) to manage or reduce risk, (ii) and to increase returns.

Options are very popular financial derivatives. A call/put option is a contract which gives the holder the right to buy/sell an asset at a certain time for a certain price (strike price). Deciding a fair value for an option is an extremely difficult task: the stock pricing process depends on many parameters; the option payoff can be greatly influenced by difficult-to-predict events in the future; options can be simple (European or American) but their payoff can also depend in a very complex way on price movements (exotic options). Several models have been developed for option pricing, most notably the Black-Scholes (BS) model [4], the Binomial option pricing model [11], the Trinomial Model [12], the Barndorff-Nielsen – Shephard model [13], and jump-diffusion model [14]. Using these models to value an option is a computationally intensive task, except for vanilla European options where BS model provides a closed-form solution. Additionally, calculating the Greek parameters for an option is an adamant requirement for traders: the Greek parameters quantify the sensitivity of the option value to changes in market conditions. Calculation of option value and Greek parameters involves parameters derived from historical or current market prices. Correct estimation of these parameters is of utmost importance for option valuation.

B. Economic impact of derivatives market

The objective of a derivatives market is to maximize investor risk protection by offering hedging and risk management mechanisms. Additionally, derivatives markets have a significant economic role in price discovery. Price discovery is the way in which a market establishes the price for items traded in that market, and then disseminates those prices as information throughout the market and the economy as a whole [15]. Derivatives markets provide the links amongst cash markets, hedgers, and speculators and thus contribute to the development of the national financial infrastructure, stimulate national economic growth, and help development of financial markets. Organized markets for derivatives were typical of developed economies until the late 1980s, but the last two decades of the 20th century witnessed an increased interest in the launch of derivative markets in emerging economies. Financial derivatives are currently not traded on MSE and the stock exchanges in neighbouring countries. The process of globalization of world economy implies the need for developing countries to establish and develop financial derivatives trading, as a condition for further development of the national economy. Introduction of financial derivatives on stock exchanges in developing countries might attract foreign investments in listed companies, will stimulate further development of stock exchanges, will have a positive impact on stock trading and stock liquidity, and might even motivate new IPOs.

This paper aims to discover whether the option pricing models are valid for emerging financial markets.

C. Models for valuation of options

At the heart of the BS model [4] for option valuation are the following assumptions:

- Market direction cannot be predicted. Geometric random walk is assumed in the BS model.
- Risk-free interest rate r remains constant over option duration.
- Stock returns are normally distributed
- Volatility σ of stock prices is constant over time.

The assumptions underlying the BS model are widely considered as being too strict and even unrealistic [16] [17]. Fisher Black [16] admits that BS formula depends on unrealistic assumptions, but there is no other formula that gives better results in a wide range of circumstances. Fallout of Long Term Capital Management (LTCM) hedge fund in 1998 [17] was mostly due to small-arbitrage trading strategies combined with high leverage. Myron Scholes and Robert C. Merton were amongst the partners of LTCM, and their mathematical risk models were used by LTCM. The inaccuracies in these models were amplified by the high leverage ratio of more than 250-to-1 [17] used by LTCM, and most certainly contributed to the fallout of LTCM.

BS option pricing formulae use r and σ. Stock's expected return doesn't appear in the BS formula (see Eq.(6)-Eq.(9)), and it doesn't affect the option's valuation according to the BS formula. However, it is intuitive that a higher (lower) expected return on the stock means a higher (lower) expected return on the option. Therefore, in the next sections 0.D and 0.E we describe the process of derivation of parameters r, σ and return rate on stock prices μ from stock pricing data.

Then, in section 0 we test the calculated statistics of r, σ, and μ for MSE against the assumptions of BS formula.

D. Derivation of stochastic parameters

Valuation of financial derivatives depends strongly on volatility estimates of the underlying stock. There are two broad approaches: *historical* and *implied* volatility [18] [19]. The historical approach assumes that past holds predictive power for the future. On the other hand, *implied* volatility is calculated from the assumption that the market prices implicitly contain a consensus estimate of volatility.

Even for historical volatility there exist several models. Different models can result in different estimates. For example, GARCH model and its special case - EWMA incorporate the dynamic structure of volatility, and are capable of forecasting future behaviour of risk. Historical approaches have two steps in common: (i) Calculate the series of periodic returns; (ii) Apply a weighting scheme.

First, for each day, we take the natural log of the ratio of stock prices.

$$\mu_i = \ln\frac{s_i}{s_{i-1}} \tag{1}$$

This produces a series of m-1 daily returns e.g. from μ_2 to μ_m, if there are price measurements for m days $s_1, s_2, s_3, \ldots, s_m$. Daily returns are expressed in continually compounded terms. Then we calculate the average return for the whole measurement period

$$\mu = \frac{1}{m-1}\sum_{i=2}^{i=m}\mu_i \tag{2}$$

Second step estimates the variance from the same series of daily returns. We are using the exponentially weighted moving average (EWMA), in which more recent returns have greater weight on the variance. EWMA is more commonly used in risk management calculations, and the variance is given by the square root of

$$\hat{\sigma}_{t+1}^2 = \lambda\hat{\sigma}_t^2 + (1-\lambda)(\mu_t - \mu)^2 \tag{3}$$

where λ is the decay factor, also known as the smoothing constant. In this method, the weights are geometrically declining, hence giving more weight to the most recent observation compared to older ones. This weighting scheme helps to capture the dynamic properties of the data. Commonly, the smoothing constants are 0.94 for daily data and 0.97 for monthly data [19].

E. Kurtosis

Kurtosis [20] characterises the relative peakedness or flatness of a distribution compared with the normal distribution. For a random variable x kurtosis is defined as

$$\mathrm{Kurt}[x] = \frac{E[(x-\bar{x})^4]}{\sigma^4} - 3 \tag{4}$$

where $E[(x-\bar{x})^4]$ is the fourth moment around the mean, and σ is the standard deviation of x. For a data series e.g. daily returns $\{\mu_i\}$, kurtosis $\mathrm{Kurt}[\mu_i]$ is calculated as

$$\mathrm{Kurt}[\mu_i] = \left(\frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum_{i=2}^{i=m}\frac{(\mu_i-\mu)^4}{\sigma^4}\right) - \frac{3(n-1)^2}{(n-2)(n-3)}$$

Distributions with zero kurtosis are called *mesokurtic*. Normal distribution has zero kurtosis. Distributions with high kurtosis are called *leptokurtic*, and tend to have a distinct peak near the mean, decline rather rapidly, and have heavy tails. Distributions with negative kurtosis (*platykurtic*) have a flat top near the mean and shorter, thinner tails.

In the following sections histograms will be calculated for daily stock returns, and then kurtosis will be used to measure accuracy of the assumption that the stock returns are normally distributed.

Analysis of real market data

Capital market in Macedonia is relatively young compared with capital markets in developed countries. Trading on the MSE (www.mse.com.mk) started in 1996. Market's short history makes challenging the calculation of historical averages for market risk premium, as well other market data like beta, industry averages. MSE's short history means short time series, which has significant negative influence on calculation of indicators and ratios necessary for security valuation such as: beta calculation, historical risk premiums determination, rates of growth, industry averages etc. Historical trading data are available from the following link http://www.mse.com.mk/Statistics.aspx?MenuId=9.

Following set of figures give the stochastic parameters for the three stocks from MSE. **Figure 2** presents the histograms of daily returns for ALK, MPT and KMB. Solid violet line with cross markers gives the Gaussian distribution. Histograms of the daily return series for all stocks are leptokurtic and do not follow the Gaussian curve, which is a significant blow to the BS model [16]. This means that significant variations in the daily prices are much more common than estimated by the Gaussian distribution. The impact of higher curtosis for MSE stocks will become more obvious in the section on accuracy of BS formulae.

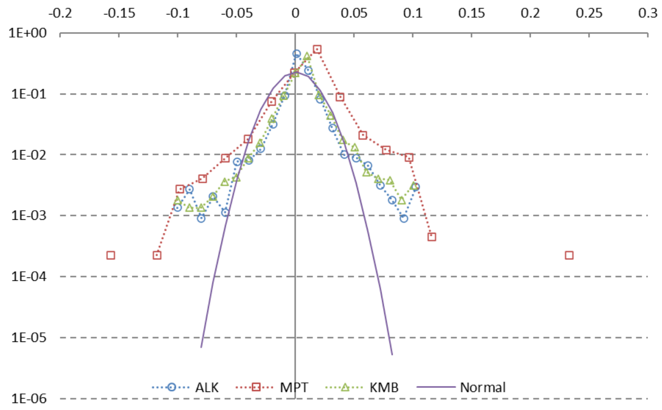

Figure 2. Histogram of daily returns for ALK, KMB and MPT.

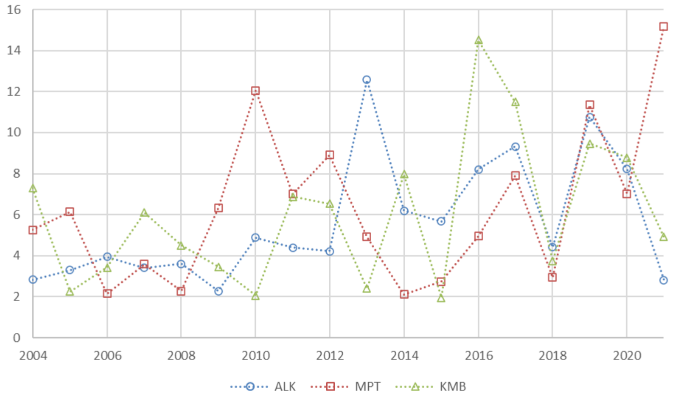

Figure 3. Kurtosis *vs.* time for ALK, KMB and MPT.

gives the kurtosis for daily returns for the period June 2003 to May 2021 calculated annually. We do not observe any downward trend in kurtosis values for the past 18 years, which means that the daily returns did not become more Gaussian-like during the past 18 years.

Following figure depicts EWMA estimator of the volatility σ_{EWMA}. As we can see in the following figure, volatility varies with time, which is in collision with the assumptions of the BS model. Over the past 18 years there were several periods of significant increases in volatility e.g. first half of 2020 which coincides with the Covid-19 outbreak and quarantine measures.

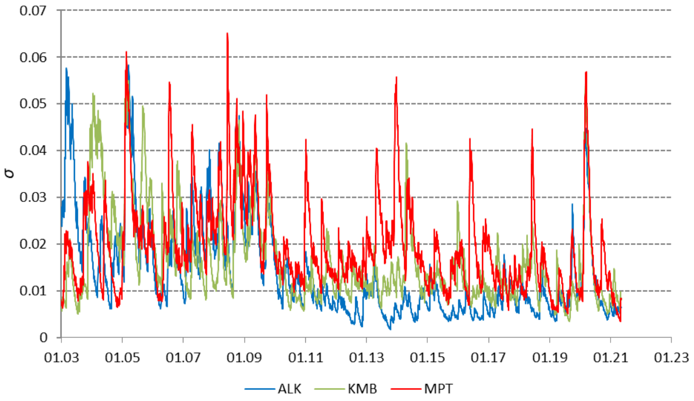

Figure 4. EWMA estimator of the volatility σ_{EWMA} for KMB stock on MSE.

Accuracy of Black-Scholes formulae

In this section, we evaluate applicability and accuracy of BS formulae for stock options on MSE. BS formulae for pricing European call and put options at time *t*=0 are:

$$C = S_0 N(d_1) - Xe^{-rT} N(d_2) \tag{5}$$

$$P = Xe^{-rT} N(-d_2) - S_0 N(-d_1) \tag{6}$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{7}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{8}$$

where C is value of a call option, P is value of a put option, *N*() is Cumulative Normal Distribution function, S_0 is current price of the underlying asset, X is exercise price, T is expiry time, r is continuously compounded risk-free interest rate, and σ is historical volatility for the underlying asset. As a risk-free interest rate we take the interest rate of treasury bonds issued by the National Bank of the Republic of Macedonia (NBRM). Data is available for download from the NBRM web site https://nbstat.nbrm.mk/.

For the purpose of measuring the accuracy of BS formulae, assume a hypothetical trader that uses those formulae to value options. Every day the trader uses historical market data to calculate volatility σ using Eq. (3) and then invests a certain amount in buying options. From trader's point of view, trading is discontinuous in time with time step 1 day. Trading times will be denoted as $t = \{1, 2, 3, \ldots\}$. Trader pays no trading charges. If the assumptions of the BS formulae are valid, then in the long run the return of this investment strategy will be equal to the risk-free interest rate. But we know for a fact that many of these assumptions are not valid [16], and they serve only to simplify the model for the stock pricing process. Then, what is the profit or loss the trader will make on real markets? What is the impact of the expiry period on the profitability of this strategy? Does the ratio between the exercise price and spot price has a consistent impact on the profitability of the investment strategy? These are the questions that we address in the remainder of this section.

Our hypothetical trader uses an investment strategy where every trading day a fixed number of options is bought. For sake of simplicity we assume that 1 option is bought every trading day. Without losing any generality of the obtained results, we also assume that

- the trader always buys options with same expiry period T,
- exercise price is calculated as a multiple of the spot price $X_{t+T} = kS_t$ by a fixed factor k.

Total investment at time *t* is calculated as the sum of present value of all previous investments

$$I_t = I_{t-1}e^{rT_M} + C_t;\ I_0 = 0 \tag{9}$$

where C_t is the premium for a call option at time *t*, $T_M = 1/252$ is one trading day.

Earning at time *t* for an option with expiry time *t* and exercise price $X_t = kS_{t-T}$ is calculated as $\max(S_t - X_t, 0)$. Total earning up to time *t* is calculated as

$$E_t = E_{t-1}e^{rT_M} + \max(S_t - X_t, 0);\ E_0 = 0 \tag{10}$$

Profit at time *t* is calculated as $\max(S_t - X_t, 0) - C_{t-T}e^T$, and total profit at time *t* is calculated as the sum of present value of all previous profits

$$P_t = P_{t-1}e^{rT_M} + \max(S_t - X_t, 0) - C_{t-T}e^T;\ P_0 = 0 \tag{11}$$

Finally, we calculate the return-on-investment $ROI = P_t/I_t$. Similar equations are valid for put options.

We repeat above calculations for $T \in$ {0.0183, 0.25, 0.5, 0.75, 1} and $k \in$ {0.6, 0.7, 0.8, 0.9, 1, 1.1, 1.2, 1.3, 1.4}. $T = 1$ means expiry period of 1 year. We use the calculations to analyse the profitability of call and put options for stocks traded on MSE. **Figure 5** gives the return-on-investment $ROI(t, k) = P_t(T, k)/I_t\,(T, k)$ as a function of *k*, and parameterised for the expiry period T for KMB call and KMB put options. For ITM options, that is, *k*<1 for call options and k> 1 for put options, the profit is moderate, which means that BS formula gives a reasonable estimate of option's value. However, our trader is making significant profits for all OTM options, except for put options with expiry period of 1 month (T= 0.0183) and k= 0.6. This is a direct consequence of the leptokurtic nature of the daily returns distribution. The heavy tails of the distribution of the daily returns will cause the deep OTM options to be exercised more frequently than expected according to the BS model. The deeper an option is out of the money, the more it is underestimated by the BS model. As a consequence, ROI grows as as k approaches 1.5 for call options, and as k decreases towards 0.6 for put options.

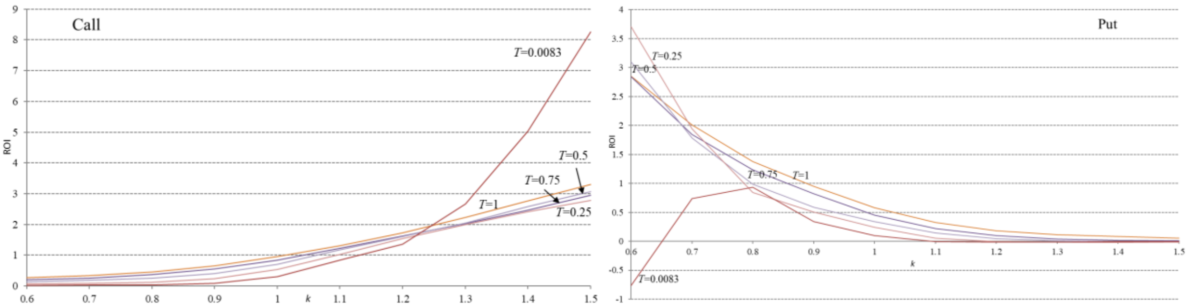

Figure 5. ROI *vs* strike price for KMB call and put options. $T \in$ {0.0183, 0.25, 0.5, 0.75, 1} is curves' parameter.

How can both call and put options be highly profitable in the same time period? The explanation is actually very simple. Namely, KMB stock price experienced strong rises and drops in several periods between 2003 to 2010, and then again around February-March 2020 coinciding with the Covid 19 outbreak. As a consequence deep OTM call and put options were exercised more frequently than anticipated by the Brownian motion model and the log-normal distribution for the stock price changes. To emphasise this phenomenon, in **Figure 6**we give the earning of individual call and put options for $T = 1$ month. Several periods of strong 1-month stock price rises in 2004-2006 make the call options highly profitable. In mid-2008 a bubble burst occurred which made the 1-month put options very profitable. At the end of February 2020, there was a Covid-19 induced drop in the share price, and consequently the OTM put options were exercised. Only a month later, the KMB prices recovered strongly, and hence the OTM call options were exercised.

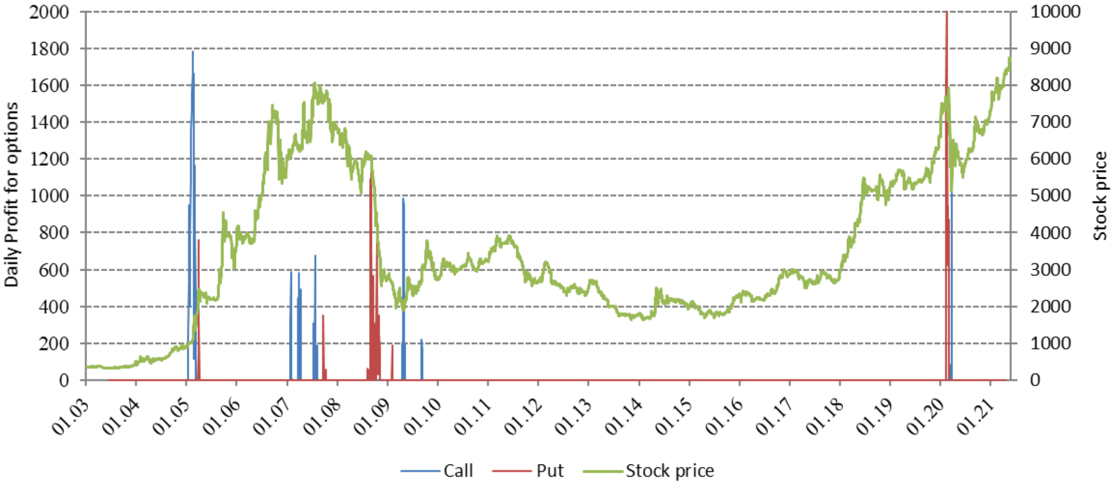

Figure 6. Daily KMB prices, and daily earning of individual KMB call and put options for $k = 1.3$ and $k = 0.8$, respectively. Expiry period is $T = 1$ month.

Phenomena depicted in **Figure 5** were observed for ALK and MPT stocks too, as illustrated in **Figure 7** and **Figure 8**, respectively. Namely, deep OTM options are heavily undervalued and can raise significant ROI for the option owners. ITM options are correctly valued by the BM formulae, and do not favour neither the option buyer nor the option seller.

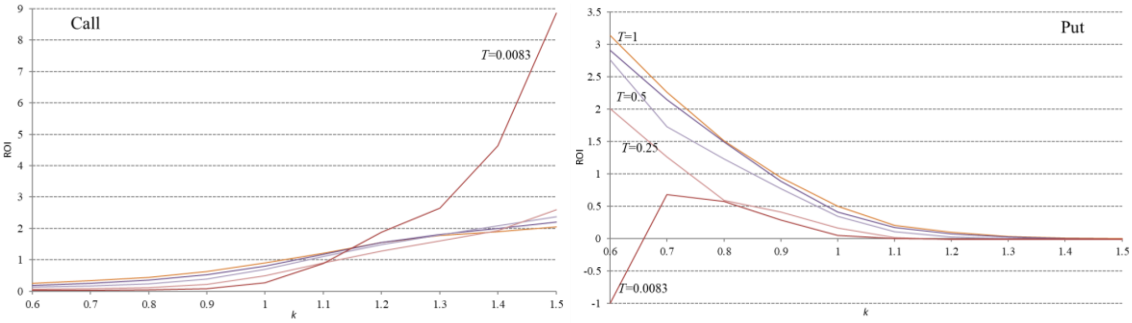

Figure 7. ROI *vs* strike price for ALK call and put options. $T \in \{0.0183, 0.25, 0.5, 0.75, 1\}$ is curves' parameter.

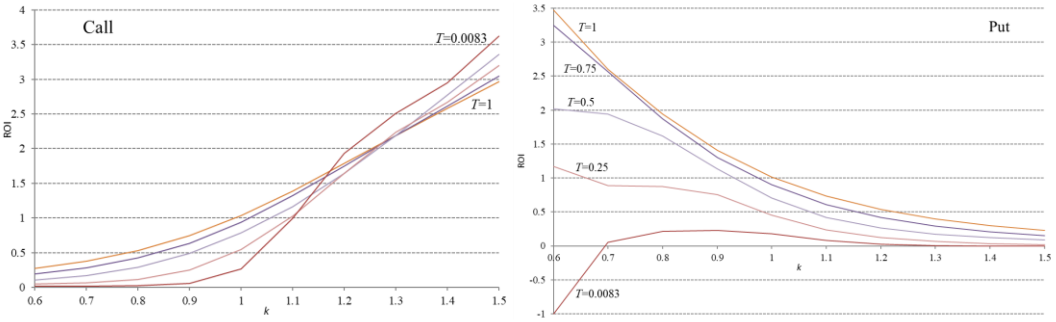

Figure 8. ROI *vs* strike price for MPT call and put options. *T*∈ {0.0183, 0.25, 0.5, 0.75, 1} is curves' parameter.

Conclusion

Accuracy of assumptions of Brownian motion model for stock pricing process are tested in this paper. We conclude that the two main assumptions of the stock pricing model underlying the BS formula are not valid for MSE and in general for developing markets First, daily returns do not follow log-normal distribution, with high kurtosis values. Significant positive (bullish) runs and significant negative (bearish) runs are more likely on MSE than expected by the Brownian motion model. Second, volatility of daily returns is not constant over time. The invalidity of the main assumptions has severe consequences on the applicability of the BS model on MSE.

When applied to MSE, BS formulae severely underestimates the likelihood of occurrence of significant jumps and drops in stock prices. Deep OTM call and put options, which can be bought for very low premiums, are frequently exercised and bring extremely high earnings to the owner. Thus, on incomplete and developing stock exchanges BS formula puts options holders in a winning position.

We recommend that the BS formulae can be reliably used only for ITM options e.g. k > 1.1 for put options and k < 0.9 for call options. For the valuation of the OTM options at incomplete emerging stock markets, whose representative is MSE, one needs to use a model for stock pricing process which takes into account high likelihood of significant drops and jumps in stock prices e.g. jump-diffusion process, and occurrence of long positive and negative runs. Another option for the evaluation of ITM options is to use Monte-Carlo simulations based on the distribution of daily returns is derived from the historical stock prices.

References

[1] L. Bachelier, "Théorie de la spéculation," Phd Thesis, Sorbonne, 1900.

[2] H. Markowitz, Portfolio Selection: Efficient Diversification of Investments, New York: John Wiley & Sons, 1959.

[3] W. F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance*, vol. 19, pp. 425-442, 1964.

[4] F. Black and M. S. Scholes, "The pricing of options and corporate liabilities," *Journal of Political Economy*, vol. 81, no. 3, pp. 637-654, May-June 1973..

[5] B. G. Malkiel, "The Efficient Market Hypothesis and Its Critics," *CEPS Working Paper No.91*, 2003.

[6] B. B. Mandelbrot and R. L. Hudson, The (Mis)behavior of Markets, Basic Books, 2006.

[7] J. C. Duan, "Conditionally fat tailed distribution and the volatility smile in options," University of Toronto, Toronto, 1999.

[8] E. Vasile and D. Armeanu, "Empirical Study on the Performances of Black-Scholes Model for Evaluating European Options," *Romanian Journal of Economic Forecasting*, vol. 10, no. 1, pp. 48-62, 2009.

[9] S. McKenzie, D. Gerace and Z. Subedar, “An empirical investigation of the Black-Scholes model: evidence from the Australian Stock Exchange,” *Australasian Accounting Business and Finance Journal*, vol. 1, no. 4, pp. 71-82, 2007.

[10] J. C. Hull, Options, Futures, and Other Derivatives, 8th ed., Prentice Hall, 2011.

[11] J. Cox, S. Ross and M. Rubinstein, “Option pricing: A simplified approach,” *Journal of Financial Economics,* vol. 7, no. 3, pp. 229-263, September 1979.

[12] P. Boyle, “Option valuation using a three jump process,” *International Options Journal,* vol. 3, pp. 7-12, 1986.

[13] O. Barndorff-Nielsen and N. Shephard, “Non-gaussian Ornstein–Uhlenbeck-based models and some of their uses in financial economics,” *Journal of the Royal Statistical Society. Series B,* vol. 63, no. 2, pp. 167-241, 2001.

[14] R. Merton, “Option Pricing when Underlying Stock Returns Are Discontinuous,” *Journal of Financial Economics,* vol. 3, no. 1-2, pp. 125-144, January-March 1976.

[15] R. Dodd, “Derivatives Markets: Sources of Vulnerability in U.S. Financial Markets,” Financial Policy Forum - Derivatives Study Center, Washington D.C., 2006.

[16] F. Black, “How to use the holes in Black-Scholes,” *Journal of Applied Corporate Finance,* vol. 1, no. 4, pp. 67-73, 1989.

[17] R. Lowenstein, When Genius Failed: The Rise and Fall of Long-Term Capital Management, Random House Trade Paperbacks, 2001.

[18] S. Figlewski, “Forecasting Volatility Using Historical Data,” New York University - Stern School of Business, May 1994.

[19] R. Suganuma, “Reality Check for Volatility Models”.

[20] J. F. Kenney and E. S. Keeping, Mathematics of Statistics, 2nd ed., vol. 2, Princeton, NJ: Van Nostrand, 1951.