Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper
https://doi.org/10.46763/IJEMT233106j

ENTREPRENEURSHIP WITH FINANCIAL AND UNEMPLOYMENT FRICTIONS

Dushko Josheski[1*], Natasha Miteva[2], Tatjana Boshkov[3]
[1]Associate professor Goce Delcev University of Stip, Faculty of tourism and business logistics, R.North Macedonia, dusko.josevski@ugd.edu.mk
[2]Associate professor, Goce Delcev University of Stip, Faculty of tourism and business logistics, R.North Macedonia, tatjana.dzaleva@ugd.edu.mk
**Corresponding author:* dusko.josevski@ugd.edu.mk
[3]Assistant professor, Goce Delcev University of Stip, Faculty of tourism and business logistics, R.North Macedonia, natasa.miteva@ugd.edu.mk

**Corresponding author:* dusko.josevski@ugd.edu.mk

Abstract

Financial frictions for capital and unemployment frictions for labor is said to be main drivers of the business cycle dynamics.Large literature in general equilibrium models of entrepreneurship exists whereby financial frictions play major role in the underlying incentives and motives to trade. In these models entrepreneur wealth shock has different effect on the baseline (frictionless) economy and the economy with financial and unemployment frictions.Entrepreneurs are iintroduced in the models and they posses different abilities (measured by low,medium,high productivity) which in turn affects aggregate savings and aggregate wealth of the economies.In these models financial frictions are introduced following assumption that working capital loans are frictionless. Each entrepreneur faces probability of exit from economy (bankruptcy probability) which is identical for all entrepreneurs in the economy. In the endogenous entrepreneurship and financial frictions model it is shown that occupation choice is different for worker or entrepreneur based on productivity: low (worker) or high (entrepreneur) which in turn affects the aggregate wealth of the economy. Also, in this model it is shown that interest rate in the economy is determined by the intersection of total wealth with public and private capital.

Key words: financial frictions, employment frictions, entrepreneurship, business cycles
JEL Classification: E00,E30,G00,J64

Introduction

Financial crisis from 2008 has made it clear that business cycles modeling can no longer abstract itself from financial factors that appear *prima facie* to be main mechanism of the economic downturn (see Christiano et al. (2011)). Entrepreneurs developers of private firms are central actors in modern economies, and the anemic growth of firms and their plants is one of the unfortunate features of the underdeveloped economies (see, Hsieh,Klenow (2014), Buera et al.(2015)).Poor countries also have low levels of financial development. With lesser access to financial services, such as savings accounts and bank loans, and measures of externa; finance to GDP that can be of order of magnitude smaller than those of advanced economies (see Banerjee, Duflo (2005), King , Levine (1993)).One particular and common explanation of low performance of entrepreneurs in developing countries is their inability to obtain credit to expand their scale of operation. This literature shows that wealth of entrepreneurs is highly undiversified, suggesting limits to the feasibility of external financing. Some forms of contractual frictions or borrowing constraints must be at work to explain why entrepreneurs take so much risk, (see Quadrini (2009)). Several studies in macroeconomics have extended the basic model of precautionary savings, (see Carroll (1997)[1], Huggett (1993),

[1] This is a significant body of literature whose goal is to understand the differential saving behavior between rich and poor households.

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

Aiyagari (1994)) to incorporate entrepreneurs. These studies are quantitative in nature and they all share the following features: first the choice to become an entrepreneur is endogenous, second feature is that entrepreneurs generate income with the input of capital, and third external financing is limited due to some form of financial frictions.There is a list of papers that share some of the issues before mentioned: Akyol,Athreya (2007), Bohacek (2006), Buera (2007), Cagetti ,DeNardi (2004,2006), Li (2000), Meh (2005), Meh,Terajima (2007), Quadrini (2000),Terajima (2006).Entrepreneurship models can also study more specific policies such as subsidies and taxes. Examples of previous are such as: Cagetti ,DeNardi (2004), Kitao (2008), Li (2000), Meh (2005). Another issue investigated in these models is whether financial imperfections have positive or negative effect on aggregate accumulation of capital, and in the representative model with uninsurable idiosyncratic risks to earnings, market incompleteness leads to higher accumulation of capital. About the incomplete markets we know that Storesletten et al. (2001) showed that liquidity-constrained households are hit particularly hard by aggregate productivity shocks. Arrow (1951) and Arrow,Debreu (1954),proved that competitive equilibrium in Arrow-Debreu economy is Pareto optimal and discovered class of convex Arrow-Debreu economies for which competitive equilibria always exist. In the case of incomplete (see Geanakoplos (1990)) markets this equilibrium may (will) not be efficient see Geanakoplos (1986) or the will be suboptimal constrained. Newer literature includes :Guntin,Kochen(2022), that relates to the literature of financial frictions and misallocation as a source of low total factor productivity see also Hsieh , Klenow, (2009) Buera, Kaboski, Shin, (2011); Midrigan ,Xu, (2014).These models have also been used to analyze business cycle fluctuation, particularly in the aftermath od the great depression or 2008 financial crisis (see Achdou et al.(2014), Bassetto et al. (2013), Buera ,Moll (2012), Buera et al. (2014), Kiyotaki , Moore (2012), Shourideh,Zetlin-Jones(2014), with entrepreneurs playing an important role relative to corporations because of the interaction of consumption, saving, and risk that is linked with investment.In the unemployment section of this research Diamond-Mortensen-Pissarides framework (DMP model)[2] has been used. Important standard textbook in macroeconomics use DMP framework these include: Carlin ,Soskice (2006); Williamson (2013); Chugh (2015)[3].DMP model has been accepted throughout macroeconomics in the economics of business cycles, Merz (1995) ;Andolfatto (1996), in the New Keynesian model, see Gertler, Trigari (2009),in the area of monetary policy, see Blanchard and Gali (2010);and in the field of endogenous disasters, Petrosky-Nadeau, Zhang, and Kuehn (2015) ,see Petrosky-Nadeau, Zhang(2017).As per Hall (2012), DMP model is a central component of modern macroeconomics. Model by Christiano et al. (2011) implements financial frictions in the accumulation and management of capital similar to Bernanke et al. (1999) and Christiano et al. (2003, 2008). The financial frictions that are introduced in the paper by Christiano et al. (2011) show that borrowers and lenders are different agents, and that they have different information. Thus, they introduce “entrepreneurs”. These agents own and manage the capital stock, financed both by internal and borrowed funds. Only the entrepreneurs costlessly observe their own idiosyncratic productivity. The presence of asymmetric information in financing the capital stock leads to a role for the balance sheets of entrepreneurs. So, this paper is structured in the following way: First the processes used in the MATLAB,DYNARE and PYTHON will be explained namely Ornstein-Uhlenbeck process, Hamilton-Jacobi-Bellman equation, Finite difference method, and Euler equation. After these, three examples will be used to show effects financial frictions,

[2] Even before this paper large literature existed on job rationing and matching frictions .Models of job ration include efficiency wage models, Solow (1979), gift-exchange model Akerlof (1982), insider-outsider models such as Lindbeck ; Snower (1988),and social norm models Akerlof (1980).

[3] Also wage-setting frictions have an impact on the effort of an employer in recruiting new employees. Accordingly, the setup is not vulnerable to the Barro (1977) critique that wages cannot be allocational in ongoing employer–employee relationships (see Hall, (2005))

Manuscript received: 29.03.2023
Accepted: 20.04.2023

and unemployment frictions (with entrepreneurs) on the economy. Codes used in those parts are written by: Frederic Martenet (PYTHON model)[4], Christiano et al. (2011) a Dynare code [5],and MATLAB code example by Benjamin Moll[6] based on Achdou et.al (2014).In the end we will draw conclusion on models of entrepreneurship with financial and unemployment frictions.

Ornstein–Uhlenbeck process

The Ornstein–Uhlenbeck [7]process x_t is defined as follows:

equation 1

$$dx = \mu F dt + \sqrt{2D} dW$$

Where x drifts with velocity μF, combining drift with an unbiased random walk with average step size$\sqrt{2Ddt}$. Suppose $x_1,(t), x_2(t), \dots x_d(t)$ are d dependent Ornstein-Uhlenbeck processes where $dx_i(t) = -\frac{1}{2}\alpha x_i(t)dt + \sqrt{\alpha} dB_i(t)$, where $B_i(t)$ are standard Brownian motions , $\sqrt{\alpha}$ represents volatility and $-\frac{1}{2}\alpha$ is a mean-reversion rate.Here $x(t)$ follows normal distribution with $\left(x_i(0)e^{-\frac{\alpha t}{2}}; 1 - e^{-\alpha t}\right)$.

The squared radius of the vector $x(t)$ is $R(t) = \sum_{i=1}^{d} x_i(t)^2 \rightarrow dR(t) = \sum_{i=1}^{d} \left(2x_i(t)dx_i(t) + d(x_i)(t)\right) = \alpha\left(d - R(t)\right)dt + \sqrt{4\alpha} d\widetilde{W_i}(t)$; $\theta = \frac{4\alpha}{\sigma^2}$ and $d = \frac{4\alpha\mu}{\sigma^2}$ and we have that $r(t) = \frac{R(t)}{\theta}$.

Definition: Let $(\mathcal{W}_t, \mathcal{F}_t)_{t\in(0,\infty)}$ be an $\mathbb{R}$-valued continuous stochastic process in probability space $(\Omega, \mathcal{F}, \mathcal{P})$,then $(\mathcal{W}_t, \mathcal{F}_t)_{t\in(0,\infty)}$ is called standard Brownian motion if:$\mathcal{W} = 0$; $\mathcal{W}_t - \mathcal{W}_s \sim \mathcal{N}(0, t-s)$; $\mathcal{W}_t - \mathcal{W}_s \perp \mathcal{F}_s$.An $\mathbb{R}^T$ valued process $\mathbb{W}_t$ is called T-dimensional Brownian motion with initial value $x \in \mathbb{R}^T$ if $\mathbb{W}_t = x + (W_t^1, \dots, W_t^T), \forall t \in (0, \infty)$, where $\mathcal{W}_t^i$ are standard Brownian motions, see Ewald (2003).From here we can obtain Fokker-Planck equation[8] for the PDF of finding Brownian particle at $\frac{dx}{dt}$

equation 2

$$\frac{\partial \rho}{\partial t} = \mu \frac{\partial}{\partial x}(\phi'(x)\rho) + D\frac{\partial \rho^2}{\partial x^2}$$

Furthermore we can write previous as follows :

equation 3

$$\frac{\partial P(X,t)}{\partial t} = \int_{n=1}^{\infty} \left(-\frac{\partial}{\partial X}\right)^n [D^n(X)P(X,t)]d(t)$$

Where :

equation 4

$$D^n(X_0) = \frac{1}{n!} \lim_{\Delta t \to 0} \frac{1}{\Delta t} [X(t + \Delta t) - X(t)^n]|_{t=0}$$

Which is commonly referred to as Kramers–Moyal expansion (see Kramers (1940), Moyal (1949)):$\frac{\partial p(x,t)}{\partial t} = \int dx'[W(x|x')p(x',t) - W(x'|x)p(x,t)]$.Where $p(x,t|x_0,t_0)$ is probability or transition probability density to an infinite order PDE (see Gardiner (2009)):

[4] see: https://github.com/FredericMartenet/entrepreneurs

[5] DYNARE code : https://faculty.wcas.northwestern.edu/lchrist/course/Korea_2012/CTW.html

[6] Benjamin Moll codes https://benjaminmoll.com/codes/
[7] See Uhlenbeck, G. E.; Ornstein, L. S. (1930).
[8] See Fokker (1914), Planck (1917), Kolmogorov (1931).

Manuscript received: 29.03.2023
Accepted: 20.04.2023
International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

equation 5

$$\frac{\partial p(x,t)}{\partial t}=\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\frac{\partial^n}{\partial x^n}[a_n(x)p(x,t)]$$

Where $a_n(x)=\int_{-\infty}^{+\infty}(x'-x)^n W(x'|x)dx'$.So now lets get back to our previous equation: $\frac{\partial \rho}{\partial t}=\mu\frac{\partial}{\partial x}(\phi'(x)\rho)+D\frac{\partial \rho^2}{\partial x^2}$ which in the context of Brownian motion this is called Smoluchowski equation (see Smoluchowski, M. (1916)):

equation 6

$$\eta\frac{\partial w}{\partial t}=\nabla(w\nabla U)+T\nabla^2 w$$

Where $U(q,t)$ is time dependent potential,$w(q,t)$ are moments of probability density,

equation 7

$$m\,\ddot{q}+\eta\,\dot{q}+\frac{\partial U(q,t)}{\partial q}=\tilde{\mathcal{F}}(t)$$

Where :

equation 8

$$m\,\ddot{\tilde{q}}+n\,\dot{\tilde{q}}+k(t)\tilde{q}=\tilde{\mathcal{F}}(t);k(t)\equiv\frac{\partial^2}{\partial q^2}U(q(t),t)$$

The total probability is given as:

equation 9

$$W(t)=\int_{well's\ bottom}w(q,t)dq$$

Where barriers height of Kramer's problem is given as: $U_0=U(q_2)-U(q_1)$,the Boltzmann distribution $w\ \propto\left\{-\frac{U(q)}{T}\right\}$ under decay law $\dot{W}=-\frac{W}{\tau}$ where the lifetime τ has to obey the Arrhenius law[9]:

equation 10

$$\tau=\tau_A\exp\left\{\frac{U_0}{T}\right\}$$

So in the context of our previous equation $\frac{\partial \rho}{\partial t}=\mu\frac{\partial}{\partial x}(\phi'(x)\rho)+D\frac{\partial \rho^2}{\partial x^2}$ it is convenient to write this equation in terms of probability of current S:

equation 11

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho}{\partial x}=0;S=-\mu\phi'(x)\rho-D\frac{\partial \rho}{\partial x}=-D\exp\left(-\frac{\phi}{k_BT}\right)\frac{\partial}{\partial x}\left[\exp\left(\frac{\phi}{k_BT}\right)\rho\right]$$

Suppose a steady-state exists for the distribution i.e. some equilibrium. The state would $\exists\rightarrow S=0$ so:

equation 12

$$\rho(x)=\rho_0\exp\left(\frac{\phi}{k_BT}\right)$$

This exactly is Boltzmann formula.Or in most simple terms Ornstein- Uhlenbeck process is:

equation 13

$$dx=\mu dt+\sigma dW$$

Which is not-stationary random walk , but the following process is :

equation 14

$$dx=\theta(\bar{x}-x)dt+\sigma dW$$

Analogue to AR(1) process is autocorrelation $e^{-\theta}\approx 1-\theta$ or : $x_{t+1}=\theta\,\bar{x}+(1-\theta)x_t+\sigma\varepsilon_t$

---

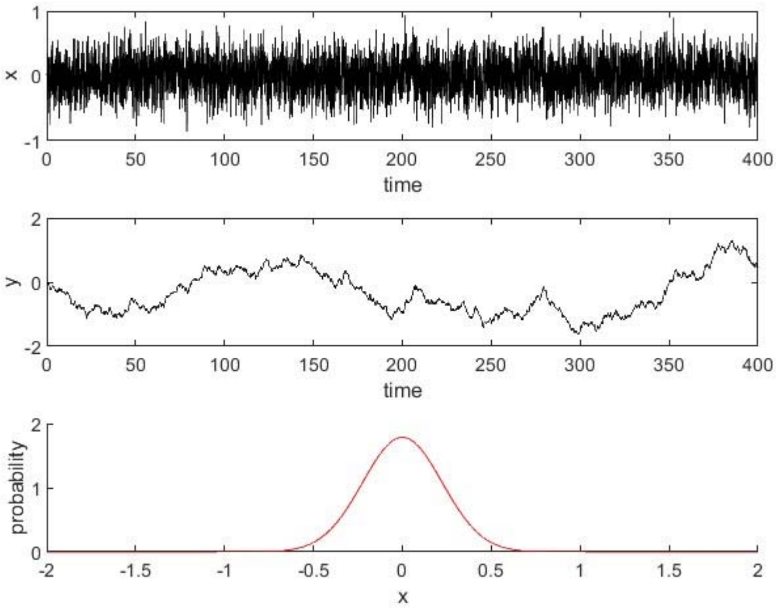
[9] $k=Ae^{\frac{E_a}{RT}}$; where A is the pre-exponential factor, k is the rate constant (frequency of collisions resulting in a reaction),$R$is universal constant.

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

So now we chose $\mu(x) = \theta(\bar{x} - x)$ and we get a nice stationary process, which is called Ornstein-Uhlenbeck process.

Figure 1 Ornstein-Uhlenbeck process



Source : code provided by Travis Kupsche (2023). histNorm (https://www.mathworks.com/matlabcentral/fileexchange/27316-histnorm), MATLAB Central File Exchange. Retrieved March 6, 2023.

HJB equation

HJB equation is modeled as in Achdou et al.(2022). The deterministic optimal control problem is given as:

equation 15

$V(x_0) = \max_{u(t)_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} h(x(t), u(t)dt$ s.t. $\dot{x}(t) = g\big(x(t)\big), u(t), u(t) \in U\,; t \geq 0, x(0) = x_0$

In previous expression:$\rho \geq 0$ is the discount rate, $x \in X \subseteq \mathbb{R}^m$ is a state vector; $u \in U \subseteq \mathbb{R}^n$ is a control vector, and $h: X \times U \to R$. The value function of the generic optimal control problem satisfies the Hamilton-Jacobi-Bellman equation, i.e.:

equation 16

$$\rho V(x) = \max_{u \in U} h(x, u) + V'(x) \cdot g(x, u)$$

In the case with more than one state variable $m > 1, V'(x) \in \mathbb{R}^m$ is the gradient of the value function. Now for the derivation of the discrete-time Bellman eq. we have: time periods of length Δ ,discount factor $\beta(\Delta) = e^{-\rho\Delta}$, here we can note that $\lim_{\Delta\to\infty} \beta(\Delta) = 0$ and $\lim_{\Delta\to 0} \beta(\Delta) = 1$.Now that discrete Bellman equation is given as:

equation 17

$V(k_t) = \max_{c_t} \Delta U(c_t) + e^{-\rho\Delta} V(k_{t+\Delta})$ s.t. $k_{t+\Delta} = \Delta[F(k_t) - \delta k_t - c_t] + k_t$

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

For a small $\Delta = 0$ we can make: $e^{-\rho\Delta} = 1 - \rho\Delta$, so that $V(k_t) = \max_{c_t} \Delta U(c_t) + (1 - \rho\Delta,)V(k_{t+\Delta})$,if we subtract $(1 - \rho\Delta,)V(k_t)$ from both sides and divide by Δ and manipulate the last term we get : $\rho V(k_t) = \max_{c_t} \Delta U(c_t) + (1 - \rho\Delta,)[V(k_{t+\Delta}) - V(k_t)\,]$ we get :

equation 18

$$\rho V(k_t) = \max_{c_t} \Delta U(c_t) + (1 - \rho\Delta,)\frac{[V(k_{t+\Delta}) - V(k_t)\,]}{k_{t+\Delta} + k_t}\frac{k_{t+\Delta} - k_t}{\Delta}$$

If $\Delta \to 0$ then $\rho V(k_t) = \max_{c_t} \Delta U(c_t) + V'(k_t)\ \dot{k}_t$. Hamilton-Jacobi-Bellman equation in stochastic settings is given as:

equation 19

$V(x_0)) = \max_{u(t)_{t=0}^{\infty}} \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} h\big(x(t), u(t)\big)dt$ s.t.$dx(t) = g\big(x(t), u(t)\big)dt + \sigma\big(x(t)\big)dW(t)\,, u(t) \in U\,; t \geq 0, x(0) = x_0$

In previous expression $x \in \mathbb{R}^m; u \in \mathbb{R}^n$. HJB equation without derivation is :

equation 20

$$\rho V(x) = \max_{u \in U} h(x,u) + V'(x)g(x,u) + \frac{1}{2}V''(x)\sigma^2(x)$$

In the multivariate case: for fixed x we define $m \times m$ covariance matrix, $\sigma^2(x) = \sigma(x)\sigma(x)'$ which is a function of $\sigma^2: \mathbb{R}^m \to \mathbb{R}^m \times \mathbb{R}^m$. HJB equation now is given as:

equation 21

$$\rho V(x) = \max_{u \in U} h(x,u) + \sum_{i=1}^{m} \frac{\partial V(x)}{\partial x_i} g_i(x,u) + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\partial^2 V(x)}{\partial x_i \partial x_j}\sigma_{ij}^2(x)$$

In vector notation previous is given as:

equation 22

$$\rho V(x) = \max_{u \in U} h(x,u) + \nabla_x V(x) \cdot g(x,u) + \frac{1}{2} tr\big(\Delta_x V(x)\sigma^2(x)\big)$$

Where $\nabla_x V(x)$: gradient of V (dimension $m \times 1$) ; $\Delta_x V(x)$: Hessian matrix of V (dimension $m \times m$).By Ito's lemma[10]:

equation 23

$$df(x) = \left(\sum_{i=1}^{n} \mu_i(x)\frac{\partial f(x)}{\partial x_i} + \frac{1}{2}\sum_{i=1}^{m}\sum_{i=1}^{m}\sigma_{ij}^2(x)\frac{\partial^2 f(x)}{\partial x_i \partial x_j}\right)dt + \sum_{i=1}^{m}\sigma_I(x)\frac{\partial f(x)}{\partial x_i}dW_i$$

In vector notation:

equation 24

$$df(x) = \left(\nabla_x f(x) \cdot \mu(x) + \frac{1}{2}tr\big(\Delta_x f(x)\sigma^2(x)\big)\right)dt + \nabla_x f(x) \cdot \sigma(x)dW$$

Now for the Kolmogorov Forward (Fokker-Planck[11]) equation we have folowing: let x be a scalar diffusion

equation 25

$$dx = \mu(x)dt + \sigma(x)dW, x(0) = x_0$$

[10] Itô's lemma is an identity used in Itô calculus to find the differential of a time-dependent function of a stochastic process. It serves as the stochastic calculus counterpart of the chain rule, see Kiyosi Itô (1951).

[11] See Fokker (1914), Planck (1917), Kolmogorov (1931).

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

Let's suppose that we are interested in the evolution of the distribution of $x, f(x,t)$ and $\lim_{t\to\infty} f(x,t)$ .So, given an initial distribution $f(x,0) = f_0(x), f(x,t)$ satisfies PDE :

equation 26

$$\frac{\partial f(x,t)}{\partial t} = -\frac{\partial}{\partial x}[\mu(x)f(x,t)] + \frac{1}{2}\frac{\partial^2}{\partial x^2}[\sigma^2(x)f(x,t)]$$

Previous PDE is called "Kolmogorov Forward Equation" or "Fokker-Planck Equation".

Corollary 1: if a stationary equilibrium exists $\lim_{t\to\infty} f(x,t) = f(x)$, it satisfies ODE

equation 27

$$0 - \frac{d}{dx}[\mu(x)f(x)] + \frac{1}{2}\frac{d^2}{dx^2}[\sigma^2(x)f(x)]$$

In the multivariate case Kolmogorov Forward Equation is given as:

equation 28

$$\frac{\partial f(x,t)}{\partial t} = -\sum_{i=1}^{m}\frac{\partial}{\partial x_i}[\mu(x)f(x,t)] + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{\partial^2}{\partial x^2}\left[\sigma_{ij}^2(x)f(x,t)\right]$$

Comparison between Hamiltnian and HJB equation gives:

equation 29

$\mathcal{H}(x,u,\lambda) = h(x,u) + \lambda g(x,u)$ (Hamiltonian)

equation 30

$\rho V(x) = \max_{u\in U} h(x,u) + V'(x)g(x,u)$ (Bellman)

Connection i.e. co-state value is $\lambda(t) = V'\big(x(t)\big)$ which is a shadow value. Bellman can be written as:

equation 31

$$\rho V(x) = \max_{u\in U} \mathcal{H}\big(x,u,V'(x)\big)$$

Finite difference method of HJB equation

Finite difference method of HJB equation is given as:

As in Achdou et al.(2022), two functions v_1, v_2 at I discrete points in the space dimension a_i , $i = 1, \dots, I$. Equispaced grids are denoted by Δa_ias the distance by the grid points, and shot hand notation used is $v_{i,j} \equiv v_j(a_i)$ and so on. Backward difference approximation is given as:

equation 32

$$\begin{cases} v_j'(a_i) \approx \dfrac{v_{i+1,j} - v_{i,j}}{\Delta a} \equiv v_{i,j,F}' \\ v_j'(a_i) \approx \dfrac{v_{i+1,j} - v_{i-1,j}}{\Delta a} \equiv v_{i,j,B}' \end{cases}$$

Two basic equations to explain Huggett economy are :

equation 33

$$\begin{cases} \rho v_1(a) = \max_c u(c) + v_1'(a)(z_1 + ra - c) + \lambda_1\big(v_2(a) - v_1(a)\big) \\ \rho v_2(a) = \max_c u(c) + v_2'(a)(z_2 + ra - c) + \lambda_2\big(v_1(a) - v_2(a)\big) \end{cases}$$

Where $\rho \geq 0$ represents the discount factor for the future consumption c_t (Individuals have standard preferences over utility flows), a represents wealth in form of bonds that evolve according to :

equation 34

$$\dot{a} = y_t + r_t a_t - c_t$$

y_t is the income of individual, which is endowment of economy's final good, and r_t represents the interest rate. Equilibrium in this Huggett (1993) economy is given as:

Manuscript received: 29.03.2023
Accepted: 20.04.2023

equation 35

$$\int_{\underline{a}}^{\infty} a g_1(a,t)da + \int_{\underline{a}}^{\infty} a g_2(a,t)da = B$$

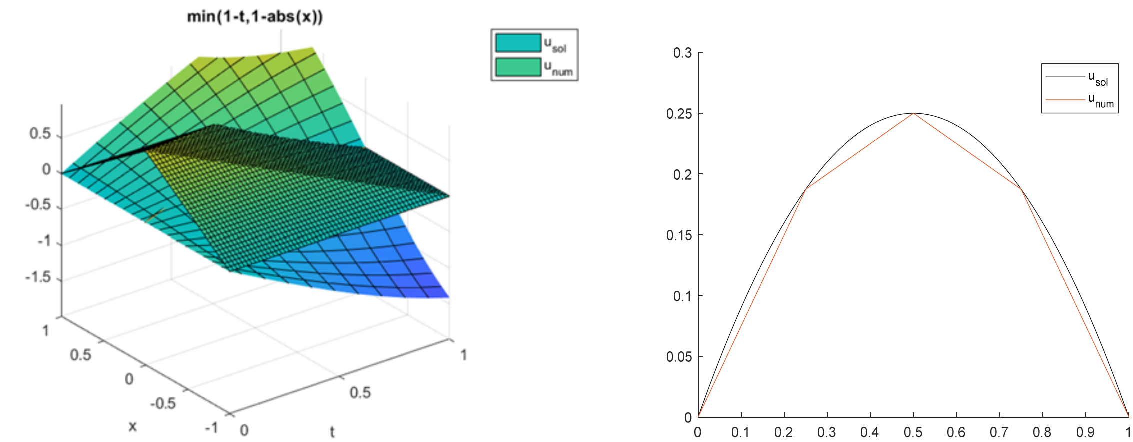
Where in previous expression $0 \leq B \leq \infty$ and when $B = 0$ that means that bonds are zero net supply. So the finite difference method approx. to $\begin{cases} \rho v_1(a) = \max_c u(c) + v_1'(a)(z_1 + ra - c) + \lambda_1(v_2(a) - v_1(a)) \\ \rho v_2(a) = \max_c u(c) + v_2'(a)(z_2 + ra - c) + \lambda_2(v_1(a) - v_2(a)) \end{cases}$ is given as:

equation 36

$$\rho v_{i,j} = u(c_{i,j}) + v_{i,j}'(z_j + ra_i + c_{i,j}) + \lambda_j(v_{i,-j} - v_{i,j}), j = 1,2$$

$$c_{i,j} = (u')^{-1}(v_{i,j}')$$

Figure 2 (a,b) Finite difference method



Source: Greif, Constantin, "Numerical Methods for Hamilton-Jacobi-Bellman Equations" (2017). Theses and Dissertations. 1480.

CRRA utility

An example of Constant relative risk aversion is given as:

equation 37

$$u(x) = \frac{x^{1-\rho} - 1}{1-\rho}$$

Note that $u'(x) = x^{-\rho}$ and $u''(x) = -\rho x^{\rho-1}$; $R(x) = xA(x) = -\frac{xu''(x)}{u'(x)}$.Where $A(x) = a$; and $u'(x) = ae^{-ax}$ and so $u''(x) = -a^2 e^{-ax}$, or $A(x) = \frac{-u''(x)}{u'(x)}$.

Expected utility theorem

Next will give expected utility theorem

Theorem: X is a finite set of prizes,$\Delta(X)$ is a set of lotteries on X. Let $\succcurlyeq$ be a binary relation on ,$\Delta(X)$. Then $\succcurlyeq$ is complete, reflexive , transitive, and satisfies independence . Preference relation $\gtrsim$ is a relation $\gtrsim \subset \mathbb{R}_+^l \times \mathbb{R}_+^l$.With properties $x \gtrsim x, \forall x \in \mathbb{R}_+^l$ (reflexivity), $x \gtrsim y, y \gtrsim z \Rightarrow x \gtrsim z$ (transitivity), $\gtrsim$ is a closed set (continuity), $\forall (x \gtrsim y), \exists (y \gtrsim x)$ (completeness) ,given $\gtrsim, \forall\ (x \gg 0)$ the at least good set $\{y: y \gtrsim x\}$is closed relative to R^l (boundary condition), A is convex, if $\{y: y \gtrsim x\}$is convex set for every y , $ay + (1-\lambda)x \gtrsim x$, whenever $y \gtrsim x$ and $0 < a < 1$, Mas-Colell, A. (1989).

Lemma 1: If $\succcurlyeq$ is complete, reflexive, transitive, and satisfies independence then:

1. $p \succ q$ and $0 \leq \alpha < \beta \leq 1$
   $\beta p + (-\beta + 1)q \succ \alpha p + (1-\alpha)q$
2. $p \succcurlyeq q \succcurlyeq r; p \succ r$; $\exists \alpha^* \Rightarrow q \sim \alpha^* p + (1-\alpha)r^*$

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

Well in the standard model of one risk-free asset and one risky asset, under constant relative risk aversion the fraction of wealth optimally placed in the risky asset is independent of the level of initial wealth, (see Arrow(1965)).
MIT shock
In definition given by Boppart et al. (2018) “MIT shock” is defined as:“*An “MIT shock” is an unexpected shock that hits an economy at its steady state, leading to a transition path back towards the economy’s steady state……*”.Mukoyama (2021) also follows Boppart et al. (2018) definition:”…. the probability of the shock is considered zero, and no prior (contingent) arrangement is possible for the occurrence of the MIT shock”…..The dynamic analysis that was using exogenous shocks or policy changes has been used in the literature with the earlier examples including: Abel,Blanchard (1983), Auerbach, Kotlikoff (1983), and Judd (1985).And more recent examples being: Boppart et al. (2018), Kaplan et al. (2018), Boar ,Midrigan (2020), Guerrieri et al. (2020).

Euler equation
Here following lemma applies see Achdou et al.(2022)
Lemma 2: The consumption and savings policy functions $c_j(a)$ and $s_j(a)$ for $j=1,2..$ corresponding to HJB equation : $\rho v_j(a)=\max_c u(c)+v_j'(a)(y_j+ra-c)+\lambda_j\left(v_{-j}(a)-v_j(a)\right)$ which is maximized at : $0=-\frac{d}{da}[s_j(a)g_J(a)]-\lambda_j g_j(a)+\lambda_{-jg_{-j}}(a)$ is given as:
equation 38

$$(\rho-r)u'\left(c_j(a)\right)=u''\left(c_j(a)\right)c_j'(a)s_j(a)+\lambda_j(u'\left(c_{-j}(a)\right)-u'\left(c_j(a)\right)$$
$$s_J(a)=y_J+ra-c_j(a)$$

Proof: differentiate $\rho v_j(a)=\max_c u(c)+v_j'(a)(y_j+ra-c)+\lambda_j\left(v_{-j}(a)-v_j(a)\right)$ with respect to a and use that $v_j'(a)=u'\left(c_j(a)\right)$ and hence $v_j''(a)=u''\left(c_j(a)\right)c_j'(a)$ ■
The differential equation $(\rho-r)u'\left(c_j(a)\right)=u''\left(c_j(a)\right)c_j'(a)s_j(a)+\lambda_j(u'\left(c_{-j}(a)\right)-u'\left(c_j(a)\right)$, $s_J(a)=y_J+ra-c_j(a)$ is and Euler equation , the right hand side$(\rho-r)u'\left(c_j(a)\right)$ is expected change of marginal utility of consumption $\frac{\mathbb{E}_t[du'(c_j(a_t))]}{dt}$.This uses Ito’s formula to Poisson process:
equation 39

$$\mathbb{E}_t[du'(c_j(a_t)]=\left[u''(c_j(a_t)c_j'(a_t)s_j(a_t)+\lambda_j\left(u'(c_{-j}(a_t)\right)-u'\left(c_j(a_t)\right)\right]dt$$

So, this equation $(\rho-r)u'\left(c_j(a)\right)=u''\left(c_j(a)\right)c_j'(a)s_j(a)+\lambda_j(u'\left(c_{-j}(a)\right)-u'\left(c_j(a)\right)$, $s_J(a)=y_J+ra-c_j(a)$ can be written in more standard form:
equation 40

$$\frac{\mathbb{E}_t[du'(c_j(a_t)]}{dt}=(\rho-r)dt$$

Generalized Euler equations when W is defined recursively $W_{t+1}=R(W_t-c_t)$ previously we should define that $\sum_{t=1}^{\infty}R^{-t+1}c_t\le W_1$ and gross interest rate $R=r+1$; are given in the following form:
equation 41

$$u'(c_t)=R\left[\beta\delta\left(\frac{\partial c_{t+1}(W_{t+1})}{\partial W_{t+1}}\right)+\delta\left(1-\frac{\partial c_{t+1}(W_{t+1})}{\partial W_{t+1}}\right)\right]u'(c_{t+1})$$

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

Where $\left[\beta\delta\left(\frac{\partial c_{t+1}(W_{t+1})}{\partial W_{t+1}}\right)+\delta\left(1-\frac{\partial c_{t+1}(W_{t+1})}{\partial W_{t+1}}\right)\right]$ is the effective discount factor, also $c_{t+1}(W_{t+1})$ represents the optimal consumption choice. With uncertainty Euler equation will become:

equation 42

$$u'(c_t)=\beta R\hat{E}\,[u'(c_{t+1})|I_t]$$

Where $\hat{E}\,[u'(c_{t+1})|I_t]$ represents the agents, expectation given the information set I_t.Now, taking 2nd order approx.to marginal utility in $t+1$ around c_t gives:

equation 43

$$\hat{E}\left[\frac{c_{t+1}-c_t}{c_t}|I_t\right]=\sigma_t(1-(\beta R)^{-1})+\frac{1}{2}\phi_t\hat{E}[(c_{t+1}-c_t)^2|I_t]$$

Where $\phi_t=-\frac{c_t u'''(c_t)}{u''(c_t)}$ is a coefficient of relative prudence (see Dynan (1991), expected consumption growth that rises with the real interest rate and falls with impatience.In continuous time previous would be:

equation 44

$$\frac{\dot{c}_t}{c_t}=\sigma_t(r-\rho)$$

Where $\sigma_t=-\frac{u'(c_t)}{c_t u''(c_t)}$; and $c_{t+\Delta t}=c_t+\Delta c_t, \beta=1-\rho\Delta t;\ \Delta t\to 0$ .Now, let's consider that $\varepsilon_{t+1}=u'(c_{t+1})-(\beta R)^{-1}u'(c_t)$ as in Hall (1978) . It was pointed by Hall (1978) that this equation $u'(c_t)=\beta R\hat{E}\,[u'(c_{t+1})|I_t]$ implies that $\hat{E}[\varepsilon_{t+1}z_t|I_t]=z_t\hat{E}[\varepsilon_{t+1}|I_t]$ for any $z_t\in I_t$.

Entrepreneurship and financial frictions (code example by Benjamin Moll)

This model is due to: Achdou et.al (2014); Buera and Shin (2013) and Cagetti and De Nardi (2006). A version with aggregate shocks and business cycle implications will be presented:

1. $\mathbb{E}_0\int_0^\infty e^{-\rho t}u(c_t)dt$ (Individual preferences)
2. $wz^\theta;\theta\geq 0$ (labor income of workers from z earnings)
3. $y_u=F_u(z,k,\ell)=zB_u k^\alpha \ell^\beta$ (unproductive technology)
4. $y_p=F_p(z,k,\ell)=zB_p((k-f_k)^+)^\alpha((\ell-f_\ell)^+)^\beta$ (productive technology)
5. $B_p>B_u$ per-period overhead costs f_k-F_ℓ (productivity of unproductive and productive entrepreneurs)
6. $A_t\ \equiv\ qk^{t-1}\ +\ B_t$ where B_t is the borrowers liquid savings or debt if $B_t<0$; qk^{t-1} is the value of housing services
7. $\forall x, x^+=\max\{x,0\}$
8. F_p non concave in k,ℓ
9. z:idisosyncratic shock
10. Collateral constraints are: $k\leq\lambda a\,;\lambda\geq 1$
11. $\left(1-\tau_{y_i}\right)e_i\left(k_i^a\ell_i^{1-\alpha}\right)^{1-v}=wl_i-(\delta+r)k_i\,;k_i\leq\lambda a_i$
12. financial frictions apply equally to everyone in the economy—λ has no individual subscript, τ_{y_i} has and it is individually specific.
13. τ_{y_i} –taxes/subsidies/wedges on output
14. $\tau_+(\geq 0);\tau_-(\leq 0)\forall e\,;\Pr\{\tau_y=\tau_+|e\}=1-e^{-qe}$
15. $\lambda\in\{1,\infty\}-1$ financial autarky; ∞ - perfect credit, maximum leverage ratio
16. Income maximization: $M(a,z,A;w,r)=\max\{wz^\theta,\Pi_u(a,z,A;w,r),\Pi_p(a,z,A;w,r)\}\,;\Pi_j(a,z,A;w,r)=\max_{k<-\lambda a}F_j(z,A,k,l)-(r+\delta)k-wl;j=p,u$
17. Individuals solve:\

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

equation 45

$\max_{\{c_t\}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_t)dt \ s.t \ da_t = [M(a_t, z_t, A_t; w_t, r_t) + r_t a_t - c_t]dt \ ; dzt = \mu z(t)dt + \sigma(z_t)dW_t; \ a_t \geq 0$

18. Optimal capital and labor choices corresponding to the productive technology are given as:

equation 46

$$k_p(a; z; w; r) = \min\left\{\lambda a, (zAB_p)^{\frac{1}{1-\alpha-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\alpha-\beta}} + f_k\right\};$$

$$\ell_p(a, z, w, r) = \left(\frac{\beta z \, AB_p}{w}\right)^{\frac{1}{1-\beta}} k_p(a, z, w, r)^{\frac{\alpha}{1-\beta}} + f_\ell$$

19. Equilibrium conditions are:

equation 47

$$\rho v(a, z, t) = \max_c u(c) + \partial_a v(a, z, t)\big[M\big(a, z; w(t)\big) + r(t)a - c\big] + \partial_z v(a, z, t)\mu(z) + \frac{1}{2}\partial_{z.z} v(a, z, t)\sigma^2(z) + \partial_t vv(a, z, t)$$

equation 48

$$\partial_t g(a, z, t) = -\partial_a[s(a, z, t)g(a, z, t)] - \partial_z\left[\mu(z)g(a, z, t) + \frac{1}{2}\partial_{zz}[\sigma^2(z)g(a, z, t)]\right]$$

Where g is PDF of the statistical distribution.

In previous $\partial_a v = \frac{\partial v}{\partial a}$ and the optimal savings function is given as:

equation 49

$$s(a, z, t) = M(a, z; w(t), r(t) + r(t)a - c(a, z, t)$$

In previous current assets $a =\neq (1 + r)a + p - c$; and p are social security payments,c is consumption and utility from consumption is givenas $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ as in Cagetti and De Nardi (2006) where public firms maximize:

equation 50

$$r(t) = \partial_K F_c\big(A, K_c(t), L_c(t)\big) - \delta$$
$$w(t) = \partial_L F_c(AK_c(t), L_c(t)$$

Capital and labor market clear at:

equation 51

$$K_c(t) = \int ag(a, z, t)dadz$$

$$L_c(t) = \int z^\theta \, \mathbf{1}_{\{wz^\theta > \max\{\Pi_u, \Pi_p\}\}} g(a, z, t)dadz$$

Where $\mathbf{1}_{\{wz^\theta > \max\{\Pi_u, \Pi_p\}\}}$ is an indicator function[12] and:

equation 52

$$\mathbf{1}: wz^\theta > \max\{\Pi_u, \Pi_p\}$$

Defined as:

equation 53

$$\mathbf{1}: \begin{cases} 1 \ \because wz^\theta > \max\{\Pi_u, \Pi_p\} \\ 0 \therefore z^\theta \leq \max\{\Pi_u, \Pi_p\} \end{cases}$$

Where θ represents entrepreneurial ability. Now about numerical computation given $\xi_l = K_c/L_c$ and then find (compute) $L_{c,l} = \frac{K_{c,l}}{\xi_l}$ so that "excess demand" is given as:

[12] Indicator function is a function that returns 1 if an element is present in a specified subset and 0 if absent; naturally isomorphic with a set's subsets.

Manuscript received: 29.03.2023
Accepted: 20.04.2023

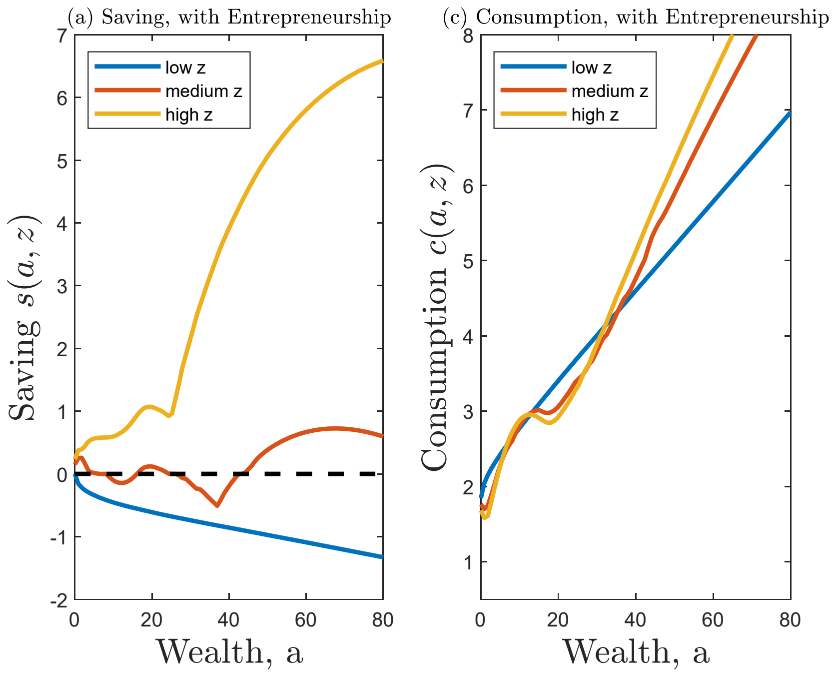
International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

equation 54

$$D_l = L_c(t) + \int l_u\big(a, z, w(t), r(t)\big)\, \mathbf{1}_{\{\Pi_u > \max\{\Pi_p, wz^\theta\}\}} g(a, z, t) dadz + \int l_p\big(a, z, w(t); r(t)\big)\, \mathbf{1}_{\{\Pi_u > \max\{\Pi_p, wz^\theta\}\}} g(a, z, t) dadz - \int z^\theta \mathbf{1}_{\{wz^\theta > \max\{\Pi_u; \Pi_p\}\}} g(a,z,t) dadz$$

Next Savings,consumption function as well as wealth function with entrepreneurship are presented.

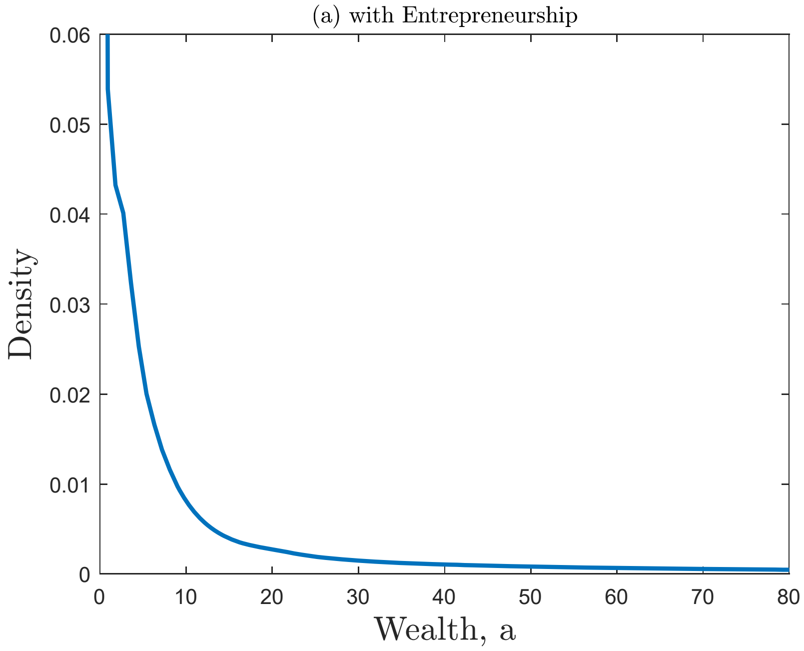
Figure 3 savings and consumption functions with entrepreneurship



Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

Figure 4 Wealth with entrepreneurship



Source code: Benjamin Moll codes https://benjaminmoll.com/codes/

Endogenous entrepreneurship and financial frictions

Code used in this part was written by Frederic Martenet [13].In this model there are continuum of individuals that differ in wealth a and entrepreneurial ability z. Entrepreneurial ability z Is drawn from Pareto distribution:

equation 55

$$\mu_{pdf}(z) = \eta z^{-\eta-1}, z \geq 1$$

Entrepreneurial ability is persistent in each period ability is drawn with probability γ .Banks collect deposits and rent out capital to entrepreneurs with rate $R = r + \delta$ where r is deposit rate. There is also representative public firm. Government taxes all entrepreneurial profits and revenues τ^{π} and τ^{y}, and rebates receipts with lump-sum payments $\mathcal{T}_t$.CRRA preferences are given as:

equation 56

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

Inverse intertemporal elasticity of substitution is σ .The budget constraint is given as:

equation 57

$$c_t + a_{t+1} \leq \max\{w_t, \pi(z_t, a_t)\} + (1 + r_t)a_t + \mathcal{T}_t$$

Profits from operation technology are given as:

equation 58

$$\pi(z_t, a_t) = \max_{l_t, k_t}\{(1 - \tau_t^{\pi})\big[(1 - \tau_t^{y})z_t Z_t (k_t^{a} l_t^{1-a})^{1-v} - w_t l_t - (\delta + r_t)k_t\big]\}$$

$$\text{s.t. } k_t \leq \lambda a_t$$

individuals with productivity z_t choose to become entrepreneurs if their wealth exceeds the threshold value $\bar{a}(z_t)$ which solves :

equation 59

$$w_t = \pi(z_t, \bar{a}(z_t))$$

[13] See: https://github.com/FredericMartenet/entrepreneurs

Manuscript received: 29.03.2023
Accepted: 20.04.2023

The technology operated by individual entrepreneurs has decreasing returns to scale :
equation 60

$$y_t = f(z,k,l) = zZ_t(k^a l^{1-a})^{1-v}$$

$Z_t = \sum z_t$ or they are aggregate TFP shocks. On the other hand, public firm operates with constant returns to scale of CRS technology:
equation 61

$$F(K_{ct}, L_{ct}) = Z_t Z_{ct} K_{ct}^a L_{ct}^{1-a}$$

FOCs are given as:
equation 62

$$r_t = aZ_t Z_{ct} K_{ct}^a L_{ct}^{-a+1}$$
$$w_t = (1-a)Z_t Z_{ct}\left(\frac{K_{ct}}{L_{ct}}\right)^a$$

Capital-labor ratio is given by:
equation 63

$$\frac{K_{ct}}{L_{ct}} = \left(\frac{aZ_t Z_{ct}}{r_t+\delta}\right)^{\frac{1}{-a+1}}$$

Labor market clearing condition is given as:
equation 64

$$L_{ct} + \int_R l_t(z,a)dD_t(a,z) - \int_W dD_t(a,z) = 0$$

Asset market (capital market) clears:
equation 65

$$K_{ct} + \int_E k_t(z,a)sD_t(a,z) - \int aD_t(a,z) = 0$$

Government budget balance is given as:
equation 66

$$\mathcal{T}_t = \int_E \left[(\tau_t^\pi + \tau_t - \tau_t^\pi \tau_t)y_t(z,k,l) - \tau_t^\pi\big(w_t l_t(z,a) + (\delta + r_t)k_t(z,a)\big)\right]dD_t(a,z)$$

$D_t(a,z)$ is some distribution of assets and skills and initial distribution is given as $D_0(a,z)$.
Now when solving for entrepreneurial profits:

$$\pi(z_t,a_t) = \max_{l_t,k_t}\left\{(1-\tau_t^\pi)^{1-v}\left[\left(1-\tau_t^y\right)z_t Z_t(k_t^a l_t^{1-a})^{1-v} - w_t l_t - (\delta + r_t)k_t\right]\right\}$$
$$\text{s.t. } k_t \le \lambda a_t$$

if collateral constraint is binding following applies : $k^c(a_t) = \lambda a_t$ or if the collateral constraint is not binding then we have:
equation 67

$$k^u(z_t) = \left[\left(1-\tau_t^y\right)z_t Z_t\right]^{\frac{1}{v}}\left(\frac{a(1-v)}{r_t+\delta}\right)^{\frac{1-(1-a)(1-v)}{v}}\left(\frac{(1-a)(1-v)}{w_t}\right)^{\frac{(1-a)(1-v)}{v}}$$

Capital policy function is given as:
equation 68

$$k(z_t,a_t) = \min\{k^c(a_t), k^u(z_t)\}$$

Labor policy function is given as:
equation 69

$$l(z_t,a_t) = \left(\frac{(1-a)(1-v)\left(1-\tau_t^y\right)z_t Z_t}{w_t}\right)^{\frac{1}{1-(1-a)(1-v)}} k(z_t,a_t)^{\frac{a(1-v)}{1-(1-a)(1-v)}}$$

Indirect profit function is given by :
equation 70

$$\pi(z_t,a_t) = (1-\tau_t^\pi)\left[\left(1-\tau_t^y\right)z_t Z_t(k(z_t,a_t)^a l(z_t,a_t)^{1-a})^{1-v} - w_t l(z_t,a_t) - (\delta + r)k(z_t,a_t)\right]$$

Manuscript received: 29.03.2023
Accepted: 20.04.2023
International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

τ_t^π does affect policies and τ_t^y does not. Bellman equation here can be written as:
equation 71

$$v_t(a_t, z_t) = \max_{c_t, a_{t+1}} \left\{ u(c_t) + \beta \left[\gamma v_{t+1}(a_{t+1}, z_t) + (1-\gamma) \int v_{t+1}(a_{t+1}. z_{t+1}) \mu(z_{t+1}) dz_{t+1} \right] \right\}$$

s.t. $c_t + a_{t+1} = M(a_{t+1}, z_t) + (1 + r_t)a_t + \mathcal{T}_t$
where $M(a_{t+1}, z_t) = \max\ \{w_t, \pi(z_t, a_t)\}$ and the corresponding Euler equation is given as:
equation 72

$$u'(c_t(z_t, a_t)) = \beta \left[\gamma \left(1 + r_{t+1}^{eff}(z_t, a_{t+1}, d_{t+1})\right) u'(c_{t+1}(z_t, a_{t+1}) + (1 - \gamma) \int \left(1 + r_{t+1}^{eff}(z_{t+1}, a_{t+1}, d_{t+1})\right) u'\left(c_{t+1}(z_{t+1}, a_{t+1}, d_{t+1})\right) u'(c_{t+1}(z_{t+1}, a_{t+1})) \mu(z_{t+1} dz_{t+1} \right]$$

Effective rate of return r_t^{eff} is defined as:
equation 73

$$r_t^{eff}(z_t, a_t) = \begin{cases} r_t \ if\ worker \\ r_t + \dfrac{\partial \pi(z_t, a_t)}{\partial a_t}\ if\ entrepreneur \end{cases}$$

Where :

$$\frac{\partial \pi(z_t, a_t)}{\partial a_t} = \begin{cases} \dfrac{\partial \pi^{const}(z_t, a_t)}{\partial a_t}\ if\ worker \\ 0\ \ if\ unconstrained\ entrepreneur \end{cases}$$

Cash on hand for the next period on the grids a, z
equation 74

$$coh(z_{t+1}, a_{t+1}) = M(z_{t+1}, a_{t+1}) + (1 + r_t)a_{t+1} + \mathcal{T}_t$$

Right hand side of Euler equation $u'(c_t(z_t, a_t)) = \beta \left[\gamma \left(1 + r_{t+1}^{eff}(z_t, a_{t+1}, d_{t+1})\right) u'(c_{t+1}(z_t, a_{t+1}) + (1-\gamma) \int \left(1 + r_{t+1}^{eff}(z_{t+1}, a_{t+1}, d_{t+1})\right) u'\left(c_{t+1}(z_{t+1}, a_{t+1}, d_{t+1})\right) u'(c_{t+1}(z_{t+1}, a_{t+1})) \mu(z_{t+1} dz_{t+1}\right]$ is given as:
equation 75

$$RHS(a_{t+1} z_t) = \beta \left[\gamma \left\{ 1 + r_{t+1}^{eff}(z_t, a_{t+1}) \right) u'\left(c_{t+1}(z_t, a_{t+1})\right) + (1-\gamma) \int \{1 + r_{t+1}^{eff}(z_{t+1}, a_{t+1}))\} \mu(z_{t+1}) d\_z_{t+1} \right]$$

This inverted will give us consumption function:
equation 76

$$c(a_{t+1}, z_t) = \frac{1}{u\left(RHS(a_{t+1} z_t)\right)}$$

Asset policy function is given as:
equation 77

$$a_t = \frac{c_t(a_{t+1}, z_t) + a_{t+1} - M(a_{t+1}, z_t) - \mathcal{T}_t}{1 + r_t}$$

Results are graphically depicted in the following two figures.
Figure 5 Occupational choice, wealth distribution

Manuscript received: 29.03.2023
Accepted: 20.04.2023

Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

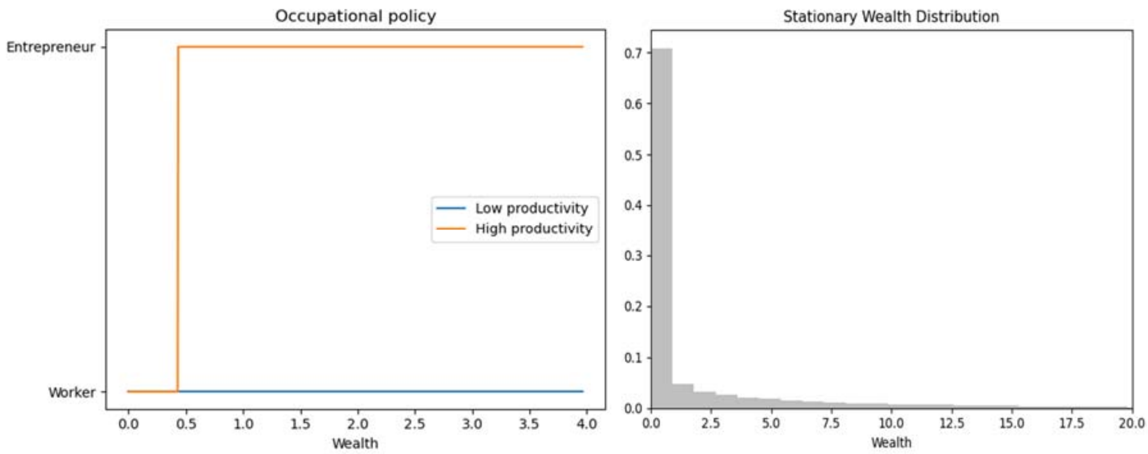
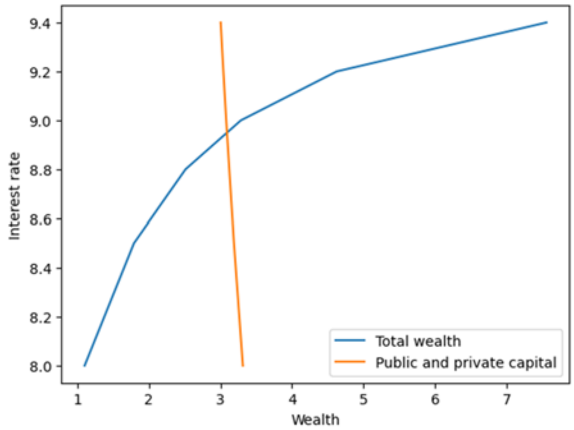


Figure 6 Equilibrium interest rate, total wealth ,and public ,private capital



Financial frictions and unemployment with entrepreneurs (snippet by Cristiano et al.2011)
This model starts with the following production function:
equation 78

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda^d}} di \right]^{\lambda_d} ;1 \leq \lambda_d < \infty$$

Where Y_t is homogenous domestic good, $\frac{1}{\lambda^d}$ is a degree of substitutability. Intermediate good $Y_{i,t}$ is :
equation 79

$$Y_{i,t} = \left(z_t, H_{i,t}\right)^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^{+} \phi$$

Where $K_{i,t}^{\alpha}$ is capital rented and its services by intermediate goods producer, $\log\left(z_{i,t}\right)$ is a technology shock whose first-difference has positive mean, and ϕ denotes production costs. And Ψ_t denotes investment specific technology shock (IST),and z_t^{+} is :
equation 80

$$z_t^{+} = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t$$

$H_{i,t}$ are homogeneous labor services, firms borrow fraction of wage bill, one unit of labor costs is denoted as $W_t R_t^f$ where:
equation 81

$$R_t^f = v^f R_t + 1 - v^f$$

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

where W_t is the aggregate wage rate, R_t is the risk-free interest rate that applies to working capital loans and v^f corresponds to the fraction that must be financed in advance.Firms marginal costs are:

$$mc_t = \tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k\right)^{\alpha} \left(\bar{w}_t R_t^f\right)^{1-\alpha} \left(\frac{1}{\epsilon_t}\right)$$

where r_t^k is the nominal rental rate of capital scaled by P_t and $\bar{w}_t = \frac{w_t}{z_t^+ P_t}$. Also, $\tau_t^d$is a tax-like shock, which affects marginal cost, but does not appear in a production function. In the linearization of a version of the model in which there are no price and wage distortions in the steady state, $\tau_t^d$is isomorphic to a disturbance in λ_d, i.e., a markup shock.Productive efficiency states that mc_t is equal to the cost of producing another unit using labor, which in turn implies :

equation 82

$$mc_t = \tau_t^d \frac{\left(\mu_{\psi,t}\right)^{\alpha} \bar{w}_t R_t^f}{\epsilon_t (1-\alpha)\left(\frac{\frac{k_{i,t}}{\mu_z^+}}{H_{i,t}}\right)^{\alpha}}$$

Price setting is subject to Calvo frictions. With probability ε_d the intermediate good firm cannot reoptimize its price, in which case following applies:

equation 83

$$P_{i,t} = \hat{\pi}_{d,t} P_{i,t-1} \, ; \hat{\pi}_{d,t} \equiv (\pi_{t-1})^{k_d} (\bar{\pi}_t^c)^{1-k_d-\chi_d} (\hat{\pi})^{\chi_d}$$

Where $k_d, \chi_d, k_d + \chi_d \in (0,1)$ are parameters π_{t-1} is lagged inflation rate and $\bar{\pi}_t^c$ is the CB target inflation rate. With probability $1 - \xi_d$ firm can change its price:

equation 84

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} \{P_{i,t+j}, Y_{i,t+j} P_{t+j} Y_{(i,t+j)}\}$$

Previous expression are discounted profits. Demand is given by:

equation 85

$$\left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_d}{\lambda_d - 1}} Y_t = Y_{i,t}$$

Domestic intermediate output good is allocated as follows:

equation 86

$$Y_t = G_t + C_t^d + I_t^d + \int_0^1 X_{i,t}^d$$

Where G_t denotes government consumption, C_t^d denotes intermediate goods used to produce final consumption goods, also I_t^ds the amount of intermediate domestic goods used in combination with imported foreign investment goods to produce a homogeneous investment good. Final consumption goods are :

equation 87

$$C_t = [(1-\omega_c)]^{\frac{1}{\eta_c}} \left(C_t^d\right)^{\frac{\eta_c - 1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} (C_t^m)^{(\eta_c - 1)/\eta_c}]^{\frac{\eta_c}{\eta_{c_1}}}$$

C_t^d is the first input for final goods consumption production[14], and has a price P_c. The input prices for representative firm are:P_t, P_t^{mc} :

equation 88

$$c_t^d = (1-\omega_c)(P_t^c)^{\eta_c} c_t$$

[14] one-for-one transformation of the homogeneous domestic good

Manuscript received: 29.03.2023
Accepted: 20.04.2023

$$c_t^m = \omega_c \left(\frac{p_t^c}{p_t^{mc}}\right) c_t$$

Where $P_t^c = p_t^c / p_t$ and $p_t^{mc} = p_t^{mc}/p_t$ and

equation 89

$$p_c^t = [(1-\omega_c) + \omega_c (p_t^{mc})^{1-\eta_c}]^{\frac{1}{1-\eta_c}}$$

The rate of inflation of consumption goods is :

equation 90

$$\pi_t^c = \frac{p_t^c}{p_{t-1}^c} = \left[\frac{(1-\omega_c) + \omega_c (p_t^{mc})^{1-\eta_c}}{(1-\omega_c) + \omega_c (p_{t-1}^{mc})^{1-\eta_c}}\right]^{\frac{1}{1-\eta_c}}$$

Investment goods are produced as:

equation 91

$$I_t + a(u_t)\bar{K}_t = \Psi_t \left[(1-\omega_i)^{\frac{1}{\eta_i}} \left(I_t^d\right)^{\frac{\eta_t - 1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i - 1}}$$

The sum of investment goods is I_t , plus investment goods in capital maintenance $a(u_t)\bar{K}_t$,where:

equation 92

$$K_t = u_t \bar{K}_t$$

And u_t denotes utilization rate of capital. Profit maximization leads to:

equation 93

$$i_t^d = \left(p_t^i\right)^{\eta_i} \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}, \mu_{z,t}^{+}}\right)$$

$$i_t^m = \omega_i \left(\frac{p_t^i}{p_t^{m,i}}\right)^{\eta_i} \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}, \mu_{z,t}^{+}}\right)$$

Where $p_t^i = \frac{\Psi_t p_i^t}{p_t}$ and $p_t^{m,i} = \frac{p_t^{m,i}}{p_t}$,the price of I_t is given as:

equation 94

$$p_t^i = \left[(1-\omega_i) + \omega_i \left(p_t^{mi}\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}$$

The rate of inflation of investment good is :

equation 95

$$\pi_t^i = \frac{p_t^i}{p_{t-1}^i} = \left[\frac{(1-\omega_i) + \omega_i \left(p_t^{mi}\right)^{1-\eta_i}}{(1-\omega_i) + \omega_i \left(p_{t-1}^{mi}\right)^{1-\eta_i}}\right]^{\frac{1}{1-\eta_i}}$$

Homogenous labor service is given as:

equation 96

$$H_t = \left[\int_0^1 \left(h_{j,t}\right)^{\frac{1}{\lambda_w}} dj\right]^{\lambda_w} ; 1 \leq \lambda_w, \infty$$

With probability $1 - \xi_w$ household jth can reoptimize its wage according to:

equation 97

$$w_{j,t+1} = \bar{\pi}_{w,t+1} W_{j,t}$$

$$\bar{\pi}_{w,t+1} = (\pi_t^c)^{k_w} (\bar{\pi}_{t+1}^c)^{1-k_w-\chi_w} (\hat{\pi})^{\chi_w} (\mu_{z^+})^{\varrho_w}$$

Where $k_w + \chi_w, k_w, \chi_w, \varrho_w \in (0,1)$.Now households reoptimize:

Manuscript received: 29.03.2023
Accepted: 20.04.2023

equation 98

$$E_t^i \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-\xi_{t+1}^h A_L \frac{(h_{j,t+1}}{1+\sigma_L} + v_{t+i} W_{j,t+1} h_{j,t+1} \frac{1-\tau^y}{1+\tau^w} \right]$$

Where τ^y and τ^w are taxes on labor income and payroll tax respectively. And:

equation 99

$$h_{j,t+1} = \left(\frac{\overline{W}_t \hat{\pi}_{w,t+1}, \dots . \hat{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_m}{1-\lambda_m}} H_{t+i}$$

Next in the model financial frictions are introduced following assumption that working capital loans are frictionless. Our strategy of introducing frictions in the accumulation and management of capital follows the variant of the BGG model implemented in Christiano et al. (2003). The discussion and derivation here borrows heavily from Christiano et al. (2008).

Entrepreneurs and financial frictions in Christiano et al.2011

Entrepreneurs net-worth is given as:N_{t+1}. Entrepreneur combines net worth with bank loan B_{t+1} :

equation 100

$$B_{t+1} = P_t P_{k',t} \overline{K}_{t+1} - N_{t+1}$$

Entrepreneur must pay gross interest rate Z_{t+1} for the bank loan in period $t+1$. Risk free interest rate is :

equation 101

$$R_{t+1}^k = \frac{(1-\tau^k)\left[u_{t+1} r_{t+1}^k - \frac{p_{t+1}^i}{\Psi_{t+1}} a(u_{t+1}) \right] P_{t+1} + (1-\delta) P_{t+1} + \tau^k \delta P_t P_{k',t}}{P_t P_{k',t}}$$

Where :

equation 102

$$a_t = \frac{S A_{t+1}^*}{P_t z_t^+}$$

A_{t+1}^* are the net-foreign assets. Real costs of assets are S_t/P_t.Cut of value for idiosyncratic productivity of entrepreneurs in order to be able to repay debt is given as:

equation 103

$$\overline{\omega}_{t+1} R_{t+1}^k P_t P_{k',t} \overline{K}_{t+1} = Z_{t+1} B_{t+1}$$

Entrepreneurs with $\omega < \overline{\omega}_{t+1}$ are bankrupt and turn over resources:

equation 104

$$R_{t+1}^k \omega P_t P_{k',t} \overline{K}_{t+1} < Z_{t+1} B_{t+1}$$

The bank monitors entrepreneur at cost:

equation 105

$$\mu R_{t+1}^k \omega P_t P_{k',t} \overline{K}_{t+1}$$

Where $\mu \geq 0$ is a parameter.For loans amounted B_{t+1} the bank receives gross interest rate $Z_{t+1} B_{t+1}$ from fraction $1 - H(\overline{\omega}_{t+1}; \sigma_t)$ of entrepreneurs who are not bankrupt, CDF is given as $(\omega, \sigma)$where σ is shock of idiosyncratic uncertainty. Zero profit condition is given as:

equation 106

$$[1 - F(\overline{\omega}_{t+1}; \sigma_t)] Z_{t+1} B_{t+1} + (1-\mu) \int_0^{\overline{\omega}_{t+1}} \omega dF(\omega; \sigma_t) R_{t+1}^k P_t \, P_{k',t} \overline{K}_{t+1} = R_t B_{t+1}$$

And after rearranging with using of $\overline{\omega}_{t+1} R_{t+1}^k P_t P_{k',t} \overline{K}_{t+1} = Z_{t+1} B_{t+1}$ we get:

equation 107

$$[\Gamma(\overline{\omega}_{t+1}; \sigma_t) - \mu_t G((\overline{\omega}_{t+1}; \sigma_t))] \frac{R_{t+1}^k}{R_t} \varrho_t = \varrho_t - 1$$

Manuscript received: 29.03.2023
Accepted: 20.04.2023

Where

equation 108

$$G((\overline{\omega}_{t+1};\sigma_t) = \int_0^{\overline{\omega}_{t+1}} \omega dF(\omega;\sigma_t)$$

$$\Gamma(\overline{\omega}_{t+1};\sigma_t) = \overline{\omega}_{t+1}[1 - F(\overline{\omega}_{t+1};\sigma_t)] + \int_0^{\overline{\omega}_{t+1}} \omega dF(\omega;\sigma_t)$$

$$\varrho_t = \frac{P_t P_{k',t}\overline{K}_{t+1}}{N_{t+1}}$$

This is leverage ratio $\varrho_t = \frac{P_t P_{k',t}\overline{K}_{t+1}}{N_{t+1}}$ and it implies that :

equation 109

$$\frac{B_{t+1}}{N_{t+1}} = \varrho_t - 1$$

And rate of interest aid by the entrepreneur is:

equation 110

$$z_{t+1} = \frac{\overline{\omega}_{t+1} R_{t+1}^k}{1 - \frac{N_{t+1}}{P_t P_{k',t}\overline{K}_{t+1}}} = \frac{\overline{\omega}_{t+1} R_{t+1}^k}{1 - \frac{1}{\varrho_t}}$$

The motion law foe every entrepreneur is given as:

equation 111

$$V_t = R_t^k P_t P_{k',t} K_t - \Gamma(\overline{\omega}_t;\sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} K_t$$

Each entrepreneur face probability to exit economy γ which is identical for all entrepreneurs. Fraction of entrepreneurs who survive bankruptcy is $\gamma_t\ \overline{V}_t$ and a fraction of $1-\gamma$ entrepreneurs arrive. Entrepreneurs who survive or are new receive transfer $W_t^e = Z_t^+ W^e$. The average net worth across all entrepreneurs is :

equation 112

$$\overline{N}_{t+1} = \gamma_t \overline{V}_t + W_t^e$$

Or alternatively:

equation 113

$$\overline{N}_{t+1} = \gamma_t\{R_t^k P_t P_{k',t-1}\overline{K}_t\} - \left[R_{t-1} + \frac{\mu \int_0^{\overline{\omega}_{t+1}} \omega dF(\omega;\sigma_{t-1}) R_t^k P_{t-1}\, P_{k',t}\overline{K}_t}{P_{t-1}P_{k',t-1}\overline{K}_t - \overline{N}_t}\right]$$

Because of its direct effect on entrepreneurial net worth, we refer to λ_t as the shock to net worth.

Labor market fictions in this model Christiano et al.2011

Economists had been using search models for more than 50 years to describe labor market more closely. And the seminal work of Diamond(1982); Pissarides (1985); and Mortensen and Pissarides (1994), had become a framework for macroeconomists to study unemployment. Matching function is given as:$mL = m(uL, vL)$,it is concave and homogenous of degree 1. Homogeneity or constant returns to scale. Where u is unemployment rate, v -vacancy rate, uL unemployed worker L-total labor force, and vL job vacancies. Vacancy to filled jobs equals $\frac{v}{u}$ is denoted to θ [15] and equals to: $\theta = m\left(\frac{u}{v}, 1\right)$.Also, δt is a small time interval during some vacant job is matched to an unemployed person,with a probability $q(\theta)\delta t$. Toa related Poisson

[15] $\theta = \frac{v}{u}$ is a market tightness, and for the firms probability of filling a vacancy is given as:$\frac{m(u,v)}{v} = m\left(\frac{1}{\theta}, 1\right) \equiv q(\theta)$, and $q'(\theta) < 0$; and for the workers probability of finding a job is: $\frac{m(u,v)}{v} = m(1,\theta) \equiv \theta q(\theta)$.There flowing applies : $\lim_{\theta\to 0}[\theta q(\theta)] = \lim_{\theta\to\infty} q(\theta) = 0$ and $\lim_{\theta\to\infty}[\theta q(\theta)] = \lim_{\theta\to\infty} q(\theta) = +\infty$

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

proces $\lambda = \frac{m(uL,vL)}{uL}$ where $\lambda = \theta q(\theta)$ and has elasticity $1-\eta(\theta) \geq 0$ .The mean duration of unemployment is $1/\theta q(\theta)$.The evolution of unemployment is given as:

equation 114

$$\dot{u} = \lambda(1-u) - \theta q(\theta)u$$

V is the present-discounted value of expected profit from a vacant job and satisfies Bellman equation:

equation 115

$$rV = -pc + q(\theta)(J - V).$$

The permanent incomes of unemployed and employed workers, in terms of the returns z and w and the discount and transition rates:

equation 116

$rU = \frac{(r+\lambda)z+\theta q(\theta)w}{r+\lambda+\theta q(\theta)}$; $rW = \frac{\lambda z+[r+\theta q(\theta)]w}{r+\lambda+\theta q(\theta)}$

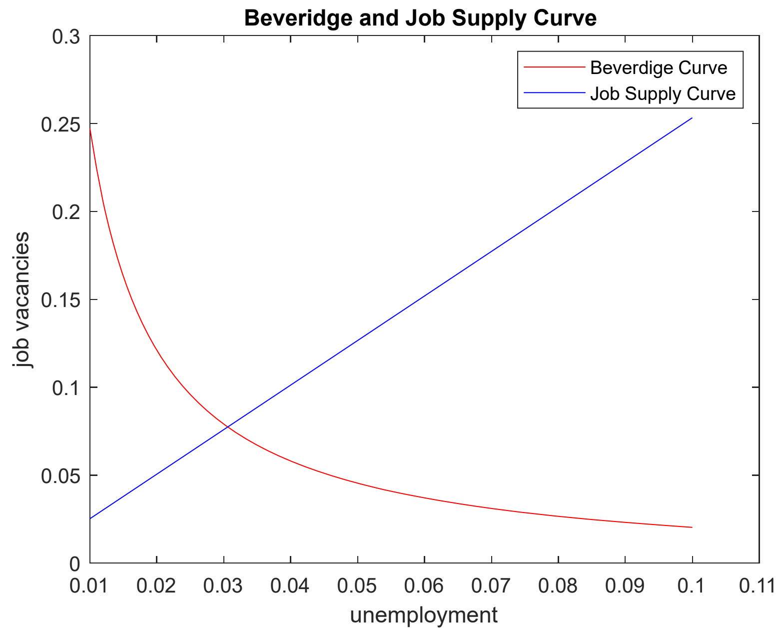
The job is worth t the worker : $rW_i = w_i - \lambda(W_i - U)$the job rate for this job satsfies :

equation 117

$$w_i = argmax(W_i - U)^{\beta}(J_i - V)^{1-\beta}$$

β is labor's share of the total surplus that an occupied job creates,$0 \leq \beta \leq 1$, $\beta = \frac{1}{2}$ is the most plausible value.Now, rU -reservation wage, $\beta(p-r)$ fraction of net surplus they create by accepting the job, product value net of what they give up[16], $rU \Rightarrow rU = z + \frac{\beta}{1-\beta}pc\theta$. Aggregate wage equation that holds in equilibrium, is given as: $w = (1-\beta)z + \beta p(1+c\theta)$.

Figure 7 DMP model without benefit shock



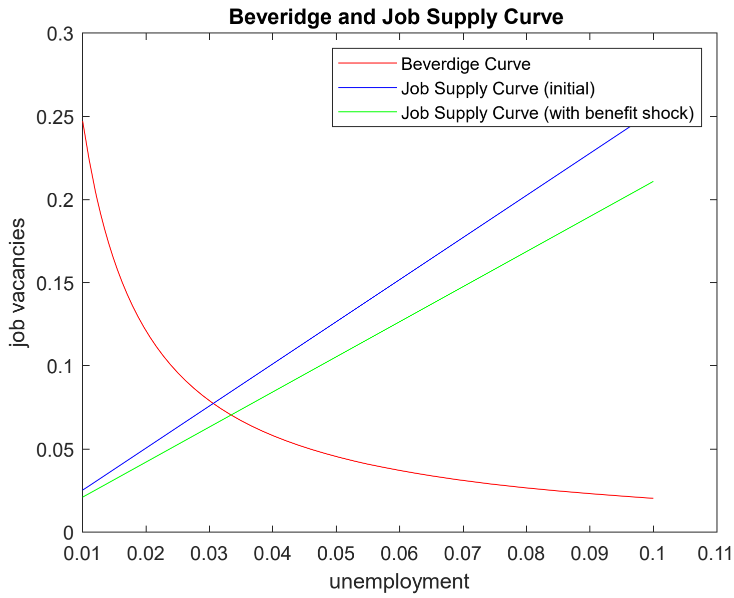
Source: author calculation based on the code published on: https://github.com/pdevlieger/MatLab-files

[16] It is intuitive for a market equilibrium if we note that $pc\theta$ is the average hiring cost for each unemployed worker (since $pc\theta = pcv/u$ and pcv is total hiring cost in the economy).

Manuscript received: 29.03.2023
Accepted: 20.04.2023

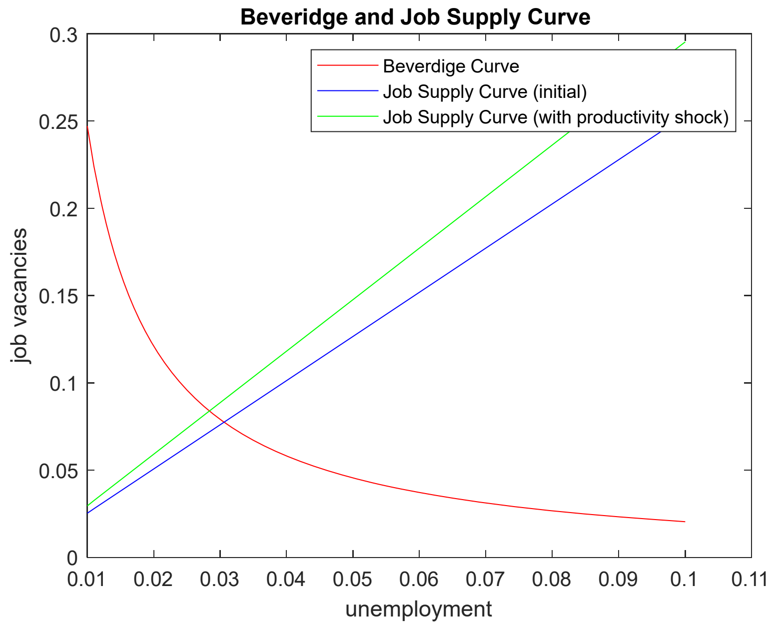
International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

Figure 8 DMP model with benefit shock



Source: author calculation based on the code published on:
https://github.com/pdevlieger/MatLab-files

Figure 9 DMP model with productivity shock



Source: author calculation based on the code published on:
https://github.com/pdevlieger/MatLab-files

That was the textbook model DMP, but in this model hours worked are presented as:

equation 118

$$E_t \sum_{t-0}^{\infty} \beta^{l-t} \left\{ \xi_{t+1}^{c} \log(C_{t+1} - bC_{t+l-1}) - \xi_{t+l}^{h} \left[\sum_{i=0}^{N-1} \frac{(\varsigma_{i,t+l})^{1+\sigma_L}}{1+\sigma_L} \left[1 - \mathcal{F}(\bar{a}_{t+1}^{i}; \sigma_{a,t+l} \right] \right] I_{t+1}^{i} \right\}$$

Where $i \in \{0, N-1\}$ represents cohort where agency belongs. And I_{t+1}^{i} represents number of workers in the cohort$i$,after endogenous separations and new arrivals. And $a_t^i < \bar{a}_t^i$ are laid of from the firm. Also:

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

equation 119

$$\mathcal{F}_t^i = \mathcal{F}(\bar{a}_t^i; \sigma_{a,t}) = \int_0^{\bar{a}_t^i} d\mathcal{F}(a; \sigma_{a,t})$$

The disutility experienced by worker form working hours is given as:
equation 120

$$\xi_t^h A_L = \frac{(\varsigma_{i,t})^{1+\sigma_L}}{1+\sigma_L}$$

each household has sufficiently many workers so that the total fraction of workers employed
equation 121

$$L_t = \sum_{i=0}^{N-1} [1 - \mathcal{F} + t^i] I_t^i$$

The household currency receipts from labor market are given as:
equation 122

$$(1-\tau^y)(1-L_t)P_t b^u z_t^+ + \sum_{i=0}^{N-1} [1-\mathcal{F}_t^i] I_t^i \varsigma_{i,t} \frac{1-\tau^y}{1+\tau^w}$$

Where $b^u z_t^+$ is a pre tax payment to the workers. Firm is posting vacancies as a:
equation 123

$$\tilde{v}_t^i \equiv \frac{Q_t^l v_t^i}{(1-\mathcal{F}_t^i) I_t^i}$$

The agency hiring rate is:$\chi_t^i = Q_t^{1-I} \tilde{v}_t^i$.Where Q_t represents probability of filing vacancy. The value function of the firms is given as:
equation 124

$$F(l_t^0, \omega_t) = \sum_{j=0}^{N-1} \beta^j E_t \frac{V_{t+j}}{v_t}, \max_{\tilde{v}_{t+j};\bar{a}_{t+j}} \left[\int_{\bar{a}_{t+j}^j}^{\infty} (W_{t+j} a - [\Gamma_{t,j} \dot{\omega}_t) \varsigma_{j,t+j} d\mathcal{F}(a) - P_{t+j} \frac{k z_t^+{}_{i+j}}{\varphi} \left(\tilde{v}_{t+j}^j\right)^{\varphi} (1 - \mathcal{F}_{t+j}^j) \right] I_{t+j}^i + \beta^N E_t \frac{v_{t+N}}{v_t} F(I_{t+N}^0, \widetilde{W}_{t+N})$$

Where :
equation 125

$$\Gamma_{t,j} = \begin{cases} \hat{\pi}_{w,t+j}, \dots, \hat{\pi}_{w,t+1}, & j > 0 \\ 1 & , j = 0 \end{cases}$$

$\Gamma_{t,j}\dot{\omega}_t$ represents the wage rate in period $t+j$ .Value function from being worker in an agency in period t is V_t^i and:
equation 126

$$V_t^i = \Gamma_{t-1,i} \widetilde{W}_{t-1} \varsigma_{i,t} \frac{1-\tau^y}{1+\tau^w} - A_L \frac{\xi_t^h \varsigma_{i,t}^{1+\sigma_L}}{(1+\sigma_L) v_t} + \beta E_t \frac{v_{t+1}}{v_t} \left[\rho \left(1 - \mathcal{F}_{t+1}^{(i+1)}\right) V_{t+1}^{i+1} + 1 - \rho + \rho \mathcal{F}_{t+1}^{(i+1)}) U_{t+1} \right]$$

$\Gamma_{t-1,i} \widetilde{W}_{t-1}$ represents the wage received by cohort i at time t. The currency value fo being unemployed is given as:
equation 127

$$U_t = P_t z_t^+ + b^u (1-\tau^y) + \beta E_t \frac{v_{t+1}}{v_t} [f_t V_{t+1}^{\chi} + (1-f_t) U_{t+1}]$$

Where f_t is the probability that an unemployed worker will land job in period $t+1$.Also V_{t+1}^{χ} is period $t+1$ function of a worker who knows that he has matched with an unemployment agency at the start of period $t+1$. And following applies:

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

equation 128

$$V_{t+1}^{\chi} = \sum_{i=0}^{N-1} \frac{\chi_t^i \left(1 - \mathcal{F}_t^i\right) I_t^i}{m_t} \tilde{V}_{t+1}^{i+1}$$

Total matching function is given as:
equation 129

$$m_t = \sum_{j=0}^{N-1} \chi_t^j \left(1 - \mathcal{F}_t^i\right) I_t^j$$

The aggregate surplus across all I_t^0 workers in the representative agency is given by:
equation 130

$$(V_t^0 - U_t)(1 - \mathcal{F}_t^0) I_t^0$$

Each fraction $1 - \mathcal{F}_t^0$ workers with $a \geq \bar{a}_t^0$ who stay in the agency experiences the same surplus $V_t^0 - U_t$. Agency surplus per worker in I_t^0 is given by $J(\dot{\omega}_t)$ and this si readily confirmed to have the following structure as:
equation 131

$$J(\dot{\omega}_t) = \max_{\bar{a}_t^0} J(\dot{\omega}_t, \bar{a}_t^0)\,(1 - \mathcal{F}_t^0)$$

Where:
equation 132

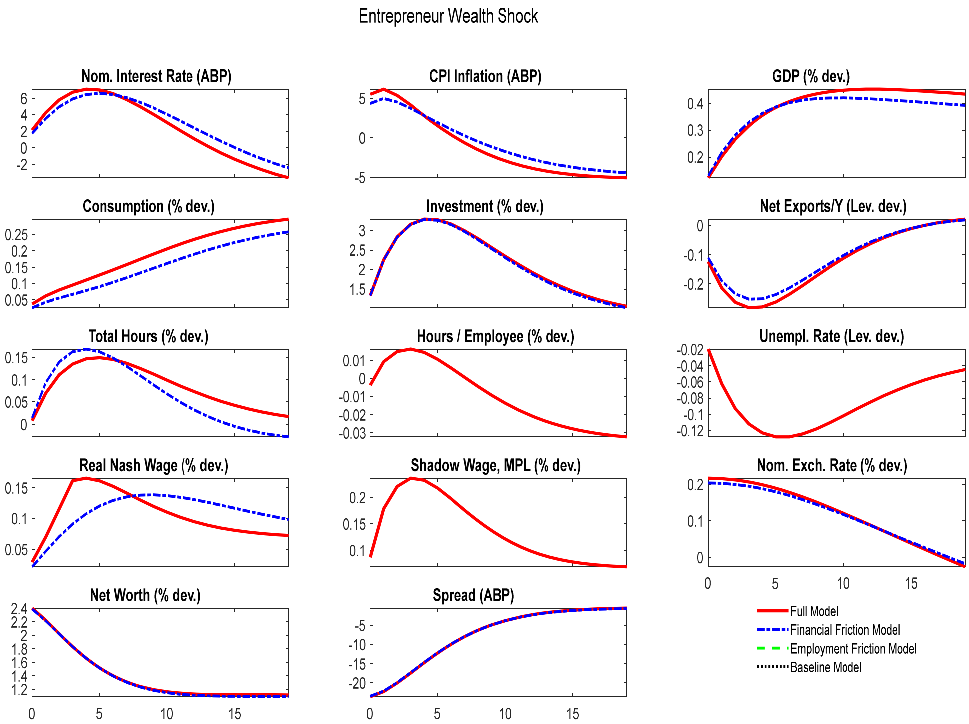
$$\tilde{J}(\dot{\omega}_t, \bar{a}_t^0) = \max_{\tilde{v}_t^0} \left\{ (W_t \varsigma_t^0 - \dot{\omega}_t) \varsigma_{0,t} - P_t\, z_t^{+} \frac{k}{\varphi} (\tilde{v}_t^0)^{\varphi} + \beta \frac{v_{t+1}}{v_t} (\chi_t^0 + \rho) J_{t+1}^1(\dot{\omega}_t) \right\}$$

Previous denotes the vaue fo an agency in cohort 0 after endogenous separatiot has taken place. Next, graphically is depicted result in the economy with entrepreneur wealth shock. Simulated are :full model, financial friction model, employment friction model and baseline model.

Manuscript received: 29.03.2023
Accepted: 20.04.2023

Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

Figure 10 Entrepreneur wealth shock in, full model, baseline economy and financial and unemployment friction economy



Source :Authors' caculation based on DYNARE code : https://faculty.wcas.northwestern.edu/lchrist/course/Korea_2012/CTW.html

Conclusion

The model with savings and consumption shows that savings and consumption policy function can be non-monotonic, and that wealth distribution has right fat tail as in Cagetti and De Nardi (2006). Also, consumption, savings and wealth distribution are affected by the productivity of entrepreneurs which may be low, medium, or high. On the other hand, endogenous entrepreneurship and financial frictions model shows that occupation choice is different for worker or entrepreneur based on productivity: low (worker) or high (entrepreneur) which in turn affects the aggregate wealth of the economy. Also, in this model it is shown that interest rate in the economy is determined by the intersection of total wealth with public and private capital. In the entrepreneurship with financial and labor market frictions model it is shown that entrepreneur wealth shock has different effect on the baseline economy and financial and unemployment friction economy. These models do prove that Financial frictions for capital and unemployment frictions for labor is said to be main drivers of the business cycle dynamics. For the open economy we will have to introduce small open economy setting in the standard New-Keynesian model.

References

1. Abel,A. B.Blanchard,O.J.(1983).An Intertemporal Model of Saving and Investment. Econometrica 51, pp. 675–692.
2. Achdou, Y. Han, J. Lasry, J-M. Lions, P-L. Moll, B.(2022). Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach, *The Review of Economic Studies*,Volume 89,Issue 1,Pages 45-86

Manuscript received: 29.03.2023
Accepted: 20.04.2023

International Journal of Economics, Management and Tourism
Vol. 3, No. 1, pp. 7-35
Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

3. Achdou, Yves, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. (2014).Wealth Distribution and the Business Cycle: The Rold of Private Firms.Princeton University Working Papers.
4. Aiyagari, S. R. (1994). Uninsured Idiosyncratic Risk and Aggregate Saving," The Quarterly Journal of Economics, 109(3), pp.659-84
5. Akerlof, G. A. (1980). A Theory of Social Custom, of Which Unemployment May be One Consequence. Quarterly Journal of Economics 94 (4): 749–75.
6. Akerlof, G. A. (1982). Labor Contracts as Partial Gift Exchange. Quarterly Journal of Economics 97 (4): 543–69
7. Akyol,A. Athreya,K.(2007).Unsecured credit and selfemployment. Unpublished manuscript, York University and Federal Reserve Bank of Richmond
8. Andolfatto, D. (1996). Business cycles and labor-market search. American Economic Review, 86, 112–132.
9. Arrow, J. (1951). An Extension of the Basic Theorems of Classical Welfare Economics. In J. Neyman (ed.), Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability (p./pp. 507--532),: University of California Press.
10. Arrow, K. J. (1965). The theory of risk aversion. Aspects of the Theory of Risk Bearing. Helsinki: Yrjo Jahnssonin Saatio. Reprinted in: Essays in the Theory of Risk Bearing. Chicago: Markham. 1971. pp. 90–109
11. Arrow, Kenneth J.; Debreu, Gérard (1954). Existence of an equilibrium for a competitive economy. Econometrica. The Econometric Society. 22 (3): 265–90. doi:10.2307/1907353. JSTOR 1907353
12. Auerbach, A. J. Kotlikoff, L. J. (1983). National Savings, Economic Welfare, and the Structure of Taxation. In M. Feldstein (Ed.), Behavioral Simulation Methods in Tax Policy Analysis, pp. 459–498. University of Chicago Press.
13. Banerjee AV, Duflo E. 2005. Growth theory through the lens of development economics. In Handbook of Economic Growth, Vol. 1A, ed. P Aghion, SN Durlauf, pp. 473–552. Amsterdam: North-Holland
14. Barro, R., (1977). Long-term contracting, sticky prices and monetary policy. Journal of Monetary Economics 3 (3), 305–316
15. Bassetto M, Cagetti M, Nardi MD. (2013). Credit crunches and credit allocation in a model of entrepreneurship. NBER Work. Pap. 19296
16. Bernanke, B., Gertler, M., Gilchrist, S., (1999). The financial accelerator in a quantitative business cycle framework. In: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics, vol. 1. , Elsevier Science, pp. 1341–1393.
17. Blanchard, O. and J. Gali (2010). Labor markets and monetary policy: A new Keynesian model with unemployment. American Economic Journal: Macroeconomics, 2, 1–30.
18. Boar, C. V. Midrigan (2020). Efficient Redistribution. mimeo. New York University.
19. Bohacek,R.(2006). Financial constraints and entrepreneurial investment. Journal of Monetary Economics, 53(8):2195–212
20. Boppart,T.,Krusell,P.Mitman,K.(2018).Exploiting MIT shocks in heterogeneous-agent economies: the impulse response as a numerical derivative. Journal of Economic Dynamics and Control,Volume 89,pp. 68-92
21. Boppart,T.,Krusell,P.Mitman,K.(2018).Exploiting MIT shocks in heterogeneous-agent economies: the impulse response as a numerical derivative. Journal of Economic Dynamics and Control,Volume 89,pp. 68-92
22. Buera FJ, Fattal-Jaef R, Shin Y. (2014). Anatomy of a credit crunch: from capital to labor markets. NBERWork.Pap. 19997
23. Buera FJ, Moll B. (2012). Aggregate implications of a credit crunch. NBER Work. Pap. 17775

Manuscript received: 29.03.2023
Accepted: 20.04.2023

24. Buera, Francisco, Kaboski, Joseph ,Shin, Yongseok. (2015). Entrepreneurship and Financial Frictions: A Macro-Development Perspective. Annual Review of Economics. 7. 150504160918000. 10.1146/annurev-economics-080614-115348.
25. Buera, Francisco J, Joseph P Kaboski, and Yongseok Shin (2011). Finance and Development: A Tale of Two Sectors. American Economic Review 101.5, pp. 1964–2002.
26. Buera, Francisco J., and Yongseok Shin. 2013. "Financial Frictions and the Persistence of History: A Quantitative Exploration." Journal of Political Economy, Forthcoming.
27. Buera. Francisco J. (2007).A dynamic model of entrepreneurship with borrowing constraints: Theory and evidence. Unpublished manuscript, Northwestern University
28. Cagetti, Marco, and Mariacristina De Nardi. 2006. "Entrepreneurship, Frictions, and Wealth." Journal of Political Economy, 114(5): 835–870.
29. Cagetti,M. DeNardi,M(2004). Taxation, entrepreneurship, and wealth. Federal Reserve Bank of Minneapolis Staff Report no. 340
30. Cagetti,M. DeNardi,M.(2006). Entrepreneurship, frictions and wealth. Journal of Political Economy, 114(5):835–70
31. Carlin, W., and D. Soskice. (2006). Macroeconomics: Imperfections, institutions and policies. Oxford: Oxford University Press.
32. Carroll,D.C.(1997). Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis, The Quarterly Journal of Economics,Vol. 112, No. 1 (Feb., 1997), pp. 1-55 (55 pages)
33. Christiano, L., Motto, R., Rostagno, M., (2003). The great depression and the Friedman–Schwartz hypothesis. Journal of Money, Credit, and Banking 35, 1119–1198.
34. Christiano, L., Motto, R., Rostagno, M., (2003). The great depression and the Friedman–Schwartz hypothesis. Journal of Money, Credit, and Banking 35, 1119–1198.
35. Christiano, L., Motto, R., Rostagno, M., (2008). Shocks, structures or monetary policies? The Euro Area and US after 2001. Journal of Economic Dynamics and Control 32 (8), 2476–2506
36. Christiano, L., Motto, R., Rostagno, M., (2008). Shocks, structures or monetary policies? The Euro Area and US after 2001. Journal of Economic Dynamics and Control 32 (8), 2476–2506.
37. Christiano, Lawrence J. & Trabandt, Mathias & Walentin, Karl,(2011). Introducing Financial Frictions and Unemployment into a Small Open Economy Model, Journal of Economic dynamics and control 35.,pp.1999-2041
38. Chugh, S. K. (2015). Modern macroeconomics. 1st ed. Cambridge, MA: MIT Press.
39. Diamond, P. A. (1982). Wage determination and efficiency in search equilibrium. Review of Economic Studies 49: 217–27
40. Dynan, K. (1993). How Prudent Are Consumers? The Journal of Political Economy, 101(6), 1104-13
41. Ewald-Christian,O.(2003). Introduction to continuous time Financial Market models. School of Economics and Finance University of St.Andrews
42. Fokker, A. D. (1914). Die mittlere Energie rotierender elektrischer Dipole im Strahlungsfeld. Ann. Phys. 348 (4. Folge 43):pp. 810–820.
43. Gardiner, C. (2009). Stochastic Methods (4th ed.). Berlin: Springer. ISBN 978-3-642-08962-6.
44. Geanakoplos, J. (1990). An introduction to general equilibrium with incomplete asset markets. Journal of Mathematical Economics, 19(1-2), pp. 1–38.
45. Geanakoplos, J.D.; Polemarchakis, H.M. (1986). Existence, regularity and constrained suboptimality of competitive allocations when the asset structure is incomplete. In Hell,

Manuscript received: 29.03.2023
Accepted: 20.04.2023

Online: ISSN **2671-3810**
UDC: 334.722:331.5]:303.72
Original scientific paper

W.P.; Starr, R.M.; Starrett, D.A. (eds.). Uncertainty, information and communication: Essays in honor of K.J. Arrow. Vol. 3. Cambridge University Press. pp. 65–95. ISBN 9780521327046.

46. Gertler, M. and A. Trigari (2009). Unemployment fluctuations with staggered Nash wage bargaining. Journal of Political Economy, 117, 38–86.
47. Greif, Constantin, (2017). Numerical Methods for Hamilton-Jacobi-Bellman Equations, Theses and Dissertations. 1480.
48. Guerrieri, V., G. Lorenzoni, L. Straub, and I. Werning (2020). Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages? NBER Working Paper 26918.
49. Guntin, R.,Kochen,F.(2022).Entrepreneurship, Financial Frictions, and the Market for Firms, NYU's 3rd year paper workshop and Universidad de Montevideo (UM) seminar.
50. Hall, R., (2005). Job loss, job finding, and unemployment in the U.S. economy over the past fifty years. In: Gertler, M., Rogoff, K. (Eds.), NBER Macroeconomics Annual, MIT Press, pp. 101–137
51. Hall, R.E. (1978). Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence. The Journal of Political Economy, 86(6), 971-987.
52. Hall, R.E.(2012). The 2010 Nobel Prize in Economics: How the DMP Model Explains Current High Unemployment. National Bureau of Economic Research
53. Hsieh C-T, Klenow P. 2014. The life cycle of plants in India and Mexico. Q. J. Econ. 129:1035–84
54. Hsieh, Chang-Tai and Peter J Klenow (2009). Misallocation and Manufacturing TFP in China and India. The Quarterly Journal of Economics 124.4, pp. 1403–1448
55. Huggett, M. (1993). The Risk-Free Rate in Heterogeneous-Agent Incomplete Insurance Economies, Journal of Economic Dynamics and Control 17, pp.953–969
56. Huggett, M. (1993). The Risk-Free Rate in Heterogeneous-Agent Incomplete Insurance Economies, Journal of Economic Dynamics and Control 17, pp.953–969
57. Judd, K. L. (1985). Short-Run Analysis of Fiscal Policy in a Simple Perfect Foresight Model.Journal of Political Economy 93, pp.298–319.
58. Kaplan, G., B. Moll, G. L. Violante (2018). Monetary Policy According to HANK. American Economic Review 108, pp.697–743.
59. King RG, Levine R. 1993. Finance, entrepreneurship and growth: theory and evidence. J. Monet. Econ.32:513–42
60. Kitao.S.(2008) Entrepreneurship, taxation and capital investment. Review of Economic Dynamics, 11(1):44–69
61. Kiyotaki N, Moore J. (2012). Liquidity, business cycles, and monetary policy. NBER Work. Pap. 17934
62. Kolmogorov,A.(1931). Über die analytischen Methoden in der Wahrscheinlichkeitstheorie [On Analytical Methods in the Theory of Probability]. Mathematische Annalen (in German). 104 (1): 415–458 [pp. 448–451].
63. Kramers, H. A. (1940). Brownian motion in a field of force and the diffusion model of chemical reactions. Physica. 7 (4): 284–304. Bibcode:1940Phy.....7..284K. doi:10.1016/S0031-8914(40)90098-2. S2CID 33337019.
64. Li,W.(2000). Entrepreneurship and government subsidies: A general equilibrium analysis. Journal of Economic Dynamics and Control, 26(11):1815–44
65. Lindbeck, Assar, and Dennis J. Snower. (1988). The Insider-Outsider Theory of Employment and unemployment. Cambridge, MA: MIT Press
66. Mas-Colell, A. (1989).The Theory of General Economic Equilibrium: A Differentiable Approach, Cambridge University Press.
67. Meh, C. A. (2005). Entrepreneurship, wealth inequality, and taxation. Review of Economic Dynamics, 8(3):688–719

Manuscript received: 29.03.2023
Accepted: 20.04.2023

68. Meh, C. A. , Terajima,Y.(2007). Personal bankruptcy and entrepreneurship. Unpublished manuscript, Bank of Canada
69. Merz, M. (1995), Search in labor market and the real business cycle. Journal of Monetary Economics, 95, 269–300.
70. Midrigan, Virgiliu ,Daniel Yi Xu (2014). Finance and Misallocation: Evidence from Plant-Level Data. American Economic Review 104.2, pp. 422–458.
71. Mortensen, Dale T., and Christopher A. Pissarides. (1994). Job Creation and Job Destruction in the Theory of Unemployment. Review of Economic studies 61 (3): 397–415.
72. Moyal, J. E. (1949). Stochastic processes and statistical physics. Journal of the Royal Statistical Society. Series B (Methodological). 11 (2): 150–210. JSTOR 2984076.
73. Petrosky-Nadeau, N., L. Zhang, and L. Kuehn (2015). Endogenous disasters. Working paper, Federal Reserve Bank of San Francisco and The Ohio State University
74. Petrosky-Nadeau, Zhang L. (2017). Solving the Diamond–Mortensen–Pissarides model accurately. Quantitative Economics 8 (2017), 611–650
75. Pissarides, C. A. (1985). Short-run equilibrium dynamics of unemployment, vacancies, and real wages. American Economic Review 75: 676-90.
76. Pissarides, C. A. 1985. Short-run equilibrium dynamics of unemployment, vacancies, and real wages. American Economic Review 75: 676-90.
77. Planck, M. (1917). Über einen Satz der statistischen Dynamik und seine Erweiterung in der Quantentheorie. Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin. 24: pp.324–341.
78. Quadrini V. 2009. Entrepreneurship in macroeconomics. Ann. Finance 5:295–311
79. Quadrini,V.(2000).Entrepreneurship, saving and social mobility. Review of Economic Dynamics, 3(1):1–40.
80. Shourideh A, Zetlin-Jones A. (2014).External financing and the role of financial frictions over the business cycle:measurement and theory. Unpublished manuscript, Carnegie Mellon Univ., Pittsburgh, PA
81. Smoluchowski, M. (1916). "Drei Vorträge über Diffusion, Brownsche Molekularbewegung und Koagulation von Kolloidteilchen". Phys. Z. (in German). 17: 557–571, 585–599.
82. Solow, Robert M. (1979). Another Possible Source of Wage Stickiness. Journal of Macroeconomics 1 (1): 79–82
83. Storesletten K, Telmer CI, Yaron A. (2001). The welfare cost of business cycles revisited: finite lives and cyclical variation in idiosyncratic risk. Eur. Econ. Rev. 45(7): pp.1311–1139
84. Terajima,Y.(2006). Education and self-employment: Changes in earnings and wealth inequality. Bank of Canada Working Paper, No. 2006-40.
85. Uhlenbeck, G. E.; Ornstein, L. S. (1930). "On the theory of Brownian Motion". Phys. Rev. 36 (5): 823–841
86. Williamson, S. D. (2013). Macroeconomics. 5th ed. New York: Pearson.