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AIYAGARI, BEWLEY, HUGGETT, IMROHOROĞLU (ABHI) ECONOMIES: LITERATURE REVIEW AND COMPUTATIONAL EXAMPLES

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Abstract
This paper will review ABHI models in economics. Namely those are collection of models Aiyagari-Bewley-Hugget-Imrohoroglu economies where there is precautionary savings amongst the economic agents, liquidity constraints, and where Markets are exogenously incomplete. There is incompleteness by assumption as opposed to limited commitment and limited enforcement models. In Huggett (1993) model there is diminishing marginal savings rates for some agents and negative marginal savings rate for other agents with an increase in their wealth (assets). In the incomplete markets in general equilibrium cash-on hand $R_a + y$ is more in consumption with lower assets, this applies even more so in partial equilibrium model. And in the period with lowest: consumption, income and assets incomplete markets in partial equilibrium model predicts highest savings rate.

Key words: *Aiyagari, Bewley, Huggett, Imrohoroğlu (ABHI) economies,incomplete markets, heteoregenous agents,pure credit model*
JEL Classification: D14, D31, E21

Introduction

Models with heterogenous agents have become dominant workhorse in macroeconomics since seminal works by: Bewley(1986), Hopenhayn(1992),Huggett (1993), Aiyagari (1994)[1].Agents in these economies make choices by taking some aggregate variables that depend on distribution of individuals in the economy. Their choices with idiosyncratic shocks will determine the evolution of distribution of savings, consumption, and wealth in the economy. The equilibrium in these economies is being characterized by the dynamic programming equation that descries intertemporal problem of each agent by the law of motion of distribution and by the market clearing conditions which link individual choices to aggregate variables, see Galo,Moll (2018). Models with perfect insurance estimated small magnitude of cost effects on business cycles (0.1% of total consumption in US). But some studies such as Imrohoroğlu (1989) precluded perfect insurance exogenously such as in Scheinkman,Weiss(1986)[2].There is evidence of liquidity constraint at micro level, see Zeldes(1989).Tobin and Dolde (1971) have examined the implications of liquidity constraints in a deterministic framework and showed that capital accumulation in the economy analyzed increases by a factor of two because of liquidity constraints. Many studies have used calibrated versions of Bewley models to give quantitative answers to questions including the welfare

[1] More complete review of this literature could be read in Heathcote et al.(2009)
[2] Other studies departed from the assumption of perfect insurance by : limiting insurance arrangements endogenously by using moral hazard or incomplete information models as pursued in Green (1987), Atkeson (1988), and Townsend (1988) ,see Galo,Moll (2018).

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costs of inflation Imrohoroğlu,(1992), the risk-sharing benefits of unfunded social security systems Imrohoroğlu, Imrohoroğlu, Joines (1995), the benefits of insuring unemployed people Hansen , Imrohoroğlu (1992), and the welfare costs of taxing capital Aiyagari, (1995). See Kaplan, Violante (2010) for a quantitative study of how much insurance consumers seem to attain beyond the self-insurance allowed in Bewley models. Heathcote, Storesletten, and Violante (2012) combine ideas of Bewley with those of Constantinides and Duffie (1996) to build a model of partial insurance. Aiyagari (1994) model belongs to a class of models that involves a considerable number of individual dynamics, uncertainty, and asset trading which is the main mechanism by which individuals attempt to smooth consumption. Aggregate variables are unchanging so that is the main difference with representative agent models[3].Bewley (1986) developed on the idea that short run consumers may act as if their marginal utilities of money were constant. Also "The model is of a pure exchange economy with immortal consumers who hold money to offset fluctuations in their endowments and utility functions. It is also assumed that there is a continuum of consumers and that the fluctuations in their utilities and endowments are independent. "Huggett model was based on the enormous literature that up until then was done on "….heterogenous-agent-incomplete-insurance models of asset pricing….", some of the references here include : Bewley (1980), Lucas (1980), Taub (1988). But the research paper in Huggett (1993) paper was motivated by the work of Mehra ,Prescott (1985).As to question :Why heterogeneity of agents matters in macroeconomics? Asked and answered partially by Boppart et al. (2018) his answer states: Marginal decisions made by households, regarding: consumption, hours worked, and investments in various types of assets "vary quite substantially" in population. Arrow (1951) and Arrow,Debreu (1954),proved that competitive equilibrium in Arrow-Debreu economy is Pareto optimal and discovered class of convex Arrow-Debreu economies for which competitive equilibria always exist. In the case of incomplete, see Geanakoplos (1990)markets this equilibrium may (will) not be efficient see Geanakoplos (1986) or the will be suboptimal constrained. The purpose of Imrohoroğlu (1989) was to "develop tools for computing the equilibria for economies with two different forms of incomplete insurance markets and to apply these tools to estimate the magnitude of the costs of business cycles." This paper will review all these issues and will include computational models in its final form. To the satisfaction of the authors and readers it will be to increase the knowledge on these models and later to go to HANK models[4]. This paper is organized as follows: First, it will be explained mathematically Aiyagari model (1994) in Discrete and Continuous Time, second, we will explain Bewley economy with assets in positive supply. Then Huggett model with a example of pure credit model will be looked at, followed by Imrohoroğlu model (1989). Computation examples will include Aiyagari economy with idiosyncratic Brownian motion and random deaths, and Human capital model which will correlate policy, physical capital and human capital and fraction of time for learning. This will be followed by computational examples on Solving the incomplete

[3] This paper exposition was built around the Brock,Mirman (1972)standard growth model modified to include a role for uninsured idiosyncratic risk and liquidity/borrowing constraints. Second goal of this model were to study the role of individual risk and its importance for aggregate saving. As literature suggests precautionary savings may be quantitatively important component of aggregate saving. Modigliani (1988) argues that pure bequest motive is important only for people in the highest income and wealth brackets and that "some portion of bequests, especially in lower income brackets, is not due to a pure bequest motive but rather to a precautionary motive reflecting uncertainty about the length of life, although it is not possible at present to pinpoint the size of this component." See Aiyagari(1994).

[4] Debortoli D. Galí,J(2017) identify three channels at work in Heterogeneous Agent New Keynesian (HANK) models: (i) changes in the average consumption gap between constrained and unconstrained households, (ii) variations in consumption dispersion within unconstrained households, and (iii) changes in the share of constrained households

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markets model in general equilibrium and solving the incomplete markets model in partial equilibrium.

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Aiyagari model (1994) in Discrete and Continuous Time

The material in this notebook is based on Achdou et al. (2022). In the discrete version of this model we have following problem:

equation 1

$$\max \mathbb{E}_{t=0}^{\infty} \beta^t u(c_t)$$

s.t.

$$a_{t+1} + c_t \leq w z_t + (1+r) a_t, c_t \geq 0, a_t \geq -\mathcal{B}$$

c_t is current consumption.a_t is assets,$z_t$is an exogenous component of labor income capturing stochastic unemployment risk, w is a wage rate,r_t is a net interest rate,$\mathcal{B}$ is the maximum amount that the agent is allowed to borrow,Here z_t follows Markov chain process [5]with matrix P[6]. For the firms in the discrete time economy, we have:

equation 2

$$Y = AK_t^a L_t^{1-a}$$

Where in previous $A > 0; a \in (0,1)$ .Now, the firm maximizes:

equation 3

$$\max_{K,N} [AK_t^a N^{1-a} - (r+\delta)K - wN\ ]$$

Where δ is depreciation rate, from the FOC with respect to capital, the firm's inverse demand for capital is given as:

equation 4

$$r = Aa\left(\frac{N}{K}\right)^{1-a} - \delta$$

Equilibrium wage rate is given as:

equation 5

$$w(r) = A(1-a)\left(\frac{Aa}{r+\delta}\right)^{\frac{a}{1-a}}$$

In continuous time this economy can be represented as the next equation where : $z_1 < z_2$ and $s_j(a) = wz_j + ra - c_j(a)$ and $c_j(a) = (u')^{-1}\left(v_j(a)\right)$. There is state-constraint $a \geq \underline{a}$.The FOC here is:$u'\left(c_j(\underline{a})\right) = v_j'(\underline{a})$ still holds at the borrowing constraint. In order to respect constraint we have to :$s_j(\underline{a}) = z_j + ra - c_j(\underline{a}) \geq 0$ .Combining the FOC the state constraint motivates boundary condition:$v_j'(\underline{a}) \geq u'(z_j + r\underline{a}); j = 1,2$.

[5] $z_{chain} = \text{MarkovChain}([0.9\ 0.1;\ 0.1\ 0.9], [0.1;\ 1.0])$

[6] $\mathrm{P}(\mathrm{x_n} = \mathrm{a_{i_n}} | \mathrm{x_{n-1}} = \mathrm{a_{i_{n-1}}}, \ldots, \mathrm{x_1} = \mathrm{a_{i_1}}) = \mathrm{P}(\mathrm{x_n} = \mathrm{a\_i_n} | \mathrm{x_{n-1}} = \mathrm{a_{i_{n-1}}})$,x_n is a Markov chain, Papoulis,(1984).

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equation 6

$$\rho v_1(a) = \max_c u(c) + v_1'(a)(wz_1 + ra - c) + \lambda_1(v_2(a) - v_1(a))$$
$$\rho v_2(a) = \max_c u(c) + v_2'(a)(wz_2 + ra - c) + \lambda_1(v_1(a) - v_2(a))$$
$$0 = -\frac{d}{da}[s_1(a)g_1(a)] - \lambda_1 g_1(a) + \lambda_2 g_2(a)$$
$$0 = -\frac{d}{da}[s_2(a)g_2(a)] - \lambda_2 g_2(a) + \lambda_1 g_1(a)$$
$$1 = \int_{\underline{a}}^{\infty} g_1(a)da + \int_{\underline{a}}^{\infty} g_2(a)da$$
$$K = \int_{\underline{a}}^{\infty} ag_1(a)da + \int_{\underline{a}}^{\infty} ag_2(a)da$$
$$r = aK^{a-1} - \delta; w = (1-a)K^a$$

Bewley economy: Assets in Positive Supply

Due to Bewley (1977),there is a class of incomplete markets general equilibrium model[7].In this model asset supply:$A(r) = 0$ or:

equation 7

$$A(r) = \sum_j \int_{\underline{a}}^{\infty} ag(a, y_j; r)da$$

Where $\underline{a} = A(-1); r = \beta^{-1} - 1$ or $r = \rho$ asset explode $A(r) \to \infty$.Government issues bonds $\mathcal{B}$ and finances interest payments according to a tax function $\tau(a, y)$ and total tax revenues are given as:

equation 8

$$T(r) = \sum_j \int_a \tau(a, y_j) g(a, y_j; r)da$$

Subject to government budget constraint:$G + r\mathcal{B} = T(r)$ and market clearing condition is $A(r) = \mathcal{B}$.If $\mathcal{B}$ is exogenous then we determine:$G(r) = T(r) - r\mathcal{B}$ as residual, provided $G(r) \geq 0$.And for computation with exogenous G we solve :

equation 9

$$A(r) = \frac{T(r) - G}{r}$$

Asset grid in this model is given as:$A = [0 < \overline{a}_1 < \overline{a_2} < \cdots . < \overline{a_n}]$; and so the household choose policy $\{c_t, a_{t+1}\}_{t=0}^{\infty}$ see Ljungqvist,L. Sargent,T.J.(2018). And so households maximize:

equation 10

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

[7] In economics, incomplete markets are markets in which there does not exist an Arrow–Debreu security for every possible state of nature.About Arrow -Debreu securities: It posits that under certain economic assumptions (convex preferences, perfect competition, and demand independence) there must be a set of prices such that aggregate supplies will equal aggregate demands for every commodity in the economy, see Arrow,Debreu(1954).

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Subject to constraint: $c_t + a_{t+1} = (1+r)a_t + ws_t$ and $a_{t+1} \in A$.Where $u(c)$ is strictly increasing, and β is discount factor, $u(c)$ is also twice differentiable that satisfies Inada conditions[8]:$\lim_{c\downarrow 0} u'(c) = +\infty\,; \beta(1+r) < 1$ .The Bellman equation for each $i \in \{i, \dots \dots, m\} \forall h \in [1, \dots, n]$is given as:

equation 11

$$v(\bar{a}_h, \bar{s}_i) = \max_{a' \in A} \left\{ u[(1+r)\bar{a}_h + w\bar{s}_i - a'] + \beta \sum_{j=1}^{m} \mathcal{P}(i,j) v(a'\bar{s}_j) \right\}$$

Where in previous a' is previous period asset holding. Wealth and employment distribution in this model are given as:

equation 12

$$Prob(a_{t+1} = a', s_{t+1} = s') = \sum_{a_t} \sum_{s_t} Prob(a_{t+1} = a' | a_t = 1, s_t = s)$$
$$Prob(s_{t+1} = s' | s_t = s) \cdot Prob(a_t = a, s_t = s)$$
$$\lambda_{t+1}(a' \cdot s') = \sum_{a} \sum_{s} \lambda_t(a,s) Prob(s_{t+1} = s' | s_t = s) \cdot \mathcal{I}(a', s, a)$$

Where the indicator function $\mathcal{I}(a', a, s) = 1 \wedge a' = g(a,s)$. This assumption exploits the fact that the optimal policy is a deterministic function of the state, which comes from the concavity of the objective function and the convexity of the constraint set. The indicator function $\mathcal{I}(a', a, s) = 1$ identifies the time t states a, s that are sent into a' at time $t+1$.The proceeding equation can be presented as:

equation 13

$$\lambda_{t+1}(a' \cdot s') = \sum_{s} \sum_{\{a: a' = g(a,s)\}} \lambda_t(a,s) \mathcal{P}(s,s')$$

A time-invariant probability distribution λ that solves equation $\lambda_{t+1}(a' \cdot s') = \sum_s \sum_{\{a:a'=g(a,s)\}} \lambda_t(a,s)\mathcal{P}(s,s')$(i.e., one for which $\lambda_{t+1} = \lambda_t$) is called a stationary distribution.The optimal policy function $a' = g(a,s)$ and the Markov chain $\mathcal{P}$ on s induce a Markov chain for x via the equation:

equation 14

$$Prob[(a_{t+1} = a', s_{t+1} = s']|(a_t = a, s_t = s)$$
$$= Prob(a_{t+1} = a' | a_t = a, s_t = s) \cdot Prob\ (s_{t+1} = s' | s_t = s) = \mathcal{I}(a', a, s)\mathcal{P}(s, s')$$

Where $\mathcal{I}(a', a, s) = 1$.Now, suppose that the Markov chain associated with $\mathcal{P}$ is asymptotically stationary and has a unique invariant distribution π_∞. Typically, all states in the Markov chain will be recurrent, and the individual will occasionally revisit each state. Then the distribution π_∞ tells the fraction of time that the household spends in each state.Now, we are deducing probability measure $\lambda(\bar{a}_i, \bar{s}_h) = Prob((a_t = \bar{a}_i, s_t = \bar{s}_h)$ over $(\bar{a}_i, \bar{s}_h)$:

equation 15

$$\lambda(\bar{a}_i, \bar{s}_h) = Prob((a_t = \bar{a}_i, s_t = \bar{s}_h) = \pi_\infty(j)$$

Where $\pi_\infty(j)$ is the j-th component of vector π_∞ and $j = (i-1)m + h$.Where we have given interest rate r , the population mean:

---

[8] Given $f: X \to Y$ where $X = \{x: x \in \mathbb{R}^n_+\}$ and $Y = \{y: y \in \mathbb{R}^n_+\}$ and the conditions are :$f(0) = 0$;the Hessian matrix $H_{ij} = \left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right)$ needs to be negative semidefinite i.e. $n \times n$ symmetric matrix $\mathcal{M}$ is negative semi-definite or non-positive semi-definite if: $x^T M x < 0; \forall x \neq 0 \in \mathbb{R}^n \setminus \{0\}$,see Inada (1963), and for Hessian matrix see Gradshteyn,Ryzhik(2000).

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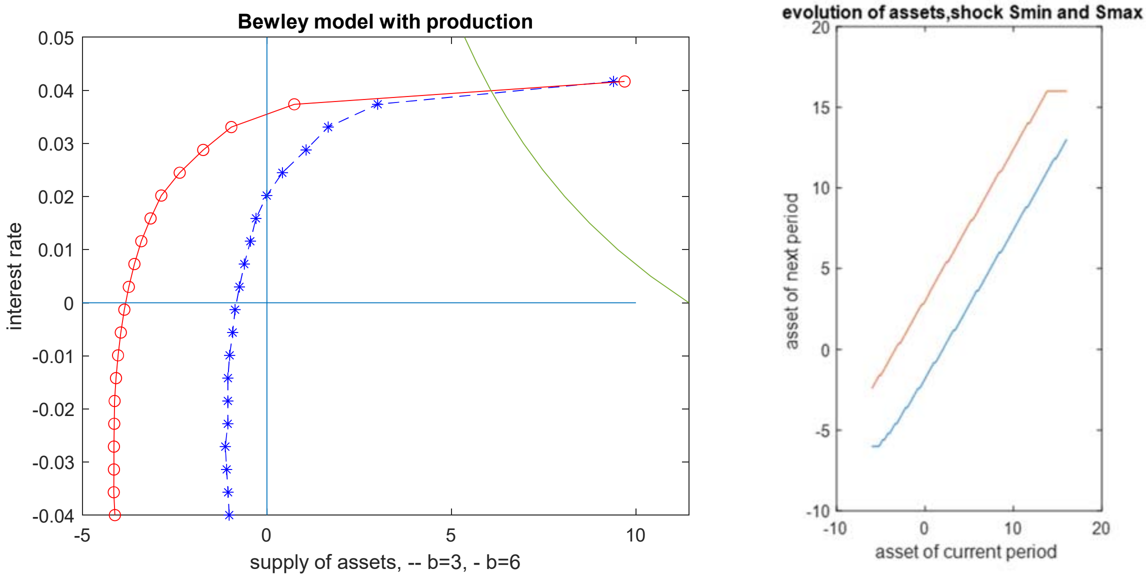
equation 16

$$E(a)(r) = \sum_{a,s} \lambda(a,s) g(a,s)$$

This first foremost is the average asset level experienced by every household, where the average is across the time. Second it is the average asset level of the economy as a whole.

Figure 1 Bewley model with production and the evolution of assets, and shocks



Source: Authors own calculations based on a code available at: https://dge.repec.org/codes/sargent/bewley/

Huggett (1993) model : a Pure credit model

This part is based on a Huggett (1993).The problem here is to maximize:

equation 17

$$E\left[\sum_{=0}^{\infty} \beta^t u(c_t)\right]$$

Subject to :$a_{t+1} + c_t \leq e_t + ra_t$ .Where in previous e_t is a stochastic process, and c_t is constrained to be adapted to the filtration generated by previous process. We impose two constraints: $a_0 \geq 0\,; a_t \geq \underline{a}$ $\forall t$.We call this borrowing limit or credit constraint, or liquidity constraint[9]. Now this stochastic Euler equation will become:

equation 18

$$u'(c_t) = \beta r E_t[u'(c_{t+1})]$$

[9] DSGE models were also critiqued by Stiglitz (2018) for this. His critique is that pre-crisis DSGE models did not allow for financial frictions and liquidity constrained consumers, though than existing literature denies this as Galí, López-Salido, and Vallés (2007) investigate the implications of the assumption that some consumers are liquidityconstrained. They find that liquidity constraints magnify the effects of government spending. Previously, Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999) develop DSGE models that incorporate credit market frictions.Zeldes (1989), confirms borrowing constraints seem empirically plausible and formal econometric tests indicate so.

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About Euler equation:
Here following lemma applies see Achdou et al.(2022)

Lemma 1: The consumption and savings policy functions $c_j(a)$ and $s_j(a)$ for $j = 1,2..$ corresponding to HJB equation : $\rho v_j(a) = \max_c u(c) + v_j'(a)(y_j + ra - c) + \lambda_j \left(v_{-j}(a) - v_j(a)\right)$ which is maximized at : $0 = -\frac{d}{da}[s_j(a)g_J(a)] - \lambda_j g_j(a) + \lambda_{-jg_{-j}}(a)$ is given as:

equation 19

$$(\rho - r)u'\left(c_j(a)\right) = u''\left(c_j(a)\right)c_j'(a)s_j(a) + \lambda_j(u'\left(c_{-j}(a)\right) - u'\left(c_j(a)\right)$$
$$s_J(a) = y_J + ra - c_j(a)$$

Proof: differentiate $\rho v_j(a) = \max_c u(c) + v_j'(a)(y_j + ra - c) + \lambda_j \left(v_{-j}(a) - v_j(a)\right)$ with respect to a and use that $v_j'(a) = u'\left(c_j(a)\right)$ and hence $v_j''(a) = u''\left(c_j(a)\right)c_j'(a)$ ■

The differential equation:
equation 20

$$(\rho - r)u'\left(c_j(a)\right) = u''\left(c_j(a)\right)c_j'(a)s_j(a) + \lambda_j(u'\left(c_{-j}(a)\right) - u'\left(c_j(a)\right)$$
$$s_J(a) = y_J + ra - c_j(a)$$

is and Euler equation , the right hand side$(\rho - r)u'\left(c_j(a)\right)$ is expected change of marginal utility of consumption $\frac{\mathbb{E}_t[du\prime(c_j(a_t)]}{dt}$.This uses Ito's formula to Poisson process:
equation 21

$$\mathbb{E}_t[du'(c_j(a_t)] = \left[u''(c_j(a_t)c_j'(a_t)s_j(a_t) + \lambda_j\left(u'(c_{-j}(a_t)\right) - u'\left(c_j(a_t)\right)\right]dt$$

So, this equation $(\rho - r)u'\left(c_j(a)\right) = u''\left(c_j(a)\right)c_j'(a)s_j(a) + \lambda_j(u'\left(c_{-j}(a)\right) - u'\left(c_j(a)\right)$, $s_J(a) = y_J + ra - c_j(a)$ can be written in more standard form:
equation 22

$$\frac{\mathbb{E}_t[du'(c_j(a_t)]}{dt} = (\rho - r)dt$$

Now,let's suppose that e_t is a time-homogenous Markov process.Then optimal savings function is given as:
equation 23

$$a_{t+1} = h(a_t, e_t)$$

Consumption function is given as:
equation 24

$$c(a, e) = e + ra - h(a, e)$$

If we assume period CRRA utility function we have:
equation 25

$$[c(a, e)]^{-\sigma} = \beta r E[[c(ra + e - c(a, e), e']^{-\sigma} | \ e]$$

Krussel, Smith (1998) explain that linearity of consumption and saving policy functions with CRRA utility functions, explains their finding that the business cycle properties of baseline heterogeneous agent model

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are virtually indistinguishable from its representative agent counterpart. Now MPC and MPS will be given as: $MPS_{\tau}(a) = e^{-\eta\tau} \approx 1-\eta\tau$ and $MPC_{\tau}(a) = 1 - e^{-\eta\tau} + \tau r \approx \tau(\eta + r), \eta := \frac{\rho - r}{\gamma}$.

Lemma 2. The conditional expectation of consumption $c_{j,\tau}(a)$ defined previously as $c'_{j,\tau}(a) = \mathbb{E}\left[\int_0^{\tau} c_j(a_t)dt \,\middle|a_0 = a, y_0 = y_j)\right]$ can be computed as $c_{j.\tau}(a) = \mathcal{P}_j(a,0)$.In previous expression $\mathcal{P}_j$ satisfies system of two PDE's.

equation 26

$$0 = c_j(a) + \partial_a \mathcal{P}_j(a,\tau)s_j(a) + \lambda_j\left(\mathcal{P}_{-j}(a,\tau) - \mathcal{P}_j(a,\tau)\right) + \partial_{\tau}\mathcal{P}_j(a,\tau), j = 1,2..\mathcal{P}_j(a,\tau) = \forall a$$

1. *Proof:* per Achdou et al.(2022) follows directly from application of Feynman-Kac formula for computing conditional expectations as solutions to PDE's. So, since $c'_{j,\tau}(a) = \mathbb{E}\left[\int_0^{\tau} c_j(a_t)dt \,\middle|a_0 = a, y_0 = y_j)\right]$ and if A is infinitesimal generator (Feller process or Levy process, or Ornstein–Uhlenbeck process): **Feller process**-Let E be a LCCB (locally compact with countable base) and $E \subset \mathbb{R}^n, \exists n \in N$ and $C_0(E) = C_0(E,\mathbb{R})$ be the space of continuous function that vanishes in inf. A Feller semigroup $C_0(E)$ is a family of positive linear operators $T_{\tau}, \tau \geq 0$ on $C_0(E)$
    - ✓ $T_0 = Id; \|T_{\tau}\|; \forall\tau \in T$ i.e. $\{T_{\tau}\}_{\tau\in T}$ is a family of contracting maps
    - ✓ $T_{\tau+s} = T_{\tau} \circ T_s$ (the semigroup property)
    - ✓ $\lim_{t\downarrow 0}\|T_{\tau}f - f\| \forall f \in C_0(E)$

See Revuz et al.(2005).

2. **Levy process-** L let be is an infinite divisible random variable $\forall t \in [0,\infty]$
- ✓ L can be written as the sum of a diffusion, a continuous Martingale and a pure jump process; i.e:

equation 27

$$L_t = at + \sigma B_t + \int_{|x|<1} x d\widetilde{N}_{\tau} + \int_{|x|\geq 1} x dN_{\tau}(\cdot, dx), \forall t \geq 0$$

In previous expression $a \in \Re$, B_t is the standard Brownian motion, N is defined to be the Poisson random measure of the Lèvy process

- ✓ Lèvy -Khintchine formula: from the previous property it can be shown that for $\forall\tau \geq 0$ one has that :

equation 28

$$E\left|e^{inL_t}\right| = e\hat{}(-\tau\psi(u)$$

$$\psi(u) = -iau + \frac{\sigma^2}{2}u^2 + \int_{|x|\geq 1}(1 - e^{iux})dv(x) + \int_{|x|<1}(1 + e^{iux} + iux)dv(x)$$

$a \in \Re; \sigma \in [0,\infty); v > 0$ borel measure and σ is Lèvy measure. More so $v(\cdot) = E[N_1(\cdot, A)]$

See Applebaum (2004).

3. **Ornstein–Uhlenbeck process-** The Ornstein-Uhlenbeck process is a stochastic process that satisfies the following stochastic differential equation:

equation 29

$$dx_{\tau} = k(\theta - x_{\tau})d\tau + \sigma dW_{\tau}$$

$k > 0$ is the mean rate of reversion; θ is the long term mean of the process, $\sigma > 0$ is the volatility or average magnitude, per square-root time, of the random fluctuations that are modelled as Brownian motions.

- ✓ Mean reverting property-where $dx_{\tau} = k(\theta - x)$:

equation 30

$$\frac{\theta - x_{\tau}}{\theta - x_0} = e^{-k(\tau-\tau_0)}, x_{\tau} = \theta + (x_0 - \theta)e^{-k(\tau-\tau_0)}$$

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- ✓ Solution for $\forall \tau > s \geq 0$ is given as:

equation 31

$$x_\tau = \theta + (x_s - \theta)e^{-k(\tau - s)} + \sigma \int_s^\tau e^{-k(\tau-u)} dW_u$$

See Jacobsen.M(1996) .So now partial differential equation $\frac{\partial c_{j,\tau}}{\partial \tau} = Ac_{j,\tau}(a) - c_{j,\tau}(\underline{a})(a)$ is the solution to $c'_{j,\tau}(a) = \mathbb{E}\left[\int_0^\tau c_j(a_t)dt \,\middle|a_0 = a, y_0 = y_j)\right]$ ■. Here will be presented two main approaches for solving Huggett (1993) model and problem numerically. This part is based on : Rouwenhorst (1995) and also in Kopecky and Suen (2010).Now, e_t is a two-state Markov process $e_t \in \{e_l, e_h\}$ and that transition probabilities are given as following:

equation 32

$$\Gamma = \begin{bmatrix} \gamma & 1-\gamma \\ 1-\gamma & \gamma \end{bmatrix}$$

Where in previous autocorrelation is given as: $2\gamma - 1$. Now about the two-state Euler equation process:

1. In the low earnings state:

equation 33

$$\begin{aligned}(e_l + ra - h(a, e_l)^{-\sigma} \\ &= \beta r\{\gamma[e_l + rh(a, e_l) - h(h(a, e_l), e_l]^{-\sigma} \\ &+ (1-\gamma)[e_h + rh(a, e_l) - h(h(a, e_l), e_h]^{-\sigma}\}\end{aligned}$$

2. In the high earning state:

equation 34

$$\begin{aligned}e_h + ra - h(a, e_h)^{-\sigma} \\ &= \beta r\{\gamma[e_h + rh(a, e_h) - h(h(a, e_h), e_h]^{-\sigma} \\ &+ (1-\gamma)[e_l + rh(a, e_h) - h(h(a, e_h), e_l]^{-\sigma}\}\end{aligned}$$

With exogenous grid savings function is approximated as:

equation 35

$$[e_l + ra_k - y]^{-\sigma} = \beta r\left\{\gamma\left[e_l + ry - \hat{h}(y, \theta^{l,0}\right]^{-\sigma} + (1-\gamma)\left[e_h + ry - \hat{h}(y, \theta^{h,0}\right]\right\}$$

Where in previous: $\theta^0 = [\theta^{l,0}, \theta^{h,0}]$ these are vectors, and$\hat{h}(a^k, \theta) = \theta_k, \forall k$ and $\theta^l, \theta^h \in \mathbb{R}^N$. To solve for y we have:

equation 36

$$y = \frac{a^m f_k^l(a^{m+1}; \theta^0) - a^{m+1} f_k^l(a^m; \theta^0)}{f_k^l(a^{m+1}; \theta^0) - f_k^l(a^m; \theta^0)}$$

Now :

$$f_k^l(y, \theta^0) = [e^l + ra - y]^{-\sigma} = \beta r\left\{\gamma\left[e^l + ry - \hat{h}(y; \theta^{l,0}\right]^{-\sigma} + (1-\gamma)\left[e_h + ry - \hat{h}(y, \theta^{h,0}\right]^\wedge - \sigma\right\}$$

Where $f_k^l(y, \theta^0)$ is the FOC function at $a = a^k; e = e^l$.With the method of endogenous grid we have:

equation 37

$$a = \frac{[A^l(y, \theta^0)]^{-\frac{1}{\sigma}} - e^l + y}{r}$$

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Savings grid is $y = \{y^1, y^2, \dots, y^n\}$ and $y^1 = \underline{a}; y^n = \bar{a}$. We can define here:

equation 38

$$A^l(y;\theta^0) = \beta r\left\{\gamma\left[e^l + ry - \hat{h}(y;\theta^{l,0}\right]^{-\sigma} + (1-\gamma)\left[e_h + ry - \hat{h}(y,\theta^{h,0}\right]^{-\sigma}\right\} = 0$$

And for $A^l(y;\theta^0)$ we have:

equation 39

$$A^h(y;\theta^0) = \beta r\left\{\gamma\left[e^h + ry - \hat{h}(y;\theta^{h,0}\right]^{-\sigma} + (1-\gamma)\left[e_l + ry - \hat{h}(y,\theta^{l,0}\right]^{-\sigma}\right\} = 0$$

Now, the density function can be discretized:

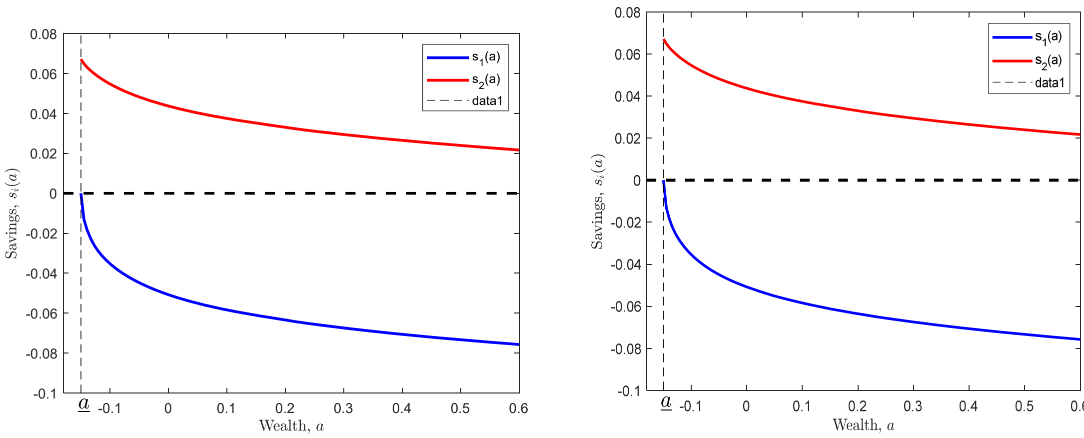
equation 40

$$f_{i,j}^1 = f_{i,j}^1 + \pi(\lambda^j|\lambda^k)\frac{a_{i+1} - h(a_l, \lambda^k)}{a_{i+1} - a^i} f_{l,k}^0$$

$$f_{i+1,j}^1 = f_{i+1,j}^1 + \pi(\lambda^j|\lambda^k)\frac{h(a_l, \lambda^k) - a^i}{a_{i+1} - a^i} f_{l,k}^0$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m} f_{i,j}^1 = 1$$

Figure 2 Huggett model economy savings and wealth



Source: Author's calculations based on code available at: https://benjaminmoll.com/codes/

Imrohoroğlu model (1989)

This part is based on Imrohoroğlu (1989). This model was instigated by studies such as hat of Lucas (1987)[10].The purpose of the study by Imrohoroğlu (1989) was to "examine whether the magnitude of the costs of business cycles in economies with incomplete insurance differs significantly from the cost estimates found in an environment with perfect insurance". In this model:

[10] Lucas (1987) estimates that magnitude of the costs of business cycles on total consumption to be remarkably small 0.1%. And this estimation is based on a assumption that of a perfect insurance of idiosyncratic risk.

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equation 41

$$E\sum_{t=0}^{\infty}\beta^t U(c_t)$$

Where $0<\beta<1$, their subjective discount factor and c_t their consumption at period t.

Utility is:
equation 42

$$U_t=\frac{c_t^{1-\sigma}}{1-\sigma},\sigma>0$$

Now, let a_{t+1} are asset holdings at the beginning of period $t+1$, and let r be the rate of return of stored assets. Then the evolution of individual assets holdings is given as:
equation 43

$$a_{t+1}=\begin{cases}(1+r)(a_t-c_t+y)\ if\ i=e\\(1+r)(a_t-c_t+\theta y)\ if\ i=u\end{cases}$$

The transition matrix pf n is a 2×2 matrix and is given as:
equation 44

$$\mathbf{P}=\begin{bmatrix}P_{11} & P_{12}\\P_{21} & P_{22}\end{bmatrix}$$

Where in previous $\Pr\{n_{t+1}=g|n_t=g\}=p_{11}$and $\Pr\{n_{t+1}=b|n_t=b\}=p_{22}$ .The transition matrix for good times for$i$ is given as $\mathbf{P}^g$ and in bad times is $\mathbf{P}^b$ now let:
equation 45

$$\mathbf{P}^g=\begin{bmatrix}p^g_{\frac{u}{u}} & p^g_{\frac{e}{u}}\\p^g_{\frac{u}{e}} & p^g_{\frac{e}{e}}\end{bmatrix},\mathbf{P}^b=\begin{bmatrix}p^b_{\frac{u}{u}} & p^b_{\frac{e}{u}}\\p^b_{\frac{u}{e}} & p^b_{\frac{e}{e}}\end{bmatrix}$$

Where $\Pr\{i_{t+1}=u^g|i_t=e\}=p^g_{\frac{u}{e}}$.No for the computation of equilibrium in economies with imperfect insurance we have the optimality equations:
equation 46

$$V(a,s)=\max\left\{U(c)+\beta\sum_{s'}\mathbf{\Pi}(s,s')V(a',s')\right\}$$

$$V_{(k+1)(a,s)}=\max\left\{U(a,s,a')+\beta\sum_{s'}\mathbf{\Pi}(s,s')V_k(a',s')\right\}$$

In this economy also :
equation 47

$$\bar{y}=ky+(1-k)\theta y$$
$$k'=k\pi^n_{\frac{e}{e}}+(1-k)\pi^n_{\frac{e}{u}}$$

Where k is a fraction of people employed in the current period and $\bar{y}^n$ be the per capita income in the current period, where $n=g,b$ .In this model :
equation 48

$$\lambda_{t+1}(x')=\sum_{(x'=a'=f(x))}\sum_{s'}\mathbf{\Pi}(s,s')\,\lambda_t k$$

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Where $\lambda_t(x)$ is the fraction of the time individual attains at a particular state (a,s) ,and state probability $x' = (a', s')$, and $a_{t+1} = f(x)$.
The budget constraint for any agent is given as:
Equation 49

$$a_t \geq 0, a_{t+1} = \begin{cases} (1+r)(a_t - c_t + y); \wedge s = e \\ (1+r)(a_t + c_t + \theta y) \wedge s = u \end{cases}$$

For this part see Imrohoroğlu et al.(1993).This is a life cycle model and economic agents must mandatory retire at certain age j^* now if $s = e$ and $n_j = \hat{h}$ all the individual receive $w_j^e = w\varepsilon_j\hat{h}$ where w, r are the wage rate and interest rate respectively, and ε_j denotes efficiency index[11], and $\hat{h}$ are the hours worked by the agent age j individual. Now, if $s = u$ then $n_j = 0$ or in the unemployed state employment is zero. After the mandatory retirement at age j^* the disposable income of the retired agent equals benefits that are presented as:
equation 50

$$b_j = \begin{cases} 0, j = 1,2, \dots, j^* - 1 \\ \theta \dfrac{\sum_{i=1}^{j^*-1} w_i^e}{j^* - 1}, j = j^*, j^* + 1, \dots, J \end{cases}$$

The only role for the government in these models is to administer the unemployment insurance and social security programs. Individual disposable income through lifetime is given as:
equation 51

$$q_j = \begin{cases} (1-\tau_s - \tau_u)w_j^e\ , j = 1,2, \dots, j^* - 1, \wedge s = e \\ w_j^u, j = 1,2, \dots, j^* - 1, \wedge s = u \\ b_j, j = 1,2, \dots, j^* - 1, \dots \dots, J \end{cases}$$

In previous θ represents fraction of some income. In previous expressions social security system is self-financing.

equation 52

$$\tau_s = \frac{\sum_{j=j^*}^{J} \sum_a \mu_j \lambda_j(a,s) b_j}{\sum_{j=1}^{j^*-1} \sum_a \mu_j \lambda_j(a, s = e) w\varepsilon_j \hat{h}} = \frac{b \sum_{j=j^*}^{J^*-1} \mu_j}{w\hat{h} \sum_{j=1}^{j^*-1} \mu_j \lambda_j(a,s)\varepsilon_j}$$

And the unemployment insurance benefit program is self-financing also:

$$\tau_u = \frac{\sum_{j=j^*}^{J} \sum_a \mu_j \lambda_j(a, s = u) \xi w \hat{h}}{\sum_{j=1}^{j^*-1} \sum_a \mu_j \lambda_j(a, s = e) w\varepsilon_j \hat{h}} = \frac{\xi \sum_{j=1}^{J} \mu_j}{w\hat{h} \sum_{j=1}^{j^*-1} \mu_j \lambda_j(a,s)\varepsilon_j}$$

The lump-sum distribution of accidental bequests is determined by :
equation 53

$$\mathcal{T}^* = \sum_j \sum_a \sum_s \mu_j \lambda_j(a,s)(1 - \psi_{j+1}) A_j(a,s)$$

Backward recursion follows:
equation 54

$$V_j(\tilde{x}_j) = \max_{\{c_j, a_j\}} \left\{ U(c_j, c_{j-1} + \beta \psi_{j+1} \sum_{s'} \Pi(s', s) V_{j+1}(\tilde{x}_{j+1} \right\}$$

[11] The number of units of work effort into which one unit can be turned, of an age j agent.

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Aiyagari economy with idiosyncratic Brownian motion and random deaths

In this economy:
equation 55

$$r_t = \frac{\alpha Y_t}{K_t} - \delta_K; w_t = \frac{(1-\alpha)Y_t}{L_t}; Y = AK^{\alpha}L^{1-\alpha}$$

Utility is given as :
equation 56

$$U_0 = \mathbb{E}_0\left[\int_0^{\infty} e^{(\rho+\eta)t}u(c_t)dt\right]$$

In this economy there is no intergeneration altruism. Individuals buy annuity in perfectly competitive insurance market that pays them a flow of ηa_t in exchange of taking control of all the assets when agent dies, see Galo,Moll (2018).Agents assets evolve according to:
equation 57

$$da_t = [w_t z_t + (r_t + \eta)a_t - c_t]dt = s(a_t, z_t, w_t, r_t, c_t)dt$$

Labor units z_t provided by the agent follow:
equation 58

$$dz_t = \theta(\hat{z} - z_t)dt + \sigma dB_t$$

$a_t \geq -\Phi$ is natural borrowing limit:
equation 59

$$-\Phi > -\underline{z}\int_t^{\infty} e^{\int_t^s r_\tau d\tau}\ w_s ds, \forall t \geq 0$$

The Hamilton-Jacobi-Bellman equation for individual problem is given as:
equation 60

$$(\rho+\eta)V = \max_{c\geq 0}\frac{c^{1-\gamma}}{1-\gamma} + s(a, z, w(t), r(t), c)\ \frac{\partial V}{\partial a} + \theta(\hat{z} - z)\frac{\partial V}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 V}{\partial z^2} + \frac{\partial V}{\partial t}$$

The state of the economy is the joint density of wealth and labor $g(t, a, z)$:
equation 61

$$\frac{\partial g}{\partial t} = -\frac{\partial}{\partial a}(s(a, z, w(t), r(t), c)g) - \frac{\partial}{\partial z}(\theta(\hat{z} - z)g) + \frac{1}{2}\frac{\partial^2}{\partial z^2}(\sigma^2 g) - \eta g + \eta\delta_0$$

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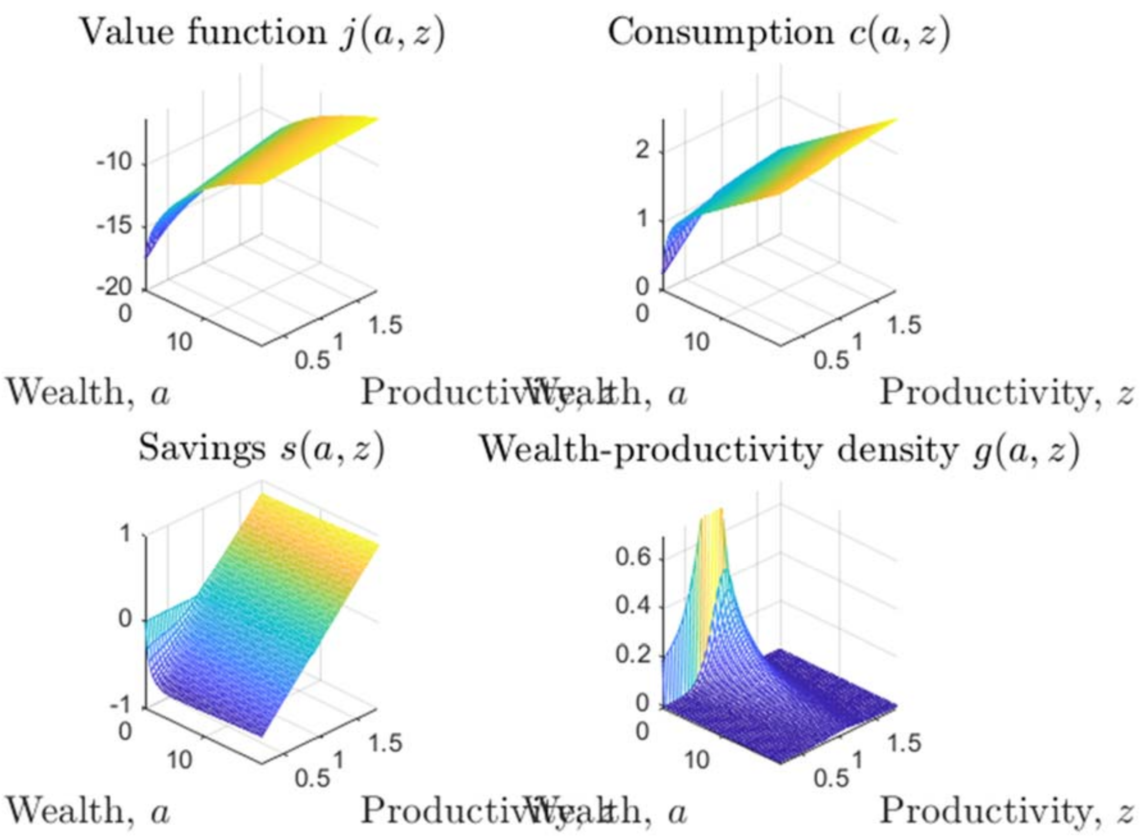
Figure 3 Value function, Consumption function ,Savings, and Wealth and productivity density in Aiyagari economy with idiosyncratic Brownian motion and random deaths



Source: Author's own calculations based on a code for paper by Galo,Moll (2018).

Social planner value function is given as[12]:
equation 62

$$\rho+\eta)j=\max_{c\geq 0}\frac{c^{1-\gamma}}{1-\gamma}+\lambda\big(a-K(t)\big)+(w(t)z+(r(t)+\eta)a-c)\frac{\partial j}{\partial a}+\theta(\hat{z}-z)\frac{\partial j}{\partial z}+\frac{\sigma^2}{2}\frac{\partial^2 j}{\partial z^2}+\frac{\partial j}{\partial t}$$

Lagrange multiplier is given as:
equation 63

$$\lambda(t)=-\frac{\alpha(1-\alpha)}{K(t)^{2-\alpha}}\int\int_{\underline{z}}^{\overline{z}}\frac{\partial j}{\partial a}(a-K(t)z)g(t,a,z)dzda$$

[12] Where discounted aggregate utility is given as: $J\big(g(0,\cdot)\big)=\max_{c(\cdot)\in\mathcal{C}(t,a,z)}\int_0^\infty e^{\rho t}\int u(c)g(t,a,z)dadzdt$

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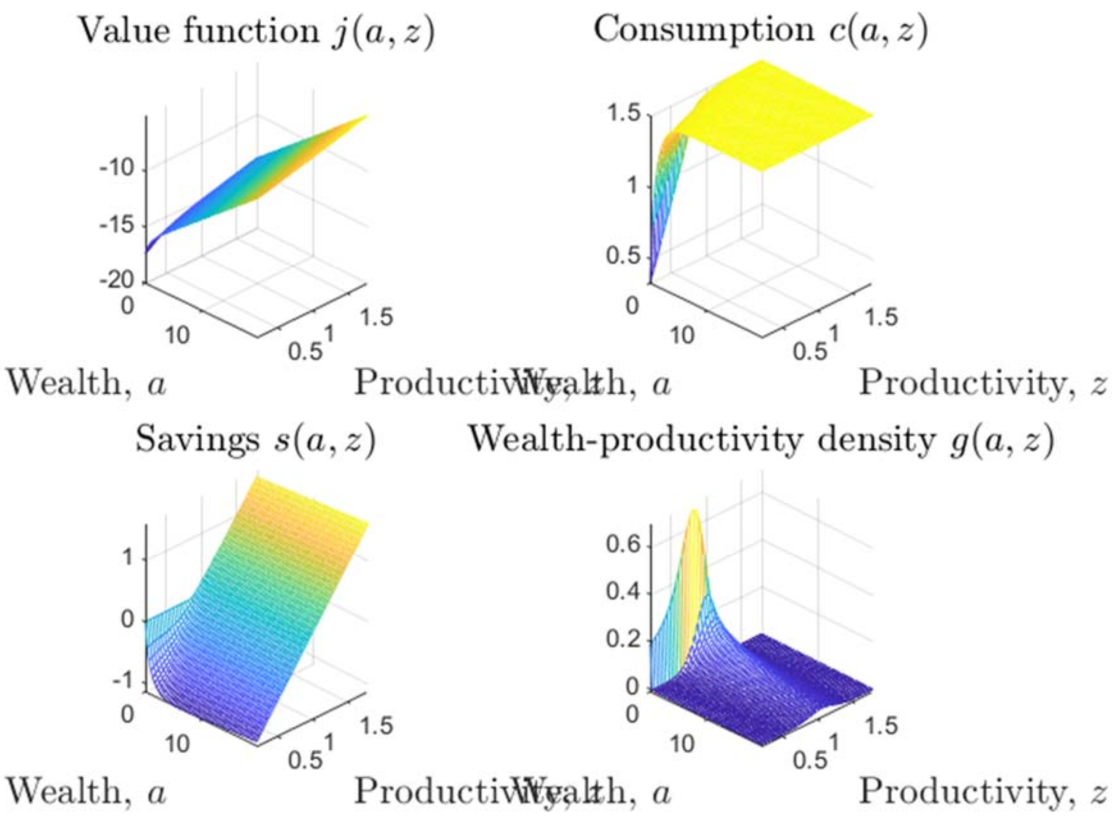
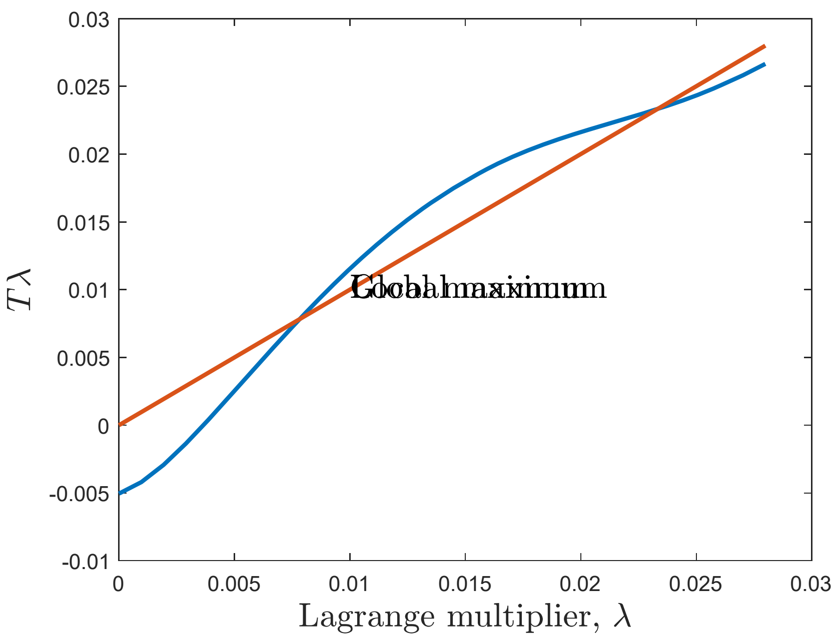
Figure 4 Value function, Consumption function ,Savings, and Wealth and productivity density in Aiyagari economy with idiosyncratic Brownian motion and random deaths



Source :Author's own calculations based on a code for paper by [Galo,Moll (2018)](#).

Figure 5 Lagrange multiplier in this economy



Source: Author's own calculations based on a code for paper by [Galo,Moll (2018)](#)

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Human Capital Model

The household solves the following problem:

equation 64

$$\max_{(c_t,s_t)_{t\geq 0}} \int_0^\infty e^{-\rho t}\, u(c_t)dt\; s.t.$$
$$\dot{a}_t = ra_t + wh_t(1-s_t) - c_t$$
$$\dot{h}_t = \theta(s_t h_t)^\alpha - \delta h_t$$
$$a_t \geq a$$

I previous expression a_t is wealth (assets),h_t is human capital,c_t is consumption,s_t is the time units spent in education. Interest rate is denoted by r and w are wages and δ is the human capital depreciaton rate and θ and α are the parameters of human capital PF (production function). There $\theta > 0\,; \alpha \in (0,1)$.There is a lower bound on wealth denoted by $\underline{a}$.Utility is CRRA with σ parameter:

equation 65

$$u(c) = \begin{cases} \log(c)\,, if\ \sigma = 1 \\ \dfrac{c^{1-\sigma}}{1-\sigma}\ if\ \sigma \neq 1 \end{cases}$$

The HJB equation for the above problem is given as:

equation 66

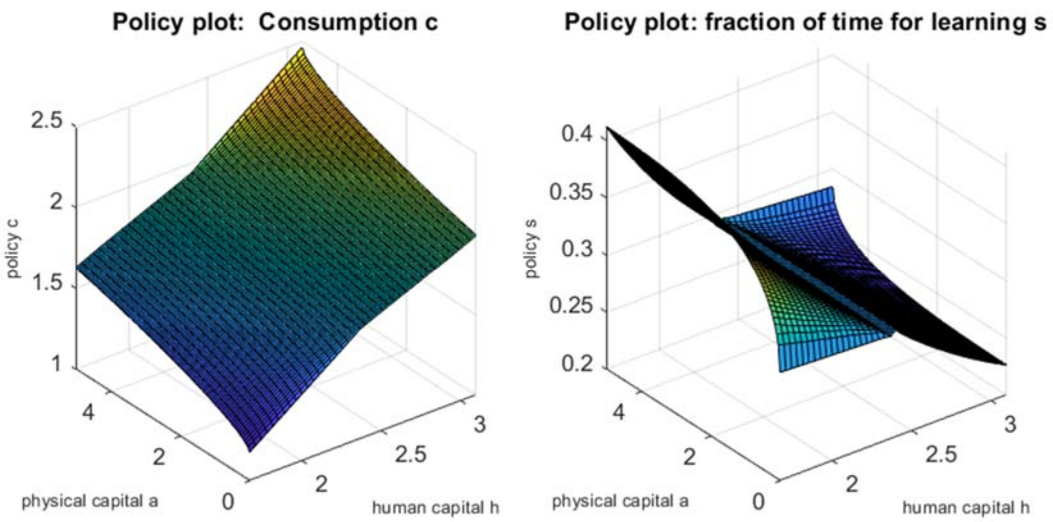
$$\rho V(a,h) = \max_{c,s} u(c) + V_a(a,h)[ra + wh(1-s) - c]\ + V_h(a,h)[\theta(sh)^\alpha - \delta h]$$

And the FOC's follow:

equation 67

$$u'(c) = V_a(a,h)$$
$$V_a(a,h)wh = V_h(a,h)[(sh)^\alpha - \delta h]$$

Figure 6 policy, physical capital and human capital and fraction of time for learning



Source: Author's own calculations based on a code available at: https://benjaminmoll.com/wp-content/uploads/2020/03/human_capital.m

Solving the incomplete markets model in general equilibrium.

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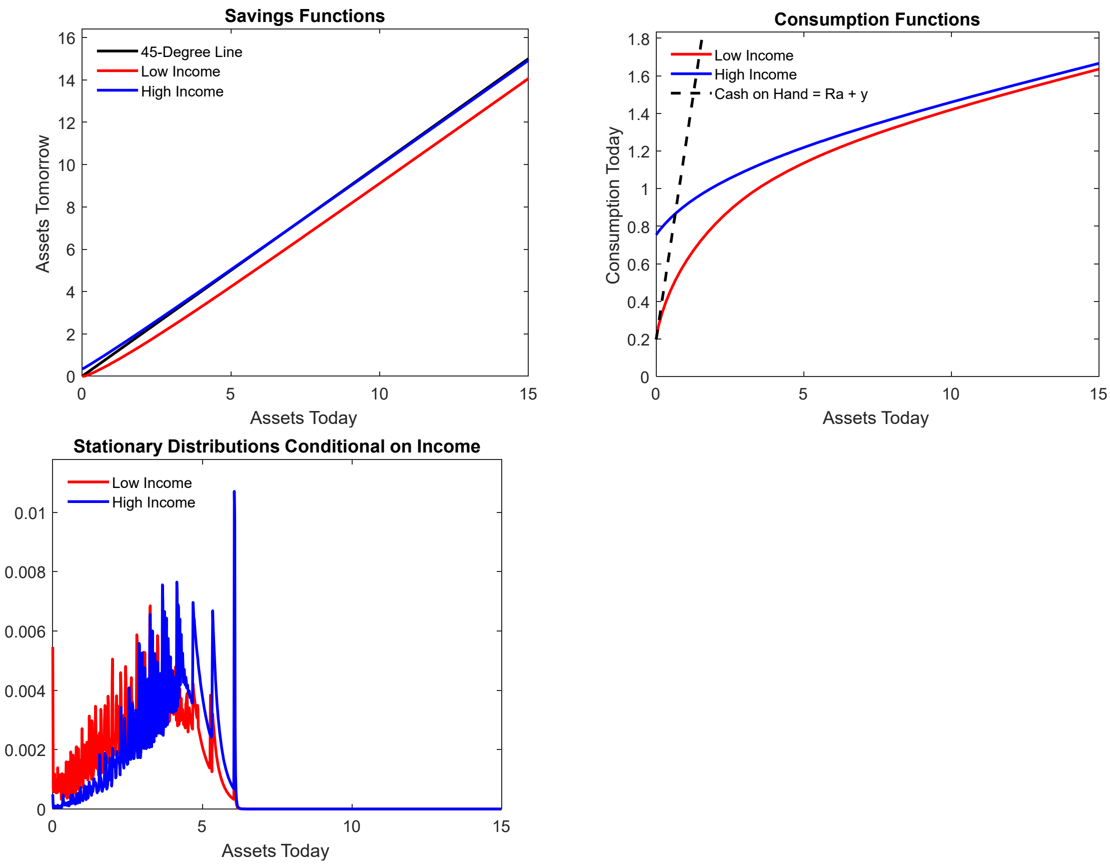
Parameters are :
β = 0.96; %HH subjective discount factor
α = 0.33; %capital share
δ = 0.1; %depreciation rate
γ = 2; %CRRA risk aversion
e_h = 1.0891; %high labor efficiency level
e_l = 0.1980; %low labor efficiency level
ph_h = 0.95; %high efficiency persistence
pl_h = 0.45; %low efficiency (inverse) persistence
$\bar{a}$ = 0; %lower bound on assets

Figure 7 Incomplete markets in General equilibrium : Savings, consumption, Stationary distribution conditional on income



Source: Authors' own calculations based on a code available at; https://github.com/stpica/EC702-Fall-TA

Solving the incomplete markets model in partial equilibrium.

Now we are presenting incomplete markets model in partial equilibrium:

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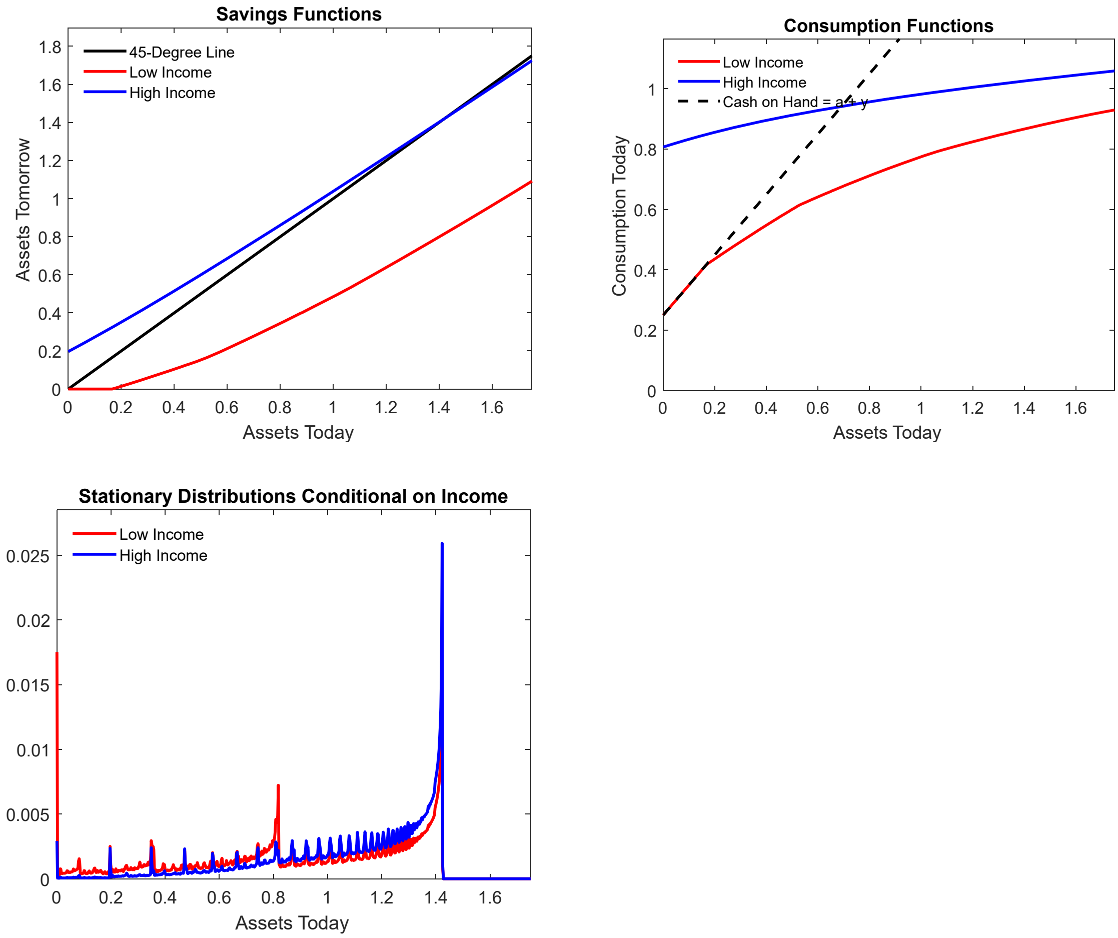
Figure 8 Incomplete markets in partial equilibrium : Savings, consumption, Stationary distribution conditional on income



Source: Authors' own calculations based on a code available at; https://github.com/stpica/EC702-Fall-TA

Parameters in this model are given as:

R = 1.02; %gross real interest rate
β = 1/1.04; %HH subjective discount rate
γ = 2; %CRRA parameter
y_l = 0.25; %low income realization
y_h = 1.0; %high income realization
pl_h = 0.7; %P(y' = yh | y = yl)
ph_h = 0.95; %P(y' = yh | y = yh)
$\bar{a}$ = 0; %borrowing constradense0nt a' >= abar

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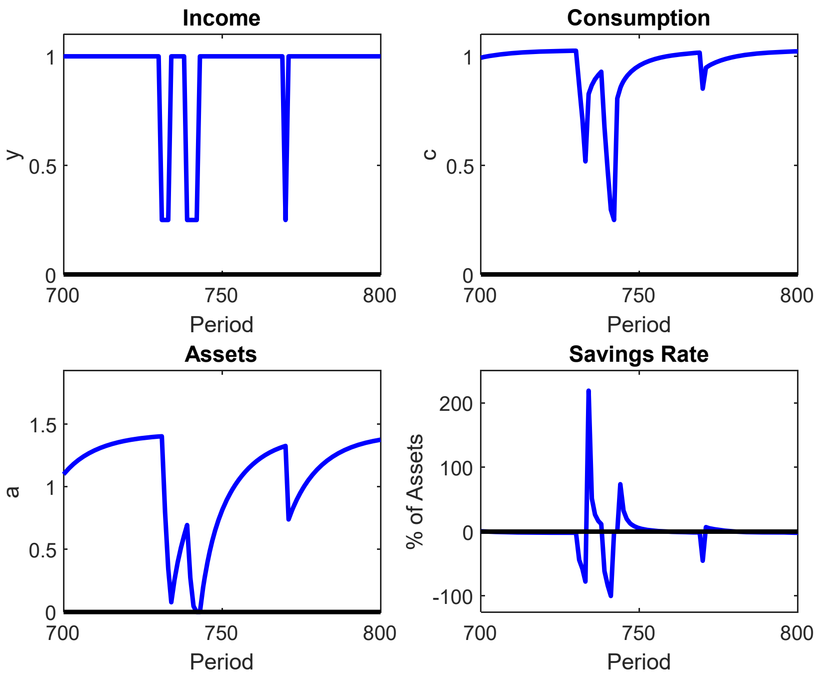
Figure 9 income ,consumption,assets,savings rate in Incomplete markets in partial equilibrium



Source: Authors' own calculations based on a code available at; https://github.com/stpica/EC702-Fall-TA

Conclusion

In the Bewley economy with assets in positive supply, interest rate is positive with positive asset supply, and there is a difference between assets of next and assets of current period,In Aiyagari model with idiosyncratic Brownian motion and random deaths the constrained efficient allocation displays under accumulation of capital in the competitive equilibrium which means that there is more capital than the first best. Competitive equilibrium displays capital overaccumulation because of precautionary savings. In the human capital model as higher is the fraction of time in learning, the lower is physical capital, and higher is policy function. In the incomplete markets in general equilibrium cash-on hand $R_a + y$ is more in consumption with lower assets, this applies even more so in partial equilibrium model. And in the period with lowest: consumption, income and assets incomplete markets in partial equilibrium model predicts highest savings rate.

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