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OPTIMAL TAXATION IN BEN-PORATH MODEL

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Abstract

This paper is about Ben-Porath model of human capital investments and non-trivial labor supply decisions throughout the lifetime of the individual. In Ben-Porath model without taxation: The time allocation condition ensures optimal trade-offs between leisure, work, and investment in human capital. Shadow price of human capital is increasing over time. In Ben-Porath-Huggett model Mirrlees taxation is the best option when skill is private knowledge, Ramsey taxation requires subsidies to prevent human capital stagnation. Pareto taxation is the second-best solution when redistribution is a goal. Ramsey taxation yields highest government revenues, Mirrlees and Pareto taxes yield highest utility.

Keywords: human capital investments, Mirrlees, Pareto, Ramsey, incomplete markets

JEL codes: J24, H21

Introduction

The process of human capital acquisition has been studied in an economic literature, starting with Becker(1964), Ben-Porath (1967), and Heckman (1976). This paper will focus on Ben-Porath (1967) model where the principal analytical assumption in this paper is that human capital operates like Harrod neutral[1] endogenous technical progress in augmenting time. On the other hand optimal taxation literature since Mirrlees(1971) and later developed by Saez,E.(2001), Kocherlakota (2005), Albanesi and Sleet (2006), Golosov, Tsyvinski, and Werning (2006), Battaglini and Coate (2008) , Farhi,Werning (2013), Golosov, Troshkin, and Tsyvinski (2013) typically assumes exogenous ability, thus abstracting from endogenous human capital investments, see Stantcheva (2017). Bovenberg and Jacobs (2005) and Stantcheva (2017) have extended the optimal taxation jointly with educational policies that considered educational decisions. Since Mirrlees(1971),optimal tax theory mostly has worked with a static model that treats heterogeneity of economic agents and uncertainty symmetrically, since redistribution can be seen as insurance behind the veil of ignorance, see Farhi,Werning (2013).Conventional wisdom in the human capital literature (at least by Ben-Porath (1967),suggests that income taxes do encourage human capital accumulation. Heckman (1976) challenged previous view stating that income tax depresses interest rate and

[1] Harrod-neutral technical change (also called labor-augmenting technical change) refers to a form of technological progress that increases the productivity of labor without affecting the marginal productivity of capital.Here $Y = K^{\alpha}(A_L L)^{1-\alpha}$ where $A_L(t) = A_{L0}e^{gt}$,where g is the growth rate of labor augmenting technology. The human capital accumulation equation now becomes:$\dot{h}(t) = A_L(t) \cdot f(e(t), h(t))$ which implies that human capital grows faster over time due to exogenous labor-augmenting technology growth.

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lowers the cost of borrowing, and because forgone earnings[2] "cost of investment may be written off when incurred, higher tax rates encourage human capital investments. On contrary a study by Trostel (1993) the study found a significant negative effect of proportional income taxation on human capital. Previous research has indicated that human capital is an important component of national wealth (see Davies,J. Whalley,J.(1991) study that suggested that stock of human capital is as three times higher than a stock of physical capital). Solow's (1956) seminal paper suggested that differences in the rates at which capital is accumulated could account for differences in output per capita. In Lucas (1988), human capital disparities were given a central role in the analysis of growth and development[3].According to Stantcheva (2017) there is two way interaction between human capital and the tax system. First, investments in human capital are influenced by tax policy which was previously recognized by Schultz(1961)[4].Taxation on labor income discourages investment in human capital by capturing part of the return to human capital but it also helps reducing the earning risk by insuring against it, thereby encouraging investment in risky human capital. Capital taxes are affecting the choice between physical and human capital. Either way investment inhuman capital affect directly tax base. Consumption taxes have an ambiguous effect: Can reduce investment if education is taxed but may encourage savings[5].Ramsey taxes effect on human capital accumulation is likely favorable[6].Also, according to Reis (2019) result which also is a common sense in accordance with Atkinson-Stiglitz theorem (see Atkinson,Stglitz (1976)) : In a Ramsey model of optimal taxation, if human capital investment can be observed separately from consumption, it is optimal not to distort human or physical capital accumulation in the long run, and only labor income taxes should be used. Jones et al. (1997) and Judd (1999) showed that "if the government can distinguish between pure consumption and human capital investment, then it can use this information to offset the distortion that labour taxation causes on human capital accumulation" see Reis(2019).Though Reis (2019) article derived that government cannot distinguish between final consumption and expenditures on human capital. So, the tax on consumption must be the same as the tax on human capital, and human capital accumulation will in general be distorted in the long run. The effect of having unobservable investment in human capital in heterogeneous agents' models has been discussed by Kapička (2006) and Kapička (2015).First paper shows that Kapička (2006) shows that the optimal income tax is significantly reduced when there is endogenous unobservable human capital[7].Kapička (2015) proves that if both ability and human capital

[2] Foregone earnings are potential earnings that could've been achieved but are absent due to charged fees, expenses, or lost time

[3] Klenow and Rodríguez-Clare (1997); Hall and Jones (1999); Parente and Prescott (2000); and Bils and Klenow (2000) argue that most of the cross-country differences in output per worker are not driven by differences in human capital (or physical capital); rather they are due to differences in a residual, total factor productivity (TFP),see also Manuelli, R. E., & Seshadri, A. (2014).

[4] Our tax laws everywhere discriminate against human capital. Although the stock of such capital has become large and even though it is obvious that human capital, like other forms of reproducible capital, depreciates, becomes obsolete and entails maintenance, our tax laws are all but blind on these matters,see Schultz (1961).

[5] Substitution Effect: Since a consumption tax discourages current consumption, households might save more. If the return to savings includes human capital investment (e.g., education, training), consumption taxes could increase human capital accumulation.Income Effect: Higher consumption taxes reduce disposable income, making education and training more expensive in real terms, which may reduce human capital investment.Intertemporal Trade-offs: If future consumption is also taxed, individuals may shift their income toward untaxed or lower-taxed investments, possibly reducing incentives to invest in human capital if wages are highly taxed.

[6] Minimizes distortions, often favoring lower taxes on human capital. Ramsey taxation might prefer higher capital or consumption taxes over direct human capital taxation, assuming labor supply and education investment are more responsive to tax policy.If labor and capital are complements in production, taxing capital (or consumption) too heavily can still reduce human capital investment indirectly

[7] Labor tax also taxes human capital this makes tax more distortionary thus reducing optimal tax.

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investment are non-observable, the optimal tax rates decrease with age, contrary to what happens if human capital is observable. Jacobs and Bovenberg (2010) discuss how a positive tax on capital income may alleviate the distortions of the labor tax on human capital accumulation in a two-period life-cycle model. Stantcheva (2015), discuss the features of the optimal subsidy for human capital expenses when these are observable. In the dynamic life cycle model by Stantcheva (2015),,there is an additional interaction between contemporaneous training and future labor supply, which is the mirror image of the interaction with the contemporaneous labor supply. This paper will draw on the theory of human capital acquisition provided in Ben-Porath (1967), and will investigate the effects of taxes human capital accumulation.

Ben-Porath model

The Ben-Porath (1967) model is a structural model of investment on the job. The model is setup as follows:Finite lived to time T,continuous time,interest rate r,earnings $E(t)$.So, people make human capital investment decisions to maximize the present value of income

equation 1

$$\int_0^T e^{-rt}E(t)dt$$

We assume that earnings take the form:

equation 2

$$E(t) = H(t)\,[1 - I(t)] - D(t)$$

$I(t)$: time spent investing in human capital $H(t)$: Human capital itself $D(t)$: Direct costs of human capital investment. Thus, the present value of earnings can be written as:

equation 3

$$\int_0^T e^{-rt}(H(t)[1-I(t)]-D(t))$$

The human capital production function is defined as

equation 4

$$\dot{H} = A(IH)^{\alpha}\ D^{\beta} - \sigma H$$

where σ is the rate of depreciation in human capital. The one other thing we need to solve this model is the initial level of human capital $H(0)$. Now we can write down the Hamiltonian as

equation 5

$$\mathcal{H} = e^{-rt}\left(H(t)[1 - I(t)] - D(t)\right) + \mu(t)\left[A(IH)^{\alpha}\ D^{\beta} - \sigma H\right]$$

FOCs:

equation 6

$$I: e^{-rt}H = \mu\alpha A I^{\alpha-1}H^{\alpha}D^{\beta}$$
$$D: e^{-rt} = \mu\beta A(IH)^{\alpha}D^{\beta-1}$$
$$\dot{\mu} = -\frac{\partial\mathcal{H}}{\partial H} = -e^{-rt}(1-I) - \mu\left[\alpha A I^{\alpha}H^{\alpha-1}D^{\beta} - \sigma\right]$$

Take the ratio of the first two first order conditions:

equation 7

$$H = \frac{\mu\alpha A I^{\alpha-1}H^{\alpha}D^{\beta}}{\mu\beta A(IH)^{\alpha}D^{\beta-1}} = \frac{\alpha D}{\beta I} \to D = \frac{\beta}{\alpha}IH$$

Since direct costs of investment D are just a multiple of time costs IH, the distinction between the two is not interesting (of course with borrowing constraints this would no longer be true).That is we can redefine the model so that:

equation 8

$$I^{*} = \left(1 + \frac{\beta}{\alpha}\right) I;\ \alpha^{*} = \alpha + \beta$$

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$$A^* = A\left(\frac{\beta}{\alpha}\right)^{\beta}\left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha+\beta}$$

With this notation we can see that:

equation 9

$$A^*(I^*H)^{a^*} = A\left(\frac{\beta}{\alpha}\right)^{\beta}\left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha+\beta}\left(\left(1+\frac{\beta}{\alpha}\right)IH\right)^{\alpha+\beta}$$
$$= A\left(\frac{\beta}{\alpha}\right)^{\beta}(IH)^{\alpha+\beta}$$
$$= A(IH)^{\alpha}\left(\frac{\beta}{\alpha}IH\right)^{\beta} = A(IH)^{\alpha}(D)^{\beta}$$

Thus there is no need to worry about D Lets abstract from it by using the redefined model (without the * notation). Then we have first order conditions:

equation 10

$$e^{-rt} = \mu\alpha AI^{\alpha-1}H^{\alpha-1}$$
$$\dot{\mu} = -e^{-rt}(1-I) - \mu\left[\alpha AI^{\alpha}H^{\alpha-1}D^{\beta} - \sigma\right] = -e^{-rt} + \sigma\mu + I\left[e^{-rt} - \mu\alpha AI^{\alpha-1}H^{\alpha-1}\right]$$
$$= -e^{-rt} + \sigma\mu$$

Define:

equation 11

$$g(t) = e^{rt}\mu$$

Then:

equation 12

$$\frac{\partial g}{\partial t} = re^{rt}\mu + e^{rt}\mu = re^{rt}\mu - 1 + e^{rt}\sigma\mu = (r+\sigma)g - 1$$

We want to solve for this differential equation, but we don't know $g(0)$. However, we do know that $\mu(T) = 0$ which implies that $g(T) = 0$ .This is straight forward to solve, it yields:$g(T) = 0$;$g(t)$ is strictly decreasing with t.From the first order condition for investment:

equation 13

$$I(t)H(t) = \left(\alpha Ag(t)\right)^{\frac{1}{1-\alpha}} \Rightarrow I(t) = \frac{\left(\alpha Ag(t)\right)^{\frac{1}{1-\alpha}}}{H(t)}$$

investment IH, is decreasing with t :IH doesn't depend on $H(0)$ (Ben-Porath neutrality), Investment is decreasing with H What happens to $H(t)$ depends on investment versus depreciation. It makes sense to impose that Investment time is bounded from above by 1

Agents' Objective

Agents maximize their lifetime utility:

equation 14

$$U = \int_0^T e^{-\rho t}\, u\big(c(t), l(t)\big)dt$$

$c(t)$ consumption at time t,$l(t)$: leisure at time t,$u(c,l)$ instantaneous utility function $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} + v(l)$,$\rho$: subjective discount rate ,T; time horizon .

Human Capital Dynamics

Human capital $h(t)$ evolves according to:

equation 15

$$\dot{h}(t)f\big(h(t), s(t)\big)$$

$\dot{h}(t)$ is the rate of human capital accumulation;$f(h(s)$ production function for human capital;$s(t)$ time spent investing in human capital ;$h(t)$ current level of human capital.
Agents face a budget constraint:

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$$c(t) \le w(t)h(t)\big(1 - s(t) - l(t)\big)$$

Where: $w(t)$ is wage rate per unit of human capital

Time Constraint

The time allocation constraint is:

equation 16

$$s(t) + l(t) + n(t) = 1$$

Where $n(t)$ is time spent working.

2. Formulating the Optimization Problem

The problem is to choose $c(t), l(t), n(t)$;Human Capital Dynamics $\dot{h}(t)f\big(h(t), s(t)\big)$; Budget constraint:$c(t) = w(t)h(t)n(t)$;Time constraint: $s(t) + l(t) + n(t) = 1$;Initial human capital: $h(0) = h_0$.

Lagrangian Formulation

The Lagrangian is:

equation 17

$$\mathcal{L} = \int_0^T e^{-\rho t} u\big(c(t), l(t)\big) dt + \int_0^T \lambda(t)\big[f h(t), s(t) - \dot{h}(t)\big] dt$$

Where $\lambda(t)$ is a costate variable (shadow price of human capital).

3. First-Order Conditions

Differentiating the Lagrangian with respect to the control variables $c(t), s(t), l(t)$ and state variable $h(t)$ gives the following FOCs : Consumption Euler equation: $\frac{\partial U}{\partial c} = \lambda(t)h(t)w(t)$;Leisure :$\frac{\partial u}{\partial l} = \lambda(t)h(t)w(t)$;Human capital investment:$\lambda(t)\frac{\partial f}{\partial s} = w(t)h(t)\lambda(t)$;Costate Equation:$\dot{\lambda}(t) = \rho\lambda(t) - \lambda(t)\frac{\partial f}{\partial h}$;Transversality Condition:$\lambda(T)h(T) = 0$.

Demand price for human capital in Ben-Porath (1967) is given as:

equation 18

$$P_t = a_0 \int_t^T e^{-(r+\delta)v} dv = \frac{a_0}{r+\delta}\big[1 - e^{-(r+\delta)(T-t)}\big]$$

Where $a_0 = \frac{Y_t}{K_t}$, here K_t is human capital,Y_t is maximum services of human capital the individual can offer at market valued by rental a_0.Discounted shadow price of human capital q is given as:

equation 19

$$\dot{q} = -\frac{\partial H}{\partial K} = -e^{-rt}(1-s)a_0 - q\left(\beta_1 \frac{Q}{K} - \delta\right)$$

In previous s_t is the fraction of the available stock of human capital allocated to the production of human capital,δ is exogenously given rate of deterioration,Q is the flow of human capital produced and:

equation 20

$$Q = \beta_0(s_t, K_t)^{\beta_1} D_t^{\beta_2}$$

Where $\beta_1, \beta_2 > 0, \beta_1 + \beta_2 < 1$,D is the quantity of purchased input. The objective of each individual is to maximize the present value of his disposable earnings:

equation 21

$$W = \int_t^T e^{-rv}[a_o K(v) - I(v)] dv$$

$I(v)$ are the investment costs. And investment costs with two components are given as:

equation 22

$$I_t = a_0 s_t K_t + P_d D_t$$

P_d is the price of purchased inputs,D_t is the quantity of purchased inputs.

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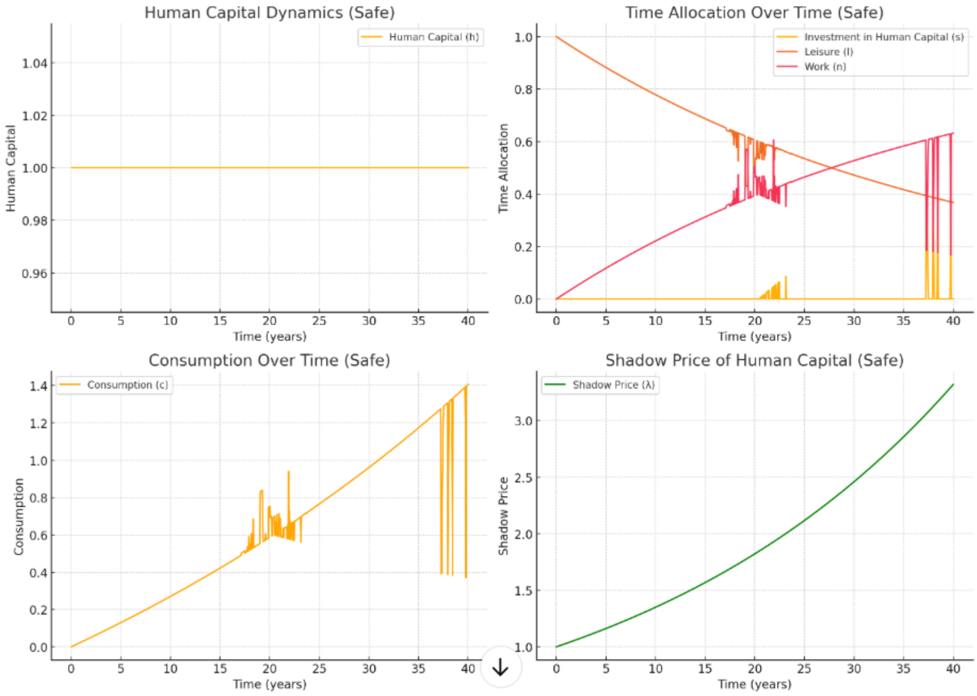

Figure 1Ben-Porath model source:Author's own calculation

From previous plots: The Euler equation balances the marginal utility of consumption with the shadow price of income derived from human capital. The costate equation describes how the value of human capital evolves over time, incorporating the discount rate and productivity. The time allocation condition ensures optimal trade-offs between leisure, work, and investment in human capital. Shadow price of human capital is increasing[8], it means that the economic value of acquiring additional skills, education, or experience grows as time progresses.

Separation theorem

Features of this theorem are:

- ✓ Partial equilibrium schooling decisions.
- ✓ Continuous time.
- ✓ Schooling decision of a single individual facing exogenously given prices for
- ✓ human capital.
- ✓ Perfect capital markets.

Theorem 1   Separation theorem: with perfect capital markets, schooling decisions will maximize the net present discounted value of wages of the individual.

Suppose $u(\cdot)$ is strictly increasing. Then the sequence $\left[\hat{c}(t), \hat{s}(t), \hat{h}(t)_{t=0}^{T}\right]$ is a solution t the maximization of

equation 23

$$max \int_0^T \exp(-(\rho + v)t) u\big(c(t)\big) dt$$

Where discount rate $\rho > 0$ and a constant flow of death $v \geq 0$ $u(c)$ is instantaneous utility and T is planning horizon $T = \infty$ is allowed. Capital markets are perfect. Previous maximization is subject to :

Evolution of human capital:

equation 24

$$\dot{h} = G\big(t, h(t), s(t)\big)$$

[8] It means that the economic value of acquiring additional skills, education, or experience grows as time progresses.

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And $s(t) \in [0,1]$ so that only full-time schooling would be possible. Exogenous sequence of wage per unit of human capital given by $[w(t)]_{t=0}^{T}$, so that his labor earnings at time t are

equation 25

$$W(t) = w(t)[1 - s(t)][h(t) + \omega(t)]$$

Here $1 - s(t)$ is the fraction of work time and $\omega(t)$ is non-human capital labor, with $[w(t)]_{t=0}^{T}$. Perfect capital markets: borrowing and lending at constant interest rate equal to r
So the life time budget constraint is given as:

equation 26

$$\int_0^T \exp(-rt)c(t)dt \leq \int_0^T \exp(-rt)w(t)[1 - s(t)]\,[h(t) + \omega(t)]dt$$

If and only if $\left[\hat{s}(t), \hat{h}(t)\right]_{t=0}^{T}$ maximizes:

equation 27

$$\int_0^T \exp(-rt)w(t)[1 - s(t)][h(t) + \omega(t)]dt$$

Subject to $\dot{h} = G\big(t, h(t), s(t)\big)$ and $s(t) \in \mathcal{S}(t) \subset [0,1]$ and $[\hat{c}(t)]_{t=0}^{T}$ maximizes $max \int_0^T \exp(-(\rho + v)t)u\big(c(t)\big)dt$ subject to $\int_0^T \exp(-rt)c(t)dt \leq \int_0^T \exp(-rt)w(t)[1 - s(t)]\,[h(t) + \omega(t)]dt$ given $\left[\hat{s}(t), \hat{h}(t)\right]_{t=0}^{T}$. That is, human capital accumulation and supply decisions can be separated from consumption decisions.

Proof:

The consumption problem is:$max \int_0^T e^{-(\rho+v)t}u\big(c(t)\big)dt$ s.t.

inequality 1

$$\int_0^T e^{-rt}c(t)dt \leq \int_0^T e^{-rt}w(t)(1 - s(t)[h(T) + w(t)]dt$$

Human capital and labor supply problem:

equation 28

$$\max \int_0^T e^{-rt}w(t)(1 - s(t)[h(T) + w(t)]dt$$

subject to the human capital accumulation constraint:

equation 29

$$\dot{h} = G\big(t, h(t), s(t)\big), s(t) \in S(t) \subset [0,1]$$

Hamiltonian for Human Capital and Labor Supply
Define the Hamiltonian for the human capital accumulation problem:

equation 30

$$\mathcal{H}_h = e^{-rt}w(t)(1 - s(t)[h + w] + \lambda G(t, h, s)$$

where λ is the costate variable associated with human capital.
The necessary first-order conditions (FOCs) for optimal human capital accumulation and labor supply are:
Optimality with respect to$s$:

equation 31

$$e^{-rt}w(t)[h + w] + \lambda\frac{\partial G}{\partial s} = 0$$

Rearranging:

equation 32

$$\lambda\frac{\partial G}{\partial s} = e^{-rt}w(t)[h + w]$$

Costate equation for human capital:

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equation 33

$$\dot{\lambda} = r\lambda - e^{-rt}w(t)(1-s)$$

These equations determine the optimal path of human capital $h(t)$ and labor supply $s(t)$ independently of consumption decisions.

Consumption Optimization

For consumption, we use the budget constraint:

equation 34

$$\int_0^T e^{-rt}c(t)dt = \int_0^T e^{-rt}w(t)(1-s(t)[h(T)+w(t)]dt$$

Define the Lagrangian for the consumption problem with a multiplier μ:

equation 35

$$\mathcal{L}_c = \int_0^T e^{-(\rho+v)t}uc(t)dt + \mu\left(\int_0^T e^{-rt}w(t)(1-s(t)[h(T)+w(t)]dt - \int_0^T e^{-rt}cdt\right)$$

FOC for c is:

equation 36

$$e^{-(\rho+v)t}u'c - \mu e^{-rt} = 0$$

Rearranging :

equation 37

$$u'c = \mu e^{(r-\rho-v)t}$$

This equation determines optimal consumption $c(t)$ independently of human capital and labor supply.■

Separation theorem II

Setup:

equation 38

$$\max \int_0^T e^{-(\rho+v)t}u\big(c(t)\big)dt$$

s.t.

evolution of human capital

equation 39

$$\dot{h}(t) = G(t, h(t), s, (t))$$

$h(t)$ is human capital,$s(t) \in [0,1]$ is the fraction of time allocated to schooling, ad $G(\cdot)$ is a human capital accumulation function.

Lifetime budget constraint is :

equation 40

$$\int_0^T \exp(-rt)c(t)dt \leq \int_0^T \exp(-rt)w(t)[1-s(t)]\,[h(t)+\omega(t)]dt$$

Sub-problem of human capital :

equation 41

$$\max \int_0^T e^{-rt}w(t)[1-s(t)][h(t)+\omega(t)]dt$$

s.t.

evolution of human capital:

equation 42

$$\dot{h} = G\big(t, h(t), s(t)\big)$$
$$s(t) \in [0,1]$$

Sub-problem for consumption

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equation 43

$$\max \int_0^T \int_0^T e^{-(\rho+v)t} u(c(t))dt$$

s.t. lifetime budget constraint:

equation 44

$$\int_0^T \mathrm{e}^{-rt}\ c(t)dt \leq \int_0^T \mathrm{e}^{-rt}\, w(t)[1-s(t)]\,[h(t)+\omega(t)]dt$$

Where $s(t)$ and $h(t)$ are given by the evolution of human capital maximization problem.

Proof:

Lagrangian for the combined problem is :

equation 45

$$\mathcal{L} = \int_0^T e^{(-\rho+v)t} u(c(t))dt - \lambda \left[\left[\int_0^T e^{-rt} c(t)dt - \int_0^T e^{-rt} w(t)[1-s(t)][h(t)+\omega(t)dt\,]\right]\right.$$

FOCs:

Consumption $c(t)$ from the Lagrangian:

equation 46

$$\frac{\partial \mathcal{L}}{\partial(c(t))} = e^{(-\rho+v)t} u'(c(t)) - \lambda e^{-rt} = 0$$

$$\Rightarrow u'(c(t)) = \lambda e^{-(r-\rho-v)t}$$

This Euler eq. shows that $c(t)$ depends on ρ, v, r and λ and not on $s(t) \vee h(t)$

Time allocation $s(t)$ from the human capital sub-problem :

equation 47

$$\frac{\partial}{\partial(s(t))}\left[e^{-rt} w(t)[1-s(t)][h(t)+\omega(t)dt\,]\right] = -e^{-rt} w(t)[h(t)+\omega(t)] + \frac{\partial G(t, h(t), s(t))}{\partial s(t)} = 0$$

This determines $s(t)$,the fraction of time allocated to schooling, independently of $c(t)$

Human capital $h(t)$ the evolution of $h(t)$ is determined by :

equation 48

$$\dot{h}(t) = G(t, h(t), s(t))$$

1. The **consumption problem** involves maximizing utility given a lifetime budget constraint. This budget constraint is determined by the optimal paths of $s(t)$ and $h(t)$, but the optimization itself does not influence those paths.
2. The **human capital and time allocation problem** involves maximizing lifetime earnings, independent of how those earnings are allocated to consumption.

Thus, the two problems are independent and separable under perfect capital markets.■

Some theory behind Pareto efficient taxation (due to Werning (2007)) [9]

Now, let ε_w^* represents the compensated elasticity of labor supply with respect to real wage. Distribution of income is given as:

equation 49

$$h(w\,) = \;k(w\,)^{-k-1}\underline{w}\,k\; for\; w\; \geq \underline{w}\;\; and\; k\; >\; 0$$

[9] See also lecture notes by Prof. James Poterba, Prof. Iván Werning: https://ocw.mit.edu/courses/14-471-public-economics-i-fall-2012/

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linear flat tax rate would be : $t(w) = t + \tau(w)$ .Where τ represents marginal tax rate and intercept t.Here we assume that ε_w^* [10]does not vary across individuals. This will be true in the case of this utility function[11]:

equation 50

$$u(c, w, \theta) = c - w\theta^{\alpha}$$

Now, starting from a general test for Pareto efficiency we will derive inequality for τ, ε_w^*, k. The starting point here is this inequality which states that marginal tax rate must be lower than 100% :

inequality 2

$$\frac{\tau(\theta)}{1-\tau(\theta)} \frac{\varepsilon_w^*}{\Phi} \left(-\frac{dlog \frac{\tau(\theta)}{1-\tau(\theta)}}{dlogw} - 1 - \frac{dlog\left(\varepsilon_w^*(w)\right)}{dlogw} - \frac{dlog(h^*(w)}{dlogw} - \frac{\partial MRS}{\partial c} w \right) \leq 1$$

the logarithm of Pareto income density is given as: $\log\left(h \cdot (w)\right) = \log\, k - (k + 1) \log w + k \log \underline{w}$. First of this log density with respect to income gives:

equation 51

$$\frac{d \log(h^*(w)}{d \log w} = \frac{d(\log\, k - (k+1) \log w + k \log\, \underline{w})}{d \log\, w} = \frac{-(k+1) d \log\, w}{d \log w} = -(k+1)$$

So the first inequality in this part $\frac{\tau(\theta)}{1-\tau(\theta)} \frac{\varepsilon_w^*}{\Phi} \left(-\frac{dlog \frac{\tau(\theta)}{1-\tau(\theta)}}{dlogw} - 1 - \frac{dlog\left(\varepsilon_w^*(w)\right)}{dlogw} - \frac{dlog(h^*(w)}{dlogw} - \frac{\partial MRS}{\partial c} w \right) \leq 1$ would become:

inequality 3

$$\frac{\tau(\theta)}{1-\tau(\theta)} \varepsilon_w^* k \leq 1$$

The parameter k has been estimated by Saez (2001) to be of value 1.6[12]. The thicker the tail of the distribution, the smaller is a.[13] Given that $\Phi(w) = 1 + we_w^*(w) \frac{\tau''(w)}{1-\tau'(w)} > 1$ .Now we have that:

equation 52

$$\frac{dlog(h^*(w)}{dlogw} = \frac{dlog(h^*(w)\Phi(\mathrm{w})^{-1}}{dlogw}$$

Now the upper bound on marginal tax rate is :

equation 53

$$\tau'(\theta) = \frac{1}{\frac{\varepsilon_w^*}{\Phi} \left(-\frac{dlog \frac{\tau(\theta)}{1-\tau(\theta)}}{dlogw} - 1 - \frac{dlog(h^*(w)}{dlogw} \right)}$$

In Werning (2007) the marginal tax rate for the Pareto optimal taxation in dual Mirrlees (1971) optimization problem is:

---

[10] The compensated elasticity of labor supply with respect to real wage ε_w^* has been estimate approximately to be 0.5 see Gruber, Saez (2002). So that $\frac{1}{\varepsilon_w^*} \in \left[\frac{1}{6}; \frac{10}{3}\right]$ or $\frac{1}{2*3} = \frac{1}{6}$ and $\frac{1}{0.2*1.5} = \frac{10}{3}$

[11] θ represents every individual's characteristics e.g. ability

[12] This value is approx.. for US incomes above 0.3 m.

[13] Pareto distribution is given as PDF lower CDF and upper CDF [13].PDF (probability density function) :$f(x, x_m, \alpha) = \frac{\alpha x_m^{\alpha}}{x^{\alpha+1}}$

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equation 54

$$\tau(\theta) = t'\big(y(\theta)\big) = 1 + \frac{u_y(c(\theta), y(\theta), \theta)}{u_c(c(\theta), y(\theta), \theta)} = 1 - \frac{\theta h'\big(y(\theta)\big)}{u'^{(c(\theta))}} = 1 - e_y(v(\theta), y(\theta), \theta)$$

Preferences are: $u(c, y, \theta) = u(c) - \theta h(y)$,where θ represents the heterogenous disutility from producing output y.Worker's utility $v(\theta)$ is maximized:

$$v(\theta) \equiv \max_y u(y - t(y), y, \theta)$$

$c(\theta) = e(v(\theta), y(\theta), \theta)$ is a consumption function dependent on workers' characteristics, $y(t) = y - t(y)$ and an allocation is resource feasible if :

inequality 4

$$\int \big(y(\theta) - c(\theta)\big) dF(\theta) + e \geq 0$$

Here e is an endowment. The social planner maximizes:

equation 55

$$\max_{\tilde{y}, \tilde{v}} \int (\tilde{y}(\theta), -e(\tilde{v}(\theta), \theta) dF(\theta) \text{ s.t.: } \tilde{v}(\theta) = \tilde{v}\big(\tilde{\theta}\,\big) - \int_\theta^{\bar{\theta}} u_\theta(e(\tilde{v}(z), \tilde{y}(z), z)\tilde{y}(z), z\,) dz$$

Constraint in previous is incentive constraint (Incentive compatibility IC), $\tilde{v}\big(\tilde{\theta}\,\big) \geq v(\theta)$ represents individual rationality. Lagrangian function is given as[14]:

$$\mathcal{L} = \int (\tilde{y}(\theta), -e(\tilde{v}(\theta), \theta) dF(\theta) + \int \left(\tilde{v}(\theta) - \tilde{v}(\bar{\theta}) + \int_\theta^{\bar{\theta}} u_\theta(e(\tilde{v}(z), \tilde{y}(z), z)\tilde{y}(z), z\,) dz \right) d\mu(\theta)$$

the FOC for $\tilde{y}(\theta)$ evaluated at $\big(y(\theta), v(\theta)\big)$ gives:

$$(1 - e_y(v(\theta), y(\theta), \theta) f(\theta) = -\mu(U_{\theta_c}(e(v(\theta), y(\theta), \theta) e_v(v(\theta), y(\theta), \theta) + u_{\theta_y}\big(e(v(\theta), y(\theta), \theta)\big)$$

Implying

equation 56

$$\mu(\theta) = \tau(\theta) \frac{f(\theta)}{h'\big(y(\theta)\big)}$$

The integral form of this efficiency condition is given as:

equation 57

$$\frac{\tau'(\theta) f(\theta)}{h' y(\theta)} + \int_\theta^{\bar{\theta}} \frac{1}{u'\Big(c\big(\tilde{\theta}\big)\Big)} f\big(\tilde{\theta}\big) d\tilde{\theta} \leq 0$$

Proposition 1 Given the utility function $u(c, y, \theta)$ and a density of skills $f(\theta)$, a differentiable tax function $t(y)$ inducing an allocation .$\big(c(\theta), y(\theta)\big)$ is *Pareto efficient* if and only if condition$\frac{\tau'(\theta) f(\theta)}{h' y(\theta)} + \int_\theta^{\bar{\theta}} \frac{1}{u'\big(c(\tilde{\theta})\big)} f\big(\tilde{\theta}\big) d\tilde{\theta} \leq 0$ holds, where $\tau(\theta) = t'\big(y(\theta)\big)$.

The Pareto distribution had a density that is a power function $g(y) = \mathcal{A} y^{-(\varphi)}$, so that these holds: $\frac{d \log g(y)}{d \log y} = -\varphi$

In $\bar{\tau} \leq \frac{\sigma + \eta - 1}{\varphi + \eta - 2}$ if $\varphi \approx 3$ as per Saez (2001) , then $\sigma < 2$ and σ cannot be interpreted as risk aversion but as control variable[15]

[14] Integrating second term by parts we have: $\mathcal{L} = \int (\tilde{y}(\theta), -e(\tilde{v}(\theta), \theta) dF(\theta) - \tilde{v}(\bar{\theta})\mu(\bar{\theta}) + \mu(\underline{\theta})\tilde{v}\big(\underline{\theta}\big) + \int \tilde{v}(\theta) d\mu + \int \mu(\theta) u_\theta\,(\tilde{v}(\theta), \tilde{y}(\theta), \theta) d\theta$

[15] A control variable (or scientific constant) in scientific experimentation is an experimental element which is constant and unchanged throughout the course of the investigation. Control variables could strongly influence experimental results, were they not held constant during the experiment in order to test the relative relationship of the dependent and independent variables. The control variables themselves are not of primary interest to the experimenter.

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Mirrlees optimal taxation (Mirrlees (1971), Diamond (1998)in Ben-Porath economy

The Mirrleesian approach to optimal taxation in a Ben-Porath economy

The Mirrleesian approach to optimal taxation in a Ben-Porath economy involves integrating the theory of optimal income taxation with human capital accumulation dynamics. Here's a step-by-step derivation and setup for the model:

1. Economic Setup

Preferences

Agents have preferences over consumption c and labor supply l:

equation 58

$$U=\int_0^T u(c_t,l_t)e^{-\rho t}dt$$

ρ is discount rate ; $u(c,l)=\frac{c^{1-\sigma}}{1-\sigma}\chi\frac{l^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}$ where σ is risk aversion , η is the Frisch elasticity of labor supply, and χ is scaling parameter of disutility of labor.

2.Human Capital

Human capital h_t evolves as:

equation 59

$$\dot{h}=g(h_t,l_t,z_t)$$

z_t : Learning effort (investment in skill development) ;$g(\cdot)$:Ben-Porath production function for human capital

3.Budget constraint

The agent faces:

equation 60

$$c_t+\tau(y_t)=\big(1-\tau(y_t)\big)y_t$$

equation 61

$$y_t=w_th_tl_t$$

$\tau(y_t)$: tax schedule

4.Planners problem

The government maximizes social welfare subject to resource constraints and individual optimization:

equation 62

$$\max_{\tau(\cdot)}\int_0^T U(c_t,l_t)f(h_t)dt$$

$f(h_t)$: Distribution of human capital

5.Resource constraint

equation 63

$$\int_0^T c_tf(h_t)dt\leq\int_0^T \tau(y_t)y_tf(h_t)dt$$

6.Incentive compatibility

Agent chooses c_t,l_t,z_t to maximize the utility given $\tau(\cdot)$ considering their human capital dynamics.

7. Characterizing Optimal Taxation

Using the first-order conditions of the agent's problem and the planner's maximization problem:

Optimal Labor Supply:

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equation 64

$$\frac{\partial u/\partial l}{\partial u/\partial c} = \left(1-\tau'(y_t)\right)w_t h_t$$

Labor supply depends on the marginal tax rate and human capital.

8. Optimal Human Capital Investment:

equation 65

$$\frac{\partial u/\partial c}{\partial g/\partial z} = shadow\ value\ of\ human\ capital\ dynamics.$$

9.Tax Schedule: The Mirrleesian tax schedule balances efficiency and equity:

equation 66

$$\tau'(y_t) = \frac{1-G(y_t)}{yt\ g(y_t)}$$

10. Including Frisch Elasticity

The Frisch elasticity η directly influences the labor supply response:

equation 67

$$l_t \propto \left(\left(1-\tau'(y_t)\right)w_t h_t\right)^{\eta}$$

In Diamond (1998) , non-linear tax formula is :

equation 68

$$\frac{\tau'(w_n)}{1-\tau'(w_n)} = \left(1+\frac{1}{e}\right)\cdot\left(\frac{\int_n^{\infty}(1-g_m)dF(m)}{nf(n)}\right)$$

Individual n chooses l_n to maximize:

equation 69

$$\max(nl - \tau n(l) - v(l))$$

In previous expression $g_m = \frac{G'(u_m)}{\lambda}$ which is the social welfare on individual m . The formula was derived in Diamond (1998) . If we denote $h(w_n)$ as density of earnings at w_n if the nonlinear tax system were replaced by linearized tax with marginal tax rate $\tau = \tau'(w_n)$ we would have that following equals $h(w_n)dw_n = f(n)dn$ and $f(n) = h(w_n)l_n(1+e)$,henceforth $nf(n) = w_n h(w_n)(1+e)$ and we can write previous equation as:

equation 70

$$\frac{\tau'(w_n)}{1-\tau'(w_n)} = \frac{1}{e}\cdot\left(\frac{\int_n^{\infty}(1-g_m)dF(m)}{w_n h(w_n)}\right) = \frac{1}{e}\cdot\left(\frac{1-H(w_n)}{w_n h(w_n)}\right)\cdot\left(1-G(w_n)\right)$$

In the previous expression $G(w_n) = \int_n^{\infty}\frac{dF(m)}{1-F(n)}$ is the average social welfare above w_n .If we change variables from $n \to w_n$, we have $G(w_n) = \int_{w_n}^{\infty}\frac{g_m dH(w_m)}{1-H(w_n)}$, see also Saez, E.,S. Stantcheva (2016).The transversality condition implies $G(w_0 = 0) = 1$.The optimal tax formula can be modified to :

equation 71

$$\frac{\tau'(w_n)}{1-\tau'(w_n)} = \left(\frac{1+e^u}{e^c}\right)\cdot\left(\frac{\eta(n)}{nf(n)}\right);\ \eta(n) = \frac{u'\left(c(n)\phi(n)\right)}{\lambda}$$

Hamiltonian for previous problem is given as:

equation 72

$$\mathcal{H} = [G(u(n) + \lambda(w(n) - \tilde{c}(\tilde{u}(n), w(n), n)]f(n) + \phi(n)\frac{w(n)}{n^2}v'\left(\frac{w(n)}{n}\right)$$

FOC's are given as:

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equation 73

$$\frac{\partial \mathcal{H}}{\partial w(n)} = \lambda \left[1 - \frac{v'\left(\frac{w(n)}{n}\right)}{nu'\big(c(n)\big)}\right] f(n) + \frac{\phi(n)}{n^2}\left[v'\left(\frac{w(n)}{n}\right) + \frac{w(n)}{n} v''\left(\frac{w(n)}{n}\right)\right] = 0$$

$$\frac{\partial \mathcal{H}}{\partial u(n)} = \left[G'(u(m) - \frac{\lambda}{u'c(m)}\right] f(n)d = -\phi'(n)$$

Ramsey taxation (due Ramsey (1927))

We consider a Ramsey taxation(Ramsey (1927)) framework within the Ben-Porath (1967) human capital model. The goal is to determine optimal Ramsey taxation of labor income, physical capital, and human capital investment while minimizing distortions to human capital accumulation. An individual maximizes lifetime utility:

equation 74

$$U = \int_0^\infty e^{-\rho t} u(c_t, d_t) dt$$

Where c_t, l_t, ρ are consumption, labor, discount rate respectively,$u(c, l)$ is the standard utility function. Human capital accumulation is:

equation 75

$$\dot{h} = f(h_t, e_t)$$

h_t, e_t are human capital and time spent investing in human capital, $f(h, e)$ is the human capital production function. Budget constraint says income comes from working and renting physical capital:

equation 76

$$\dot{a}_t = (1 - \tau_h) w_t h_t l_t + (1 - \tau_k) r_t a_t - c_t$$

Where $a_t, w_t, r_t, \tau_h, \tau_k$ are [physical capital, wage rate per unit human capital, interest rate, tax on labor income and tax on capital income. The government budget constraint is:

equation 77

$$\tau_h w_t h_t l_t + \tau_k r_t a_t = G_t$$

The Ramsey planner chooses τ_h, τ_k to maximize welfare while satisfying the government budget constraint. FOC's are:

1.Euler equation for consumption $u_c = \lambda$; λ is shadow price of wealth

2.optimal labor supply condition: $u_l = \lambda(1 - \tau_h) w_t h_t$

3. Optimal Capital Accumulation Condition:$\dot{\lambda} = \lambda(\rho - (1 - \tau_k) r_t)$

4. Optimal Human Capital Accumulation:$\frac{\partial f}{\partial e} = \frac{w_t(1-\tau_h)}{1-e_t}$

By differentiating the optimality conditions, we get the Ramsey inverse elasticity rule:

equation 78

$$\frac{\tau_h}{1 - \tau_h} = \frac{\eta_l}{\eta_c}$$

where η_l, η_c are elasticities of labor and consumption respectively. For capital taxation, the standard Chamley-Judd result holds in the long run:$\tau_k \to 0$.

Ben-Porath model and Mirrleesian optimal taxation framework with heterogeneous agents: numerical example

To derive the mathematical model of the Ben-Porath economy in a Mirrleesian optimal taxation framework with heterogeneous agents, let's break it down into steps.In the Ben-Porath model, agents invest in education, which increases their future earnings. The agents' decision about how much to invest in human capital depends on the future returns from education and the taxes they face. The key equations in this model typically include:

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1.The individual's income depends on their human capital h

Agents choose h (education level) to maximize lifetime utility.

The economy features heterogeneous agents, each with different ability levels and initial endowments of human capital.

2. Mirrleesian Optimal Taxation:

In the Mirrleesian model, the government sets a tax schedule $T(y)$, where y is income, to maximize social welfare subject to a budget constraint, while individuals respond to these taxes by choosing their labor supply and educational investment.

3. Mathematical setup

Agent's problem:Each agent maximizes utility $U(c,h)$ where c is consumption and h is the levelfo human capital:

equation 79

$$U(c,h) = \ln(c) - \frac{h^2}{2}$$

Where h is the level of human capital and c is consumption.The income ofan agent is determined by their human capital, so:

equation 80

$y = Ah^{\alpha}$

Where α is constant reflecting the return to human capital and A is productivity

Budget constraint

The agent faces budget constraint :

equation 81

$$c = (1 - \tau(y))\, y$$

Where $\tau(y)$ is a tax rate depending on income.

Agent's Problem (Maximization):

Each agent maximizes:

equation 82

$$\max_h \ln\left(\left(1 - \tau(Ah^{\alpha})\right)Ah^{\alpha}\right) - \frac{h^2}{2}$$

FOC for the optimal h is given as:

equation 83

$$\frac{\partial}{\partial h}\left(\ln((\,1 - \tau(Ah^{\alpha})\right) - \frac{h^2}{2}\right) = 0$$

Government's Problem (Mirrleesian Taxation):

The government aims to maximize social welfare, subject to the budget constraint. The welfare function is typically a utilitarian function:

equation 84

$$W = \int U(c,h)f(h)dh$$

Where $f(h)$ is the distribution of human capital in the population.The government's constraint is the revenue from taxes:

equation 85

$$\int \tau\,(Ah^{\alpha})Ah^{\alpha}f(h)dh = G$$

Where G is government spending.

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Table 1 Optimal taxes in Ben-Porath economy

Human Capital (h)	Tax Rate (Mirrlees)	Tax Rate (Pareto)	Tax Rate (Ramsey)	Consumption (Mirrlees)	Consumption (Pareto)	Consumption (Ramsey)	Revenue (Mirrlees)	Revenue (Pareto)	Revenue (Ramsey)	Utility (Mirrlees)	Utility (Pareto)	Utility (Ramsey)
1.00	0.05	0.05	0.15	1.43	1.43	1.27	0.08	0.08	0.23	17.71	17.71	11.92
1.09	0.05	0.05	0.16	1.55	1.55	1.37	0.09	0.09	0.27	21.83	21.82	15.64
1.18	0.06	0.06	0.17	1.67	1.67	1.46	0.10	0.10	0.31	25.62	25.58	19.04
1.27	0.06	0.06	0.18	1.79	1.79	1.56	0.12	0.12	0.35	29.14	29.04	22.15
1.36	0.07	0.07	0.19	1.91	1.91	1.65	0.13	0.14	0.40	32.42	32.25	25.03
1.45	0.07	0.07	0.20	2.03	2.02	1.74	0.15	0.16	0.44	35.49	35.23	27.69
1.55	0.07	0.08	0.21	2.15	2.14	1.83	0.16	0.18	0.49	38.38	38.02	30.18
1.64	0.07	0.08	0.22	2.28	2.25	1.92	0.18	0.20	0.54	41.12	40.63	32.51
1.73	0.07	0.09	0.23	2.40	2.37	2.00	0.19	0.22	0.59	43.71	43.08	34.69
1.82	0.08	0.09	0.24	2.52	2.48	2.09	0.21	0.25	0.64	46.17	45.40	36.75

Source: Author's own calculations

From previous we can see that for different levels of human capital Ramsey tax rates are highest, followed by Pareto marginal tax rates which are like Mirrlees tax rates. Consumption is highest with Mirrlees tax rates similar the one with Pareto taxes, lowest is Ramsey consumption. Ramsey revenues are highest, followed by Pareto revenues and last by Mirrlees. Utility is highest in Mirrlees followed by Pareto utility and Ramsey utility.

Ben-Porath Model: This model typically explains human capital accumulation, where individuals invest in education or training to increase their productivity over time. The economic output depends on their human capital and the returns to investment in it.

Mirrleesian Taxation: The Mirrleesian model of optimal taxation is designed to redistribute wealth in a way that maximizes social welfare, subject to the constraints imposed by individuals' incentive to work and invest in human capital.

Pareto Optimal Taxation: Pareto optimal taxation ensures that the economy reaches an allocation where no one can be made better off without making someone else worse off. This typically involves less progressive taxation than the Mirrleesian model.

Ramsey Taxation: Ramsey taxation aims to minimize the distortionary effects of taxes by taxing goods and income according to their elasticity of demand (less elastic goods should be

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taxed more). In the context of human capital, the tax rate is designed to maximize social welfare without significantly distorting labor supply or human capital accumulation.

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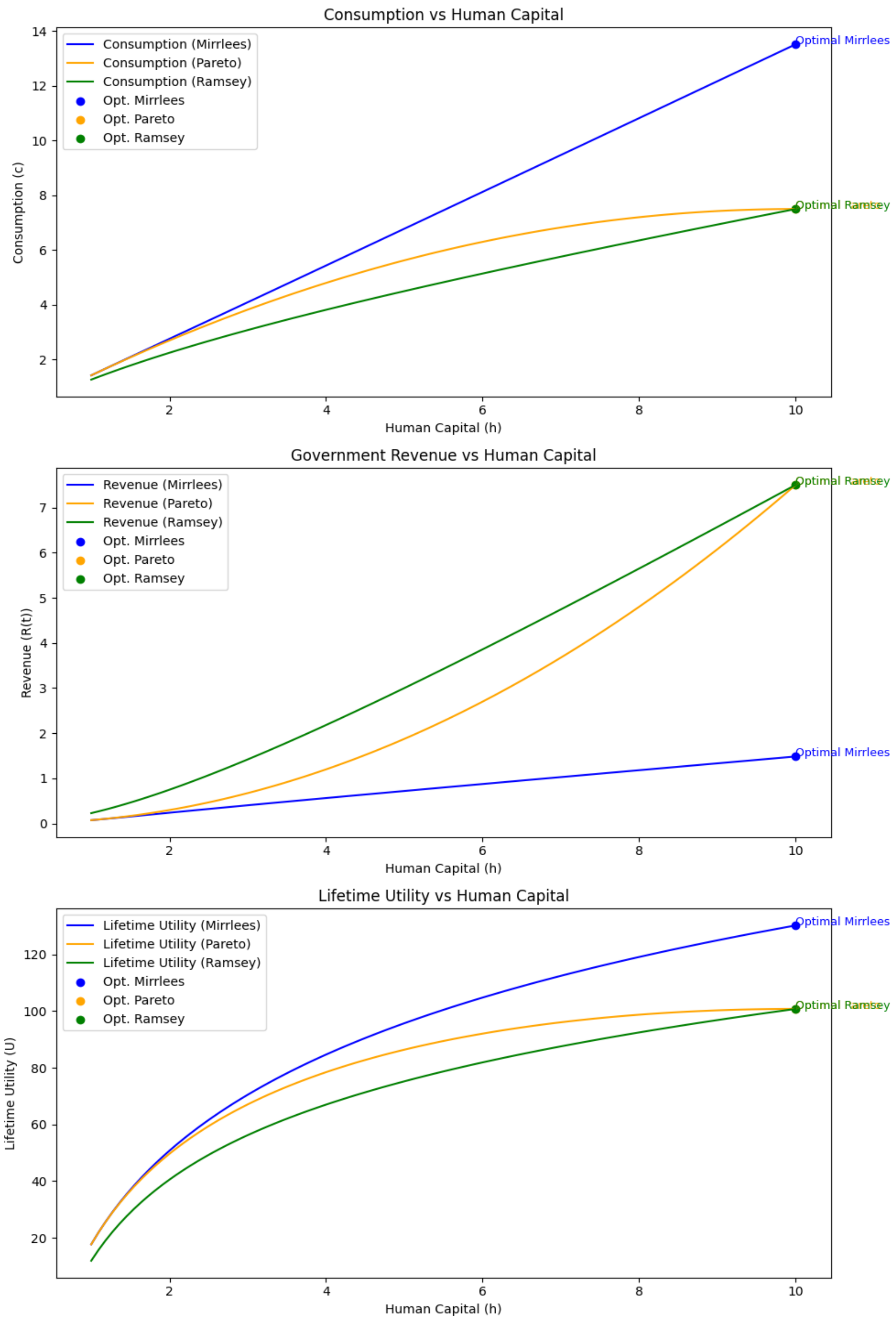

Figure 2 consumption ,government revenue, and lifetime utility vs human capital in Mirrlees,Ramsey and Pareto setting. Source: Author's own calculation

Previously used formulas for optimal tax equations provided are:

1. Mirrlees taxation

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Tax rate

equation 86

$$\tau_m(h) = \frac{ah^2}{1 + bh^2}$$

Consumption

equation 87

$$c_m(h) = \left(1 - \tau_m(h)\right) \cdot w(h)$$

Where $w(h) = \theta \cdot h$

Government Revenue:

equation 88

$$R_m(h) = \tau_m(h) \cdot w(h)$$

Lifetime utility :

equation 89

$$U_m(h) = \frac{\ln c_m(h)}{\rho}$$

2. Pareto Taxation

Tax Rate:

equation 90

$$\tau_p = \alpha \cdot h$$

Consumption:

equation 91

$$c_p(h) = \left(1 - \tau_p(h)\right) \cdot w(h)$$

Disposable income is proportional to human capital after taxation.

Government Revenue:

equation 92

$$R_p(h) = \tau_p(h) \cdot w(h)$$

Revenue is again proportional to the tax rate and income.

Utility:

equation 93

$$U_p(h) = \frac{\ln c_p(h)}{\rho}$$

3. Ramsey Taxation

Ramsey taxation minimizes the distortionary effects of taxation while achieving revenue targets.

Tax Rate:

equation 94

$$t_r(h) = \frac{h \cdot \eta}{1 + \gamma \cdot \eta}$$

Consumption:

equation 95

$$c_r(h) = \left(1 - \tau_r(h)\right) \cdot w(h)$$

Derived similarly from disposable income.

Government Revenue:

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equation 96

$$R_r(h) = \tau_r(h) \cdot w(h)$$

Follows standard revenue formulation.

Utility:

equation 97

$$U_r(h) = \frac{\ln c_r(h)}{\rho}$$

Reflects the log utility assumption in Ramsey models (Ramsey, 1927). Key Equations: Human Capital Accumulation:

equation 98

$$\dot{h}(t) = \delta h(t) - \gamma \cdot investment(t)$$

Where δ is depreciation parameter and γ is the investment efficiency parameter.

Consumption: The consumption of an agent depends on their wage and the tax rate:

equation 99

$$c(h\ ) = \big(1 - \tau(h)\big) \cdot w(h)$$

where $\tau(h)$ is the tax rate, which differs based on the taxation scheme.Lifetime Utility (using a CRRA utility function):

equation 100

$$U = \int_0^\infty \frac{c(t)^{1-\rho}}{1-\rho} e^{-\rho t} dt$$

Where ρ is the coefficient of relative risk aversion.

Mirrlees tax rate :

equation 101

$$\tau_m(h) = \frac{ah^2}{1 + bh^2}$$

Pareto tax rate:

equation 102

$$\tau_p = \alpha \cdot h$$

Ramsey tax rate :

equation 103

$$t_r(h) = \frac{h \cdot \eta}{1 + \gamma \cdot \eta}$$

Table 2 summary of marginal Mirrlees,Pareto,Ramsey tax rates in Ben-Porath economy

Feature	Mirrlees(1971)	Pareto optimality	Ramsey (1927)
Tax rate form	Marginal tax rate increases capped by b	Linear taxation proportional to$h$	Inverse relation to elasticity
Consumption	$(1 - \tau_m(h))w(h)$	$\left(1 - \tau_p(h)\right)w(h)$	$(1 - \tau_r(h))w(h)$
Utility	$\ln(c_m(h))/\rho$	$\ln\left(c_p(h)\right)/\rho$	$\ln(c_r(h))/\rho$

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Government revenue	$\tau_m(h)w(h)$	$\tau_p(h)w(h)$	$\tau_r(h)w(h)$

Source: Author's own calculation

Ben-Porath-Huggett (1993) economy: Heterogenous agents and incomplete markets

The Huggett (1993) model is a standard framework for studying incomplete markets with borrowing constraints and idiosyncratic income risk. Below is a derivation of the mathematical model. According to Achdou et al.(2022), in Huggett (1993) economy two basic equations are:

equation 104

$$\begin{cases} \rho v_1(a) = \max_c u(c) + v_1'(a)(z_1 + ra - c) + \lambda_1\big(v_2(a) - v_1(a)\big) \\ \rho v_2(a) = \max_c u(c) + v_2'(a)(z_2 + ra - c) + \lambda_2\big(v_1(a) - v_2(a)\big) \end{cases}$$

Where $\rho \geq 0$ represents the discount factor for the future consumption c_t (Individuals have standard preferences over utility flows), a represents wealth in form of bonds that evolve according to :

equation 105

$$\dot{a} = y_t + r_t a_t - c_t$$

y_t is the income of individual, which is endowment of economy's final good, and r_t represents the interest rate. Equilibrium in this Huggett (1993) economy is given as:

equation 106

$$\int_{\underline{a}}^{\infty} a g_1(a,t)da + \int_{\underline{a}}^{\infty} a g_2(a,t)da = B$$

Also:

equation 107

$$s_j(a) = z_j + ra - c_j(a)$$
$$c_j(a) = (u')^{-1}\Big(v_j(a)\Big)$$

$s_j(a), c_j(a)$are optimal savings and consumption. Where in previous expression $0 \leq B \leq \infty$ and when $B = 0$ that means that bonds are zero net supply. So the finite difference method approx. to $\begin{cases} \rho v_1(a) = \max_c u(c) + v_1'(a)(z_1 + ra - c) + \lambda_1\big(v_2(a) - v_1(a)\big) \\ \rho v_2(a) = \max_c u(c) + v_2'(a)(z_2 + ra - c) + \lambda_2\big(v_1(a) - v_2(a)\big) \end{cases}$ is given as:

equation 108

$$\rho v_{i,j} = u\big(c_{i,j}\big) + v_{i,j}'\big(z_j + ra_i + c_{i,j}\big) + \lambda_j\big(v_{i,-j} - v_{i,j}\big), j = 1,2$$
$$c_{i,j} = (u')^{-1}\big(v_{i,j}'\big)$$

This section for Huggett(1993) model will be visualized here for better understanding of the model. Parameters of this model are :

$\rho = 0.95$ # Discount factor; $r = 0.05$ # Interest rate;$z_1 = 2.0$ # Income shock for state 1; $z_2 = 1.0$ # Income shock for state 2; $B = 0$ # Net bond supply (zero bond supply condition); $\gamma = 2.0$ # Risk aversion parameter; $n_a = 100$ # Number of grid points for wealth; $a_{min} = 0.01$ # Minimum wealth (avoid zero wealth);$a_{\max} = 20$ # Maximum wealth; $max_{iter} = 500$ # Maximum iterations for solving

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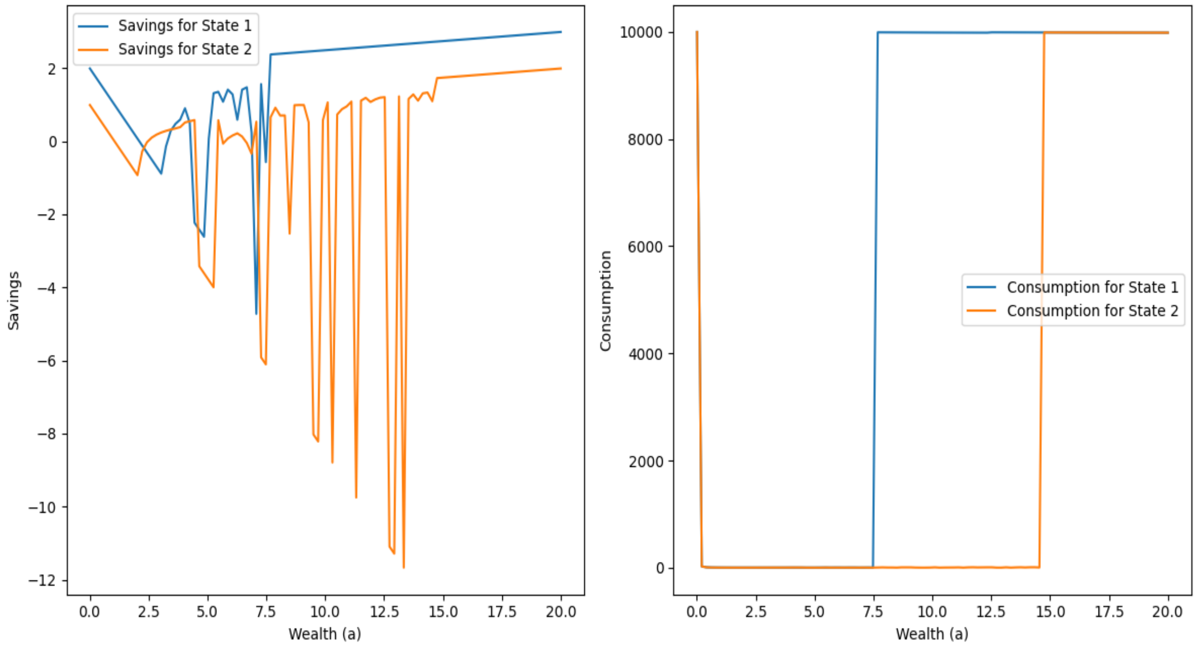

Figure 3 Savings and wealth and consumption and wealth in Huggett (1993) model. Source: Author's own calculation

Now to connect Ben-Porath (1967) and Huggett (1993) with optimal taxes, we will outline the Ben-Porath-Huggett model with taxes:

1. **Agents' problem in Ben-Porath economy:**
   An individual chooses consumption $c(t)$, labour $l(t)$ and investment in human capital $i(t)$:

equation 109

$$\max \int_0^T e^{-\rho t} u\big(c(t)\big) dt$$

subject to the human capital accumulation equation:

equation 110

$$\dot{h}(t) = f\big(h(t), i(t)\big) = \phi i(t) h(t)^{\gamma}, h(0) = h_0$$

Where: $h(t), i(t), \phi(t), \gamma$ are: human capital, investment in human capital, productivity of investment, and γ is the elasticity of human capital accumulation. The budget constraint in a complete markets economy would typically be:

equation 111

$$\dot{a}(t) = ra(t) + w(t)h(t)l(t) - c(t) - i(t)$$

Where $a(t)$ is assets and $w(t)h(t)l(t)$ is labor income.

2. **Incomplete Markets: Huggett's Model**

In the Huggett (1993) incomplete markets model, individuals face borrowing constraints and cannot fully insure against income shocks. The budget constraint modifies to:

equation 112

$$\dot{a}(t) = ra(t) + w(t)h(t)l(t) - c(t) - i(t)$$

s.t. budget constraint:

inequality 5

$$a(t) \geq a_{\min}$$

Where $a_{\min}$ is the exogenous borrowing limit.

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3. Recursive Formulation

Define the Bellman equation for an individual facing incomplete markets:

equation 113

$$V(a,h)=\max_{c,i,l}\{u(c)+e^{-\rho}\mathbb{E}V(a',h')\}$$

subject to:

equation 114

$$a'=(1+r)a+whl-c-i$$
$$h'=h+\phi ih^{\gamma}$$
$$a(t)\geq a_{\min}$$

The first-order conditions (FOCs) for consumption, human capital investment, and labor supply are:

Euler equation (consumption smoothing):

equation 115

$$u'(c)=e^{-\rho}\mathbb{E}[(1+r)u'(c')]$$

Human capital investment:

equation 116

$$\lambda=e^{-\rho}\mathbb{E}V_h(h')$$

Labor supply condition:

equation 117

$$whu'(c)=V_l(h,a)$$

Where λ is the shadow value of assets.

4. General Equilibrium in Incomplete Markets

Stationary Distribution: The distribution $\mu(a,h)$evolves via the Kolmogorov Forward Equation[16].

Interest Rate Equilibrium: The market-clearing condition for the bond market is

equation 118

$$\int a d\mu(a,h)=0$$

ensuring aggregate borrowing equals lending. Wage Equilibrium: The labor market clears

equation 119

$$w=F_L(K,L)$$

Where:

equation 120

$$L=\int hld\mu(a,h)$$

5. Policy Analysis: Mirrleesian, Pareto, and Ramsey Taxation

The introduction of taxation policies modifies the budget constraint:

[16] Now for the Kolmogorov Forward (Fokker-Planck[16]) equation we have folowing: let x be a scalar diffusion : $dx=\mu(x)dt+\sigma(x)dW, x(0)=x_0$.Let's suppose that we are interested in the evolution of the distribution of $x, f(x,t)$ and $\lim_{t\to\infty} f(x,t)$ .So, given an initial distribution $f(x,0)=f_0(x), f(x,t)$ satisfies PDE :$\frac{\partial f(x,t)}{\partial t}=-\frac{\partial}{\partial x}[\mu(x)f(x,t)]+\frac{1}{2}\frac{\partial^2}{\partial x^2}[\sigma^2(x)f(x,t)]$. Previous PDE is called "Kolmogorov Forward Equation" or "Fokker-Planck Equation

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equation 121

$$\dot{a}(t) = (1 + r)a + (1 - \tau_L)whl - (1 + \tau_c)c - i$$

Where τ_l, τ_c are labor income tax and consumption tax.

6. Deriving Mirrleesian, Ramsey, and Pareto optimal taxes in this economy

The introduction of taxation policies modifies the budget constraint once again:

equation 122

$$\dot{a}(t) = (1 + r)a(t) + (1 - \tau_l)whl - (1 + \tau_c)c - (1 - \tau_l)i$$

Where τ_i is the tax/subsidy on human capital investment. Borrowing constraint is the same as previous: $a(t) \geq a_{\min}$.

FOCs:

The household's optimization problem leads to:

Euler equation (consumption smoothing):

equation 123

$$u'(c) = e^{-\rho}\mathbb{E}[(1 + r)(1 + \tau_c)u'(c')]$$

Human capital investment decision:

equation 124

$$\lambda = e^{-\rho}\mathbb{E}V_h(h')(1 - \tau_i)$$

labor supply decision:

equation 125

$$(1 - \tau_l)whu'(c) = V_l(h, a)$$

Where λ is the shadow value of assets.

7. Social Planner Problem

The government seeks to maximize social welfare while financing government expenditure G using taxes:

equation 126

$$\max_{(\tau_l,\tau_c,\tau_I)} \int_0^T e^{-\rho t}U\big(c(t), l(t)\big)dt$$

subject to:

The government budget constraint:

equation 127

$$G = \tau_l whl + \tau_c c + \tau_i i$$

8. Deriving Optimal Taxes

Mirrlees taxation

Mirrlees taxation considers **asymmetric information** where individual productivity h is private knowledge. The planner chooses optimal nonlinear tax functions $T(w, h, l)$ to **maximize welfare while ensuring incentive compatibility**. The government maximizes:

equation 128

$$\max_{(\tau_l,\tau_c,\tau_I)} \int U(c, l)d\mu(a, h)$$

Incentive constraint: Individuals must prefer truthful reporting of h. First-best solution: If human capital is observable, set:

equation 129

$$\tau_l^*(h) = 1 - \frac{1}{\varepsilon_l(h)}$$

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Where $\varepsilon_l(h)$ is the Frisch elasticity[17].

Ramsey Optimal Taxation

The Ramsey planner chooses linear tax rates τ_l, τ_c, τ_i to maximize welfare while ensuring government revenue neutrality:

equation 130

$$\max_{(\tau_l, \tau_c, \tau_I)} \int e^{-\rho t} U(c, l) dt$$

s.t.:

equation 131

$$G = \tau_l whl + \tau_c c + \tau_i i$$

Ramsey's inverse elasticity rule gives:

equation 132

$$\frac{\tau_l}{\tau_c} = \frac{\eta_c}{\eta_l}$$

Where η_c, η_l are elasticities of consumption and labor.

Pareto Optimal Taxation

Pareto taxation balances equity and efficiency by solving:

equation 133

$$\max \lambda \int_0^T e^{-\rho t} U^{rich}(c, l) dt + (1 - \lambda) \int_0^T e^{-\rho t} U^{poor}(c, l) dt$$

for some weight λ Pareto optimal tax formula satisfies:

equation 134

$$\tau_l^P = \tau_l^{Ramsey} + \Delta\tau_l^{Redistribution}$$

where $\Delta\tau_l^{Redistribution}$ depends on inequality aversion. Next, we will code and plot this economy to draw conclusions.

[17] The Frisch elasticity measures the relative change of working hours to a one-percent increase in real wage, given the marginal utility of wealth λ .In the steady-state benchmark model is given as: $\frac{\frac{dh}{h}}{\frac{dw}{w}} = \frac{1-h}{h} \left(\frac{1-\eta}{\eta} \theta - 1 \right)^{-}$

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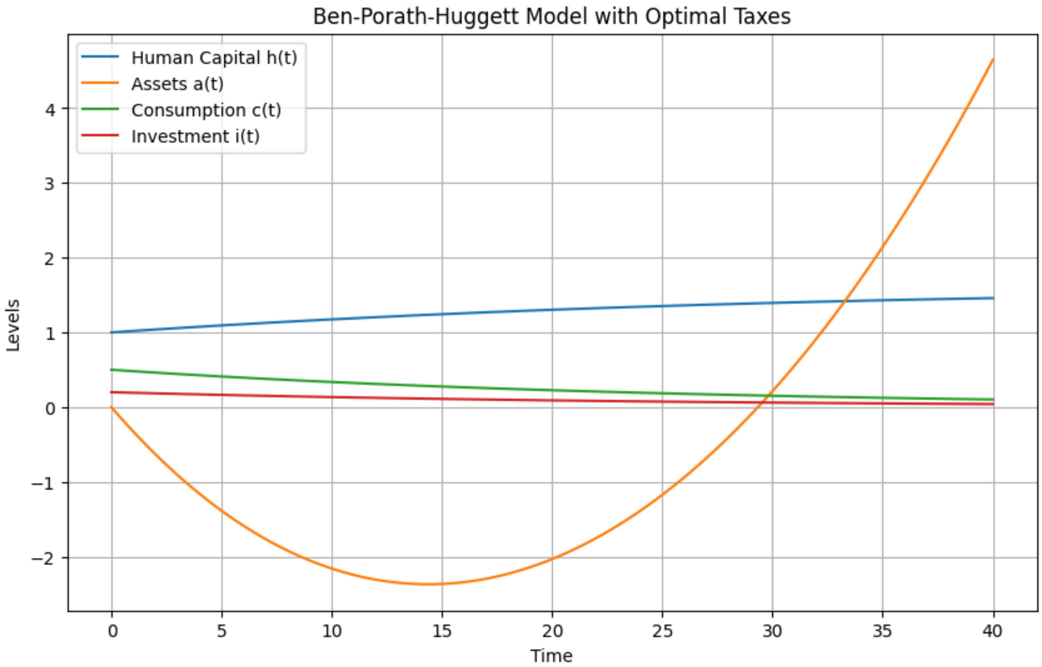

Figure 4 Ben-Porath-Huggett economy with optimal taxes

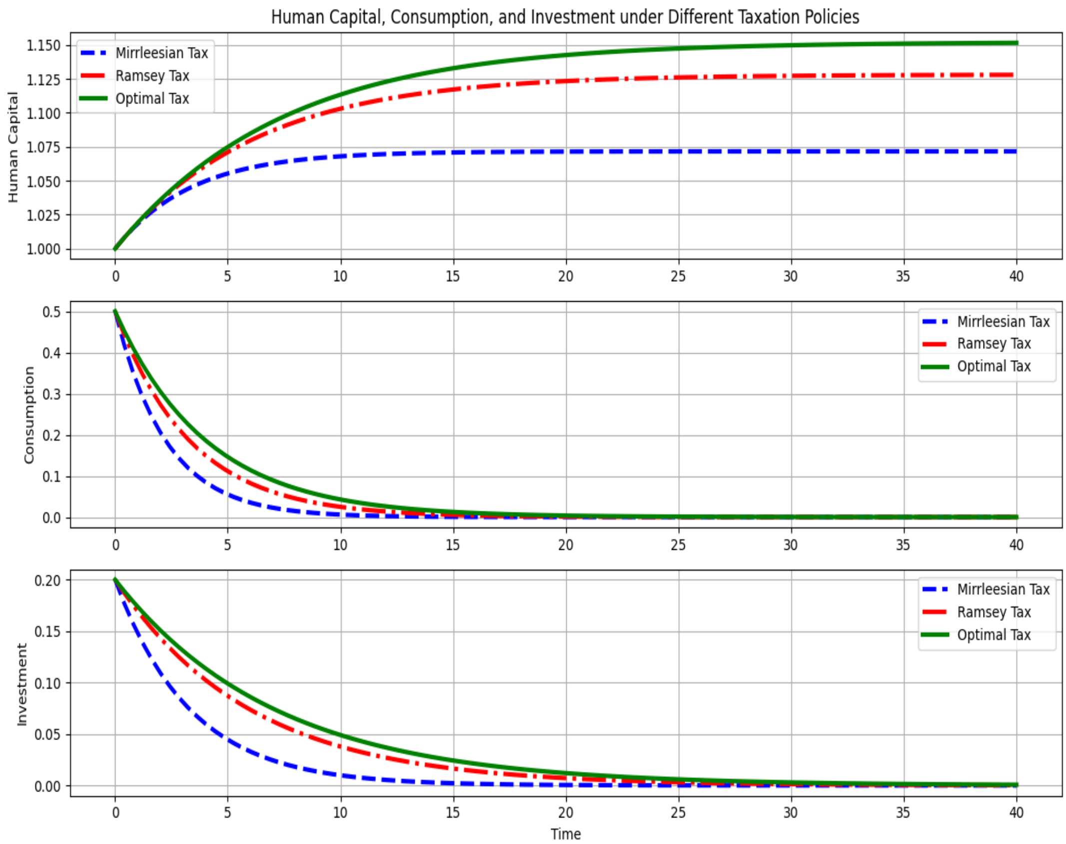

Figure 5 Human capital, consumption ,investment under different taxation policies

Table 3 Numerical Comparison of Welfare Loss (%) relative to no tax):

Policy	High-income welfare	Low-income welfare	Aggregate welfare
No tax	100%	100%	100%

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Mirrlees	95%	110%	102%
Ramsey	98%	90%	94%
Pareto	96%	108%	103%

1. **No Tax (Baseline)**
   - ✓ 100% welfare for all because there are no distortions from taxation.
   - ✓ Individuals maximize their consumption and human capital investment freely.
2. **Mirrlees Optimal Taxation**
   - ✓ High-income group: 95% welfare → Slight welfare loss due to progressive taxation reducing their after-tax income.
   - ✓ Low-income group: 110% welfare → Welfare gain due to redistribution and possibly subsidized education.
   - ✓ Aggregate Welfare: 102% → Overall, the economy benefits because taxation is designed to be least distortionary while improving equity.
3. **Ramsey Taxation**
   - ✓ High-income group: 98% welfare → Very small welfare loss, as Ramsey taxation aims to be efficient.
   - ✓ Low-income group: 90% welfare → Large welfare loss, since Ramsey taxation is generally flat and regressive, offering little redistribution.
   - ✓ Aggregate Welfare: 94% → Lower than Mirrlees because the poor suffer more without redistribution.
4. **Pareto Optimal Taxation**
   - ✓ High-income group: 96% welfare → Moderate welfare loss due to redistribution, but less than in Mirrlees.
   - ✓ Low-income group: 108% welfare → Welfare gain due to redistribution, though slightly less than in Mirrlees.
   - ✓ Aggregate Welfare: 103% → Slightly better than Mirrlees because redistribution helps low-income individuals without excessive distortions.

Instead of conclusion(s) -explanations

Key takeaway on these taxes effect on human capital is:
- ✓ Mirrlees taxation may allow for progressive taxation that does not heavily distort skill investment.
- ✓ Ramsey taxation leads to a positive education subsidy to offset labor taxation’s disincentive effect.
- ✓ Pareto taxation balances redistribution and efficiency, possibly taxing high earners more while subsidizing education for lower-income groups.

The elasticity of investment $i(t)$ with respect to τ_l is given as:

equation 135

$$\frac{di}{d\tau_l} = -\frac{\partial i}{\partial whl}\frac{\partial whl}{\partial \tau_l}$$

which is negative, meaning higher meaning higher τ_l discourages investment in human capital. Abut human capital investment tax τ_i:
- ✓ If human capital investment is taxed (or not subsidized), individuals will **under-invest in education**.
- ✓ Ramsey taxation, which seeks to minimize distortions, typically results in **subsidizing education** to offset income taxation’s negative effect on skill accumulation.

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A higher consumption tax indirectly affects education decisions by reducing disposable income.If households anticipate higher future taxes on consumption, they may increase savings and human capital investment as substitutes.

Effects on Income Inequality

Income inequality is driven by differences in human capital accumulation and the inability to fully insure against income shocks due to market incompleteness in the Huggett framework. Let σ_Y^2 represents the variance of income in the economy:

equation 136

$$\sigma_Y^2 = Var(whl)$$

- ✓ A progressive tax schedule (Mirrlees) reduces after-tax income dispersion.
- ✓ A flat labor tax (Ramsey) reduces work incentives for high-skilled individuals, leading to skill stagnation.
- ✓ Pareto taxation introduces targeted redistribution, lowering inequality at the cost of efficiency.

The Gini coefficient G captures inequality:

equation 137

$$G = \frac{\sum_i \sum_j \left|Y_i - Y_j\right|}{2N \sum_i Y}$$

- ✓ Higher τ_l **lowers $\boldsymbol{G}$**, reducing inequality.
- ✓ Higher τ_l **increases $\boldsymbol{G}$**, worsening inequality due to lower skill formation.
- ✓ Higher τ_l is regressive, increasing inequality unless offset by transfers.

In Huggett's economy, individuals face borrowing constraints, meaning they cannot smooth consumption over time.

Progressive labor taxation helps provide implicit insurance.

Education subsidies (low τ_i) allow credit-constrained individuals to invest in human capital.

- ✓ Mirrlees taxation reduces inequality without harming investment.
- ✓ Ramsey taxation can be regressive without an education subsidy.
- ✓ Pareto taxation balances redistribution and skill formation.

About the effects on welfare :

equation 138

$$W = \int_0^T e^{-\rho t} U(c,l) d\mu(a,h)$$

Where $U(c,l)$ is the individual utility, and $\mu(a,h)$ is the stationary distribution. In Mirrlees taxation, welfare is maximized as taxation is designed to minimize distortions and ensure redistribution. Less distortionary than Ramsey because taxes depend on individual ability. Ramsey taxation is : Efficient but less redistributive. Reduces welfare for low-income groups unless education is subsidized. Pareto optimal taxation: Welfare increases for lower-income individuals due to redistribution. Efficiency losses are minimized by carefully balancing taxes and subsidies. Key takeaways are:

- ✓ Mirrlees taxation maximizes welfare by targeting distortions.
- ✓ Ramsey taxation benefits high earners but hurts the poor unless subsidies exist.
- ✓ Pareto taxation improves equity with minor efficiency losses.

Key takeaways on taxes in Ben-Porath-Huggett economy are:

- ✓ Mirrlees taxation achieves the best balance—it improves overall welfare while reducing inequality.
- ✓ Ramsey taxation is efficient but regressive, leading to a larger welfare loss for low-income individuals.
- ✓ Pareto taxation provides redistribution with minimal efficiency loss, making it a good alternative when equity is a concern.
- ✓ No tax maximizes efficiency but leads to higher inequality due to lack of redistribution.

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- ✓ table 2 quantifies the trade-offs between efficiency and equity in different taxation models within the Ben-Porath - Huggett incomplete markets economy.

Furthermore, on policy implications:

- ✓ Education subsidies are crucial to mitigate distortions from labor taxation.
- ✓ Progressive labor taxation (Mirrlees) is best for reducing inequality without harming growth.
- ✓ Ramsey taxation requires education subsidies to avoid inequality worsening.
- ✓ Pareto taxation provides an optimal trade-off between equity and efficiency.

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