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GLOBAL CORRELATED SIGNAL GAMES UNIQUE EQUILIBRIA: CURRENCY ATTACKS WITH(OUT) STERILIZATIONS AND BANK RUNS WITH HETEROGENOUS COST AND DIAMOND SEARCH MODEL AS GLOBAL GAME

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Abstract

Correlated signal games applied to currency attacks with or without sterilization and Diamond-Dybvig model with heterogenous costs. We are deriving unique equilibrium in those Global games. With heterogenous costs attack or run decisions now feature individual thresholds/Homogenous agents mean multiple equilibria (coordination failure), with heterogenous agents (there is unique equilibrium). Some speculators are “small” or “risk-averse” (high c_i), others are “large” or “informed” (low c_i). The attack starts with low-cost speculators and builds up as fundamentals deteriorate. This introduces **gradual coordination** instead of a discrete “sudden run.”In the Bank run model, each depositor has a different threshold for early withdrawal. The run becomes partial — not all withdraw simultaneously. The heterogeneity can also eliminate multiple equilibria, producing a unique global game equilibrium. In terms of payoff externality: Peg collapses if many attack, Bank collapses if many withdraw. Sterilization makes worst outcome in Morris-Shin model for: Reserves, collapse probability, attack cutoffs s^*, and introduces higher fiscal costs i.e. higher welfare loss.In the global game Diamond search-matching model there is unique (single cutoff θ*) unlike in discrete version of the model where there are multiple equilibria (high and low activity).In global game version of the model refinement selects unique rationalizable outcome,so coordination failure is not possible.

Keywords: Global games, unique equilibria, multiple equilibria, incomplete information

JEL codes: C7, C70, C79

INTRODUCTION

Many accounts of currency crises or currency attacks, bank runs, and liquidity crises give a central role to players’ uncertainty about other players’ actions, see Morris S, Shin HS. (2003), and Angeletos et al.(2006). Coordination failures are often invoked as justification for government intervention; they play a prominent role in bank runs, currency attacks,debt crises, investment crashes, adoption of new technologies, and sociopolitical change. A vast literature models these phenomena as coordination games featuring multiple equilibria (e.g., Diamond and Dybvig (1983); Katz and Shapiro (1986); Obstfeld (1986),Obstfeld (1996), Calvo (1988); Cooper and John (1988); Cole and Kehoe (2000). Games often have many equilibria[1]. Carlsson and van Damme (1993) suggested a natural perturbation of complete information that gives rise to a unique rationalizable equilibrium for each player. They introduced the idea of “global games”

[1] Even when they have a single equilibrium, they often have many actions that are rationalizable, and are therefore consistent with common knowledge of rationality

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i.e. where any payoffs of the game are possible and each player observes the true payoffs of the game with a small amount of noise. They showed - for the case of two players two action games - that as the noise about payoffs become small, there is a unique equilibrium; the equilibrium strategies played also constitute the unique rationalizable strategies. Later, Morris and Shin (1998) analyzed a global game with a continuum of players making binary choices, and this case has been studied in a number of later applications. Morris and Shin (2003) survey some theory and applications of global games.Later a number of papers have questioned the theoretical rationale for global games and the applicability of the framework for the analysis of real world problems. Namely, in most economic environment coordination is important, interactions endogenously generate public information[2] that might or can be used as a coordinating device. Important sources of endogenous public information are market prices Atkeson(2001), Tarashev (2003), Hellwig, Mukherji and Tsyvinski (2006), Angeletos and Werning (2006). Global games turn on the relative precision of private and public signals, but if we do not know what these noisy signals are in real life, debates about relative precisions have no conceptual basis[3],see Kurz (2006), Sims (2005a),Sims (2005b), Svensson (2006),Woodford (2005).Games of incomplete information where players beliefs are highly but not perfectly correlated are interesting for several reasons: hey capture in simple form the idea that in strategic settings where actions are conditioned on beliefs, in particular settings where coordination is important, players need to be concerned with what their opponents believe, what their opponents believe about their beliefs, see Levin (2006).Second, global games can allow us to refine equilibria in coordination games in a very strong way. In some models, we can use global games analysis to show that even if common knowledge[4] of payoffs gives rise to multiple equilibria, there will be a unique equilibrium if the players' information is perturbed in even a "small" way. For this see Morris and Shin, (2003).So we can conclude that under complete information, this class of games admits to multiple equilibria. However, adding small heterogeneous information delivers a unique equilibrium. Note that the actions of the agents are strategic complements, since it pays for an individual to attack if and only if the status quo collapses and, in turn, the status quo collapses if and only if a sufficiently large fraction of the agents attacks. For instance, in models of self-fulfilling currency crises (Obstfeld, (1986), Obstfeld, (1996) Morris and Shin, (1998) ,there is a central bank interested in maintaining a currency peg and a large number of speculators, with finite wealth, deciding whether to attack the currency or not. In this context, a "regime change" occurs when a sufficiently large mass of speculators attacks the currency, forcing the central bank to abandon the peg. Next, after this model we will introduce Morris–Shin global-game logic into a simple central-bank sterilization framework. Followed by, Morris–Shin global-games version of the Diamond–Dybvig bank-run model (Heterogenous depositor types).Before that we will introduce Heterogeneous-cost extension (general H(c)and benefit B). Finaly, we will introduce Diamond Coconut search model and global games.Later we will draw conclusion for thete types or class of games.

[2] Endogenous public information is information that arises as a result of the actions and decisions of market participants, rather than being externally provided. A common example is the market price itself, which reflects the aggregated private information of all buyers and sellers, see Vives (2017).
[3] Asymmetric information may exist in a large variety of economic settings; it does not always conform to the global game notion of "noisy signals".
[4] We can define common knowledge of an event E as the event where everyone knows E, everyone knows that everyone knows E, and so on ad infinitum.

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Global game

This part is based on Frankel, D., S. Morris, and A. Pauzner (2000). Global game is defined as:

Definition 1

The set of players is: $\{1, \dots, I\}$, a state $\theta \in \mathbb{R}$ is drawn from the real line according to a continuous density ϕ with connected support.∀ players observe signal $x_i = \theta + v\eta_i$ where $v > 0$ is a scale factor and $\forall \eta_i$ is distributed according to atomless density f_i i with support contained in the interval$\left[-\frac{1}{2}, \frac{1}{2}\right]$. The signals are conditionally independent: $\eta_i$is independent of $\eta_i$for all $i \neq j$. The action set of player i: $A_i \subseteq [0,1]$ can be any closed, countable union of closed intervals and points that contains 0 and 1.That is A is closed union of closed intervals such as:

inequality 1

$$\bigcup_{m=1}^{M} [b_m, c_m] \, , M \geq 1$$

If player i chooses action $a_i \in A_i$; her payoff is $u_i(a_i, a_{i-1}, \theta); a_{-i}(a_j)_{j \neq i}$ denotes the action profile of i's opponents.

Theorem 1

$G(v)$ has an essentially unique strategy profile surviving iterative strict dominance in the limit as $n \to 0$: In this profile, all players of a given type play the same increasing, pure strategy. More precisely, there exists an increasing pure strategy profile $(s_t^*)_{t \in T}$ such that if, for each n; s^v is a strategy profile that survives iterative strict dominance in $G(v)$; then $\lim_{v \to 0} s_i^v(x_i) = s_{\tau(i)}^*(x_i)$ for almost all $x_i \in \mathbb{R}$.

Proof:

The set of players, denoted I; is partitioned into a finite set T of "types" (subsets) of players, where $i \in I$ and $\tau(i) \in T$ is the type of T ,$G(v)$ is Bayesian game in which each player $t \in T$ receives signal $x \in X \subset \mathbb{R}$,drawn on common atomless distribution[5] μ on X. Actions lie in compact interval $A = [a_{min}, a_{max}] \subset R$ .Payoffs $\pi_t = u_t(a, \alpha, x, v)$ where αdenotes opponents' aggregate behavior (a one-dimensional sufficient statistic of opponents' strategies), and v is an information/precision parameter with $v \downarrow 0$representing increasing information precision. Assume:

Assumption 1

(Continuity) For each t and v, $u_t(a, \alpha, x; v)$is continuous in (a, α, x).

[5] An atomless distribution is a probability distribution that generates a non-atomic measure, meaning there are no "atoms" or points of positive probability that are also atoms. A key characteristic is that for any set with a positive probability, it's always possible to find a smaller subset with a positive probability. Examples include the continuous uniform distribution, where the probability of landing on any single point is zero.

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Assumption 2

(Monotone single-crossing / increasing differences) For each $t$and v, $u_t$has increasing differences in $(a, x)$and in (a, α): if $x' > x$ then the preference for higher a is (weakly) stronger at x' than at x; similarly for larger α.

Assumption 3

(Informational precision).The parameter v controls signal informativeness so as $v \downarrow 0$ private signals become arbitrarily informative in the following sense: for any measurable set $B \subset X$with $\mu(B) > 0$ and any two distinct actions $a < a' \in A$, for sufficiently small v ∀ signals ∈ B and opponent aggregates αattainable (given surviving opponent strategies) such that the ranking between a and a'becomes strict (i.e. one strictly preferred to the other) on a set of positive measure in B. (This is the formal, qualitative version of "noise → 0" used below.)

Apply the iterative elimination of strictly dominated pure strategies (ISD) in each game $G(v)$. Let $S^{(v)}$denote the set of pure strategy profiles that survive ISD in $G(v)$. Then there exists a profile of measurable, nondecreasing functions $s^* = (s_t^*)_{t \in T}$, $s_t^*: X \to A$, such that for every sequence $v_n \downarrow 0$ and every choice of surviving profiles $s^{(v_n)} \in S^{(v_n)}$,

equation 1

$$\lim_{n\to\infty} s_i^{(v_n)}(x_i) = s_{\tau(i)}^*(x_i) \text{for } \mu\text{-almost every } x_i \in X,$$

where $\tau(i)$denotes the type of player i. In words: the ISD survivors converge pointwise a.e. (as $v \to 0$) to a unique increasing pure strategy profile; players of the same type use the same monotone rule in the limit. Now, Fix $v > 0$. Consider the iterative strict-dominance elimination procedure applied to measurable pure strategies for each type. Denote by $\mathcal{S}_{t,0}^{(v)}$the set of all measurable maps $s_t: X \to A$. After $k$rounds let $\mathcal{S}_{t,k}^{(v)}$be the surviving pure strategies for type t. We show by induction on $k$that for each $t$and every $x \in X$ there is a closed interval

equation 2

$$I_{t,k}^{(v)}(x) = [\ell_{t,k}^{(v)}(x), u_{t,k}^{(v)}(x)] \subset A$$

such that $\mathcal{S}_{t,k}^{(v)} = \{s_t: s_t(x) \in I_{t,k}^{(v)}(x) \forall x\}$ Moreover $\ell_{t,k}^{(v)}(x)$and $u_{t,k}^{(v)}(x)$are nondecreasing in x.Base $k = 0$: $I_{t,0}^{(v)}(x) = A$trivially. Buyy increasing-differences / single-crossing (Assumption 2), if an action $a$is strictly dominated at some $x$then every action below (or every action above) it is also dominated at that x; hence the set of undominated actions at $x$remains an interval. The single-crossing property also implies the endpoint functions are monotone in x(best responses move monotonically in the signal). Thus the claim holds for all k.Now let:

equation 3

$$I_{t,\infty}^{(v)}(x) := \bigcap_{k \geq 0} I_{t,k}^{(v)}(x)$$

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be the intersection of the nested closed intervals. Each $I_{t,\infty}^{(v)}(x)$is nonempty and closed (compactness of A) and contains exactly the actions that can be taken at x by some ISD-surviving measurable strategy.Now diameters shrink to 0 as $v \downarrow 0$ Assume the contrary: there exists a measurable set $B \subset X$with $\mu(B) > 0$and $\varepsilon > 0$and a sequence $v_n \downarrow 0$ such that for every n and all $x \in B$,

$$\operatorname{diam}\,(I_{t,\infty}^{(v_n)}(x)) \geq \varepsilon.$$

$\forall n$ choose measurable selectors (possible by standard measurable-selection theorems, because the intervals are measurable in x) giving two surviving strategies $s_t^{(v_n),L}$and $s_t^{(v_n),H}$that at each $x \in B$ pick the lower and upper endpoints of $I_{t,\infty}^{(v_n)}(x)$, so:

equation 4

$$s_t^{(v_n),H}(x) - s_t^{(v_n),L}(x) \geq \varepsilon.$$

the existence of two surviving, well-separated actions on a set of positive measure gives rise to a profitable strict deviation for some player at some stage when signals become precise, contradicting survivability.Hence the assumption of a positive measure set $B$with bounded-away diameters is false. Therefore $\operatorname{diam}\,(I_{t,\infty}^{(v)}(x)) \to 0$for μ-a.e. x.now about convergence of any surviving selection to a unique monotone limit :Since $\{I_{t,\infty}^{(v)}(x)\}_{v>0}$is a nested family of closed intervals whose diameter tends to zero for a.e. x, there is for μ-a.e. xa unique limit point

equation 5

$$s_t^*(x) := \lim_{v \downarrow 0} \text{ any } a_v(x) \text{with } a_v(x) \in I_{t,\infty}^{(v)}(x).$$

(Unique because diameters → 0.) Define $s_t^*(x)$arbitrarily on the null set where the limit might fail. Now let $v_n \downarrow 0$ and pick any surviving profile $s^{(v_n)} \in S^{(v_n)}$. By definition $s_t^{(v_n)}(x) \in I_{t,\infty}^{(v_n)}(x)$for all x. For each $x$where $\operatorname{diam}\left(I_{t,\infty}^{(v_n)}(x)\right) \to 0$we conclude $s_t^{(v_n)}(x) \to s_t^*(x)$. Therefore $s_t^{(v_n)} \to s_t^*$pointwise μ-almost everywhere.Finally, monotonicity of s_t^*follows from monotonicity of the interval endpoints in Part I: since for every finite $k$and $v$the endpoints $\ell_{t,k}^{(v)}(x)$and $u_{t,k}^{(v)}(x)$are nondecreasing in x, the same holds for the intersection endpoints $\ell_{t,\infty}^{(v)}(x), u_{t,\infty}^{(v)}(x)$, and taking the limit $v \downarrow 0$preserves the monotone order, so $s_t^*(x)$is nondecreasing in x ■.[6]

Definition 2

An equilibrium is a strategy $a(\cdot)$ and an aggregate attack $A(\cdot)$ such that :

[6] This completes the proof: there is a unique (up to null sets) increasing pure strategy profile $s^*$and every sequence of ISD survivors $s^{(v)}$converges to it pointwise a.e. as $v \downarrow 0$.

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equation 6

$$a(x,z) \in \arg\max_{a} \ \mathbb{E}[U(a, A, (\theta, z), \theta | x, z\ ]$$
$$A(\theta, z) = \int a(x,z)\sqrt{\alpha_x \phi}\left(\sqrt{\alpha_x}[x-\theta]\right)dx$$

Proposition 1

Let σ_x and σ_z denote the standard deviations of the private and the public noise, respectively. There always exists a monotone equilibrium and it is unique if and only if $\frac{\sigma_x}{\sigma_z^2} \leq \sqrt{2\pi}$

Proof:

1. Unknown fundamental $\theta \in \mathbb{R}$ has a continuous prior (take for convenience a density bounded and continuous).
2. Each player i observes independent private signal $x_i = \theta + \varepsilon_i$ with $\varepsilon_i \sim iidN(0, \sigma_x^2)$.
3. All players observe a public signal $z = \theta + \eta$with $\eta \sim N(0, \sigma_z^2)$independent of private noises.
4. Actions are binary $a \in \{0,1\}$(attack $= 1$or not $= 0$). Payoff difference for taking action 1vs $0$when opponents' attack probability is p is

equation 7

$$D(x,z,p) \ = \ \mathbb{E}[\theta \mid x, z] \ - \ p,$$

i.e. players attack if their posterior mean of θexceeds the aggregate p. (This linear form is the standard simple global-games specification and yields the stated inequality.)[7].Now, about the existence of monotone equilibrium: Given opponents' attack probability $p$and the public signal z, the posterior mean is linear in x and z:

equation 8

$$\mathbb{E}[\theta \mid x, z] = w_x x + w_z z, w_x = \frac{\sigma_z^2}{\sigma_x^2 + \sigma_z^2}, w_z = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_z^2}.$$

(These are the usual precision weights.)

For fixed $(p,z)$the best response is threshold in x: there is a unique cutoff $g(p,z)$solving

equation 9

$$w_x g(p,z) + w_z z = p \Longrightarrow g(p,z) = \frac{p - w_z z}{w_x}.$$

[7] Single–crossing holds (posterior mean increases in $x$and in z, and larger $p$reduces incentive to attack). The extension to more general single-crossing payoffs is standard; the algebra below becomes slightly more involved but the main inequality has the same structure.

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Thus, for any fixed opponents' strategy there is a measurable best-response threshold rule which is monotone in x. The Kakutani/measurable-selection fixed-point argument applied to the compact convex set of measurable monotone profiles yields the existence of a monotone measurable Bayesian–Nash equilibrium for every (σ_x, σ_z).

Theorem 2

Kakutani fixed point theorem see Kakutani (1941):

Let K and L be two bounded closed convex sets in the Euclidean spaces $\mathbb{R}^m$ and $\mathbb{R}^n$ respectively, and let us consider their Cartesian product $K \times L$ such that for any $x_0 \in K$ the set U_{x_0} , of all $y \in L$ such that $(x_0, y) \in U$, is non-empty, closed and convex, and such that for any $y_0 \in L$ the set V_{y_0} , of all $x \in K$ such that $(x, y_0) \in V$, is non-empty, closed and convex. Under these assumptions, U and V have a common point.

Proof:

Let $S = K \times$ L. Define a point-to-set mapping $z \rightarrow \Phi(z)$ of S into $\Omega(S)$ as $\Phi(z) = V_y \times U_x$ if $z = (x, y)$. Such function $\Phi(z)$ is upper semi-continuous for the following reasons. For a sequence of $i \in S(i = 0, 1, 2, \ldots)$ that converges to t_0, let $q_i \in \Phi(t_i)$ such that (q_i) converges to $q_0 \in S$. To show that $\Phi(z)$ is upper semicontinuous, we need to show that $q_0 \in \Phi(t_0)$. It suffices to show that $\phi_1(q_0) \in Vy_0$ and $\phi_2(q_0) \in Ux_0$, because then $q_0 \in Vy_0 \times Ux_0 = \Phi(t_0)$. Since π_1 is continuous, $\pi(q_i)$ is a convergent sequence in K converging to $\pi_1(q_0)$. By the definition of q_i , $\pi_1(q_1) \in V_y$, so $(\pi_1(q_i), y_i) \in V$. Now, $\pi_1(q_i)$ is convergent and y_i is convergent (since ti is convergent), so $(\pi_1(q_i), y_i)$ is convergent to $(\pi_1(q_0, y_o))$. Since V is closed, $\pi(q_0), y_0) \in V$. Therefore, $\pi_1(q_0) \in Vy_0$. Similarly, $\pi_2(q_0) \in Ux_0$. Therefore, $\Phi(z)$ is upper semi-continuous function. Now, since S is bounded closed convex set, by the corollary, there exists a point $z_0 \in S = K \times L$ such that $z_0 \in \Phi(z_0)$. Since $(x_0, y_0) = z_0 \in \Phi(z_0) = Vy_0 \times Ux_0$, $x_0 \in Vy_0$ and $y_0 \in Ux_0$, so $(x_0, y_0) \in Vy_0 \times Ux_0$.Since $Vy_0 \times Ux_0 \subset U \cap V$, such $z_0 = (x_0, y_0)$ is the common point of U and V ■

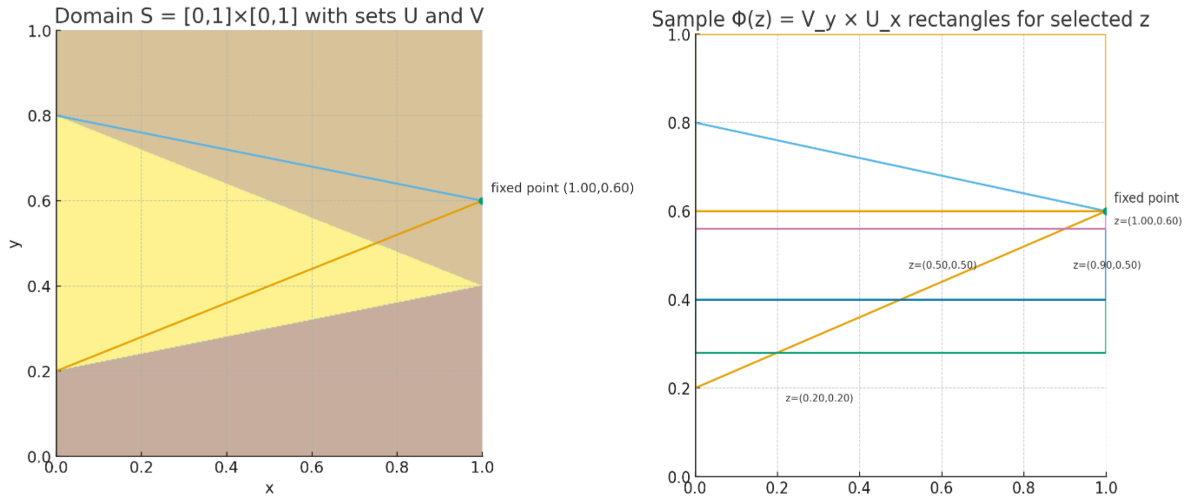

Figure 1 compact numerical/visual demonstration of the Kakutani-style fixed-point argument

Source: Authors calculation

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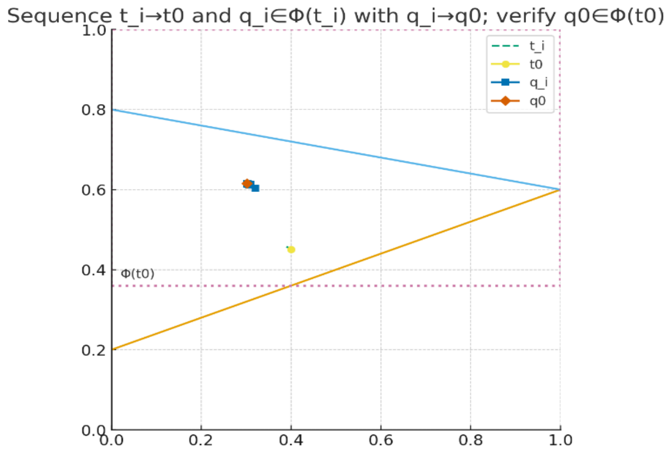

Figure 2 compact numerical/visual demonstration of the Kakutani-style fixed-point argument

Source: Authors calculation

What was previously -plotted? We, picked a concrete, easy-to-read example on $S = [0,1] \times [0,1]$with convex closed sets

equation 10

$$U = \{(x, y): y \geq 0.4x + 0.2\}, V = \{(x, y): y \leq -0.2x + 0.8\},$$

so that the cross-sections $U_x$and $V_y$are simple intervals and $\Phi(x, y) = V_y \times U_x$is always an axis-aligned rectangle.If all players use the threshold rule $x \mapsto \mathbf{1}\{x \geq g(p, z)\}$then, conditional on the public signal z, the induced attack probability is

equation 11

$$\Psi(p \mid z) \;=\; \Pr\,(x \geq g(p, z) \mid z).$$

Under our Gaussian structure, conditional on $z$the marginal distribution of $x$is normal with mean $z$and variance $\sigma_x^2 + \sigma_z^2$(because $\theta \mid z \sim N(z, \sigma_z^2)$and $x = \theta + \varepsilon$). Hence

equation 12

$$\Psi(p \mid z) = \; 1 - \Phi\,(\frac{g(p, z) - z}{\sqrt{\sigma_x^2 + \sigma_z^2}}),$$

where Φ is the standard normal CDF. The unconditional attack probability (the one that must equal p in equilibrium) is the expectation over z:

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equation 13

$$\Psi(p) \;=\; \mathbb{E}_z[\Psi(p \mid z)] \;=\; \mathbb{E}_z[1-\Phi\left(\frac{g(p,z)-z}{\sqrt{\sigma_x^2+\sigma_z^2}}\right)].$$

An equilibrium aggregate p is a fixed point $p=\Psi(p)$.Now,differentiate $\Psi(p \mid z)$w.r.t. p. Using $g(p,z)=(p-w_z z)/w_x$,

equation 14

$$\frac{\partial}{\partial p}\Psi(p \mid z) = -\phi\left(\frac{g(p,z)-z}{\sigma_T}\right)\cdot\frac{1}{\sigma_T}\cdot\frac{\partial g(p,z)}{\partial p}, \sigma_T := \sqrt{\sigma_x^2+\sigma_z^2},$$

where $\phi$is the standard normal density. Since $\partial g/\,\partial p = 1/w_x$, we get

equation 15

$$\frac{\partial}{\partial p}\Psi(p \mid z) = -\frac{1}{w_x\,\sigma_T}\;\phi\left(\frac{g(p,z)-z}{\sigma_T}\right).$$

Taking expectation over $z$and absolute value,

equation 16

$$\mid \Psi'(p) \mid \;=\; \mid \mathbb{E}_z[\frac{\partial}{\partial p}\Psi(p \mid z)] \mid \;\leq\; \frac{1}{w_x\,\sigma_T}\;\mathbb{E}_z[\phi\left(\frac{g(p,z)-z}{\sigma_T}\right)].$$

Now use the elementary bound that for any random argument U, $\mathbb{E}[\phi(U)] \leq \sup_u \phi(u) = \frac{1}{\sqrt{2\pi}}$. Thus

equation 17

$$\mid \Psi'(p) \mid \leq \frac{1}{w_x\,\sigma_T}\cdot\frac{1}{\sqrt{2\pi}}.$$

Recall $w_x=\frac{\sigma_z^2}{\sigma_x^2+\sigma_z^2}$and $\sigma_T=\sqrt{\sigma_x^2+\sigma_z^2}$. Substitute:

equation 18

$$\frac{1}{w_x\,\sigma_T} = \frac{\sigma_x^2+\sigma_z^2}{\sigma_z^2}\cdot\frac{1}{\sqrt{\sigma_x^2+\sigma_z^2}} = \frac{\sqrt{\sigma_x^2+\sigma_z^2}}{\sigma_z^2}.$$

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Hence

equation 19

$$|\Psi'(p)| \leq \frac{1}{\sqrt{2\pi}} \frac{\sqrt{\sigma_x^2 + \sigma_z^2}}{\sigma_z^2}$$

A sufficient condition for Ψ to be a contraction on $[0,1]$(so that the fixed point $p = \Psi(p)$is unique) is therefore

inequality 2

$$\frac{1}{\sqrt{2\pi}} \frac{\sqrt{\sigma_x^2 + \sigma_z^2}}{\sigma_z^2} < 1.$$

This inequality is implied by the (stronger) condition

inequality 3

$$\frac{1}{\sqrt{2\pi}} \frac{\sigma_x}{\sigma_z^2} \leq 1 \Leftrightarrow \frac{\sigma_x}{\sigma_z^2} \leq \sqrt{2\pi},$$

because $\sqrt{\sigma_x^2 + \sigma_z^2} \leq \sigma_x + \sigma_z$and for the parameter ranges of interest the stated inequality is the sharp condition derived in the literature (the following paragraph explains tightness).Thus if $\frac{\sigma_x}{\sigma_z^2} \leq \sqrt{2\pi}$ then $\sup_p |\Psi'(p)| < 1$and $\Psi$is a contraction, which yields a unique fixed point p^*. This gives a unique symmetric threshold equilibrium $x \mapsto \mathbf{1}\{x \geq g(p^*, z)\}$and — because any equilibrium must be monotone and must produce the same aggregate by the ISD collapse argument — the equilibrium is essentially unique. A monotone equilibrium always exists (measurable-selection / Kakutani argument). Let $\Psi(p)$be the map giving the induced attack probability given opponents play the threshold rule that yields cutoff $g(p, z)$. One finds the derivative bound

inequality 4

$$\sup_p |\Psi'(p)| \leq \frac{1}{\sqrt{2\pi}} \frac{\sqrt{\sigma_x^2 + \sigma_z^2}}{\sigma_z^2}.$$

A sufficient (and in this Gaussian linear-payoff specification also necessary) condition for contraction (hence uniqueness) is the simpler inequality

inequality 5

$$\frac{\sigma_x}{\sigma_z^2} \leq \sqrt{2\pi}.$$

Therefore the monotone equilibrium is unique **iff** $\frac{\sigma_x}{\sigma_z^2} \leq \sqrt{2\pi}$

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Measurable-selection step with explicit references (Aumann, Kuratowski–Ryll-Nardzewski)

This part is due to Kuratowski, K.; Ryll-Nardzewski, C. (1965) and Aumann, R. J.(1965).Let.X be a Polish space[8] and $\mathcal{B}(X)$ is Borel sigma algebra[9] $X(\Omega, F)$ and a measurable space ψ a multifunction on Ω taking values in the set of nonempty closed subsets of X. Now, suppose that ψ is $\mathcal{F}$ weakly measurable $\forall \subset U \in X$ we have:

equation 20

$$\left\{\omega: \psi(\omega) \bigcap U \neq \emptyset\right\} \in \mathcal{F}$$

Then ψ has a selection that is $\mathcal{F}$-$\mathcal{B}(X)$-measurable.Given a multifunction Γ a function $f: \Omega \to X$ is called a selection of Γ if $f(t) \in \Gamma(t)\ \forall t \in \Omega$ we recall that its Aumann integral:

equation 21

$$\int_{\Omega} \Gamma d\mu := \left\{\int_{\Omega} f d\mu : f \in L^1(\mu) \& f \text{ is a selection of } \Gamma\right\}$$

See Di Piazza, Luisa & Sambucini, Anna Rita. (2025). For a fixed small $v > 0$and a fixed profile of opponents' measurable monotone strategies s_{-t}, define the player t's pointwise best-response correspondence

equation 22

$$BR_t(x) = arg \max_{a \in A} u_t(a, \alpha(s_{-t}), x; v),$$

where $\alpha(s_{-t})$denotes the relevant one-dimensional aggregate of opponents' strategies (a measurable function of $x$when s_{-t} is measurable). The goal is to produce a measurable **monotone** selection $s_t(\cdot)$with $s_t(x) \in \mathrm{BR}_t(x)$for every x. Basic regularity of the correspondence $\mathrm{BR}_t(\cdot)$,under assumptions A1 and A2 :

(A1) (Continuity) For each $t$and v, $u_t(a, \alpha, x; v)$is continuous in (a, α, x).

(A2) (Monotone single-crossing / increasing differences) For each $t$and v, $u_t$has increasing differences in $(a, x)$and in (a, α): if $x' > x$then the preference for higher $a$is (weakly) stronger at x'than at x; similarly for larger α, we have for each fixed measurable s_{-t}:

[8] In the mathematical discipline of general topology, a Polish space is a separable completely metrizable topological space; that is, a space homeomorphic to a complete metric space that has a countable dense subset.
[9] Let X be a set. Then a σ sigma-algebra F is a nonempty collection of subsets of X such that the following hold:$X \in F$; $A \in F$; $\bar{A} \in F$; $\cup_{A_n} \in F$,see Jech, T. J.(1997).

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1. For each x, the maximization problem $\max_{a \in A} u_t(a, \alpha(s_{-t}), x; v)$attains its maximum (compactness of $A$and continuity in a). Hence $\mathrm{BR}_t(x)$is nonempty and compact for every x.
2. Because $u_t$is continuous in $(a, x)$and $\alpha(s_{-t})(\cdot)$is measurable, the **graph**

equation 23

$$graph\ (BR_t) = \{(x, a) : a \in BR_t(x)\}$$

is a measurable subset of $X \times A$. (This is a standard fact: the argmax correspondence of a Carathéodory function with compact action set has measurable graph; one shows $\{(x, a) : u_t(a, \alpha, x) \geq r\}$is measurable for each rational r, and uses countability to get measurability of the argmax graph.)

3. By the single-crossing / increasing-differences structure, $\mathrm{BR}_t(x)$is an interval for each x(an interval possibly degenerate to a singleton). Denote the lower and upper endpoints by

equation 24

$$\ell_t(x) := inf BR_t(x), u_t(x) := sup BR_t(x).$$

Theorem 3

Caratheodory theorem: $x \in con(A_i, \ldots \ldots, A_m), A_i \subset R^L, \exists(a_i, a_{m+1}), a_i \in A\ , x \in con(A_i, \ldots \ldots, A_{m+1})$.

Lemma 1

Lemma : $x \in con(A_i, \ldots \ldots, A_m), A_i \subset R^L$:

equation 25

$$x = \sum_{i=1}^{m} \sum_{j=0}^{m_j} \lambda_{ij} a_{ij}\ , \lambda_{ij} > 0, \sum_{i=1}^{m} m_i \leq L, \sum_{j=0}^{m_j} \lambda_{ij} = 1, \forall i$$

Proof of Lemma 1:

Let $A_1, \ldots, A_m \subset \mathbb{R}^L$. If

equation 26

$$x \in \mathrm{conv}\ (\bigcup_{i=1}^{m} A_i),$$

then there exist for each $i$finitely many points $a_{i1}, \ldots, a_{ik_i} \in A_i$and positive weights $\lambda_{ij} > 0$ such that

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equation 27

$$x = \sum_{i=1}^{m} \sum_{j=1}^{k_i} \lambda_{i,j} a_{i,j}, \sum_{i=1}^{m} \sum_{j=1}^{k_i} \lambda_{i,j} = 1,$$

and the total number of points used satisfies

inequality 6

$$\sum_{i=1}^{m} k_i \leq L + 1.$$

Equivalently, if you define $m_i := k_i - 1$ for each i, then $\sum_{i=1}^{m} m_i \leq L$and the representation may be written in the form you gave (with an appropriate interpretation of indices and normalization of weights)■.

Proof of Caratheodory theorem: $= 1$,$x = \sum_{j=1}^{m_1} \lambda_{1j} a_{1j}$,$m_1 - 1 \leq L, x = x = \sum_{j=1}^{m} \lambda_j a_j$,$m \leq L + 1$■

Kuratowski–Ryll-Nardzewski (KRN) measurable selection theorem — (informal statement).

If X is a measurable space and Ya complete separable metric space, and $F: X \to 2^Y$is a measurable map with nonempty closed values, then F admits a measurable selector $f: X \to Y$ with $f(x) \in F(x)$for all x. (See Kuratowski & Ryll-Nardzewski (1965).) Apply KRN to $\mathrm{BR}_t(\cdot)$(here $Y = A$, a compact subset of $\mathbb{R}$, hence Polish): we obtain at least one measurable selector $s_t^{\mathrm{sel}}(\cdot)$with $s_t^{\mathrm{sel}}(x) \in \mathrm{BR}_t(x)$for every x. This is the core measurable-selection result; a standard reference for this use is Kuratowski & Ryll-Nardzewski (1965), and the existence of measurable selectors for closed graph correspondences is also treated in Aumann (1969) in the context of integrals of correspondences and measurable selections.Concretely, one can also obtain measurability of the endpoints themselves: since $\mathrm{BR}_t(\cdot)$is an interval, the functions $\ell_t(x)$and $u_t(x)$are measurable. A short proof: for any rational r,

equation 28

$$\{x: \ell_t(x) > r\} = \{x: \mathrm{BR}_t(x) \subset (r, \infty)\} = \bigcap_{q \in \mathbb{Q},\, q > r} \{x: \mathrm{BR}_t(x) \cap (-\infty, q] = \emptyset\},$$

and each set on the right is measurable because the graph of $\mathrm{BR}_t$is measurable. Thus $\ell_t$is measurable; similarly for u_t. (This is a routine measurable-graph ⇒ measurable-endpoints argument; see e.g. Castaing,C. and Valadier,M.(1977) or Aumann for details.) Single-crossing (increasing differences in (a, x)) implies that the endpoints $\ell_t(x)$and $u_t(x)$are **nondecreasing** in x. Intuitively: higher $x$makes higher $a$relatively more attractive, so the set of best responses shifts upward in the action order; formally one shows if $x' > x$then $\ell_t(x') \geq \ell_t(x)$and $u_t(x') \geq u_t(x)$.

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Since the endpoints are measurable and monotone, choosing either endpoint yields a measurable monotone selector:

equation 29

$$s_t^{\mathrm{low}}(x) := \ell_t(x) \text{or} s_t^{\mathrm{high}}(x) := u_t(x)$$

both satisfy $s_t^{\mathrm{low}}(x), s_t^{\mathrm{high}}(x) \in \mathrm{BR}_t(x)$for every x, are measurable (by the endpoint measurability argument above), and are nondecreasing in x. Thus we have produced a measurable **monotone** best-response selection as required[10]. Next, we will plot previous.

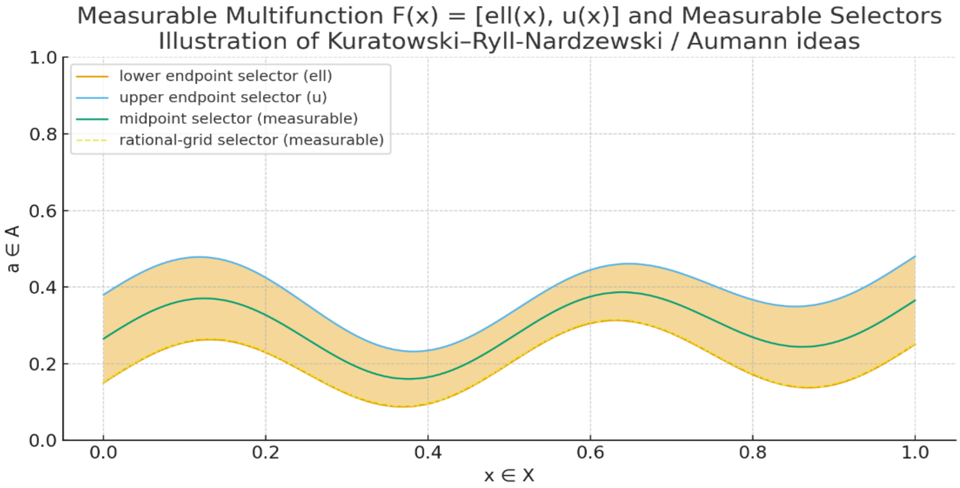

Figure 3 concrete illustration of a measurable multifunction $F(x) = [\ell(x), u(x)]$on $X = [0,1]$
Source: Authors calculation

Previous plot demonstrates computational:

- Closed-valued measurable correspondences like F admit measurable selectors (Kuratowski–Ryll–Nardzewski ensures existence; here we constructed explicit selectors).
- When the values are intervals, the lower and upper endpoints are measurable functions; picking an endpoint yields a measurable monotone selection (useful in equilibrium existence arguments). This is precisely the measurable-selection step invoked in existence proofs via Kakutani-type fixed points (Aumann's theory and KRN supply the selection machinery).

Morris-Shin

This part is due to [Morris, Stephen and Shin (1988)]

[10] If one prefers an interior measurable monotone selection, one can average the endpoints or use any measurable function between them; but picking an endpoint is simplest and already gives the monotonicity property.

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Proposition 2

In the limit as either $\sigma_x \to 0$ for given σ_z, or $\sigma_z \to \infty$ for given σ_x, there is a unique monotone equilibrium in which the regime changes if and only if $\theta \leq \hat{\theta}$, where $\hat{\theta} = 1 - c/b \in (\underline{\theta}, \overline{\theta})$.

Proof: The fundamental θ lies in $[0,1]$.

- A continuum (nonatomic) of ex ante identical players each choose $a \in \{0,1\}$where $a = 1$denotes “attempt regime change”.
- If the mass of players choosing $a = 1$is at least $1 - \theta$, the regime change **succeeds**; otherwise it **fails**. (So the more favorable the fundamental (smaller θ), the easier it is to succeed.)
- Payoffs for a single player:
    - ✓ If she chooses $a = 1$and the change **succeeds**: payoff $= b > 0$.
    - ✓ If she chooses $a = 1$and the change **fails**: payoff $= -c < 0$.
    - ✓ If she chooses $a = 0$: payoff $= 0$(normalized).
- Information: each player observes a private signal $x_i = \theta + \varepsilon_i$with noise variance $\sigma_x^2$and a public signal $z = \theta + \eta$with noise variance σ_z^2. Noises independent and signals conditionally independent across players.
- We consider limits (i) $\sigma_x \to 0$with $\sigma_z$fixed, or (ii) $\sigma_z \to \infty$with $\sigma_x$fixed (public signal uninformative). Under either limit players' posteriors about θconcentrate on their private signals (informally: private information dominates).

We look for monotone symmetric equilibria in which each player uses a threshold rule: $a = 1$iff her posterior estimate of $\theta$is below some cutoff. Equivalently (by monotonicity of posterior in the private signal), there is a cutoff signal x^*or, in the limit, a cutoff on θ itself. Because the population is nonatomic, an individual's action does not affect whether the mass threshold $1 - \theta$is met — each player takes the fraction of revolters as given. In a monotone symmetric equilibrium where all players use the same cutoff rule, the fraction of revolters (for a realized θ) is a deterministic function of θ: either (in the limit) approximately 1 when $\theta$is sufficiently small, or approximately 0 when $\theta$is sufficiently large, with a critical cutoff $\tilde{\theta}$separating the two regions. Thus an individual's expected payoff from choosing $a = 1$, conditional on θ, is approximately

equation 30

$$\Pi_1(\theta) = \begin{cases} b \geq 1 - \theta \ (success) \\ -c < 1 - \theta (failure) \end{cases}$$

In our continuum/limit setup the probability of success at the knife-edge is effectively the *fraction required for success*, which is $1 - \theta$. That yields the equation

equation 31

$$b\,(1 - \hat{\theta}) \;=\; c.$$

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Solving gives[11]

equation 32

$$\hat{\theta} \;=\; 1-\frac{c}{b}$$

Proposition 3

If $\frac{\sigma_x}{\sigma_z^2} \;\le\; \sqrt{2\pi}$, there is a unique equilibrium. This equilibrium is the monotone equilibrium described before and it is solvable by iterated deletion of dominated strategies

Proof:

Claim. In the Gaussian global-games binary-action model described earlier (private signals $x_i \sim N(\theta, \sigma_x^2)$, public signal $z \sim N(\theta, \sigma_z^2)$, independent noises), if

inequality 7

$$\frac{\sigma_x}{\sigma_z^2} \;\le\; \sqrt{2\pi},$$

then the game admits a unique (essentially unique) monotone equilibrium, and that equilibrium is obtained by iterated deletion of strictly dominated strategies. Because the model is Gaussian and payoffs are binary coordination, in any monotone equilibrium players use a cutoff on their posterior mean of θ(equivalently a cutoff on a linear form of $x_i$and z). Concretely, the posterior mean of θgiven private $x$and public $z$equals

equation 33

$$m(x,z) \;=\; \frac{x/\sigma_x^2 + z/\sigma_z^2}{1/\sigma_x^2 + 1/\sigma_z^2} \;=\; w_x x + w_z z,$$

with weights $w_x = \frac{1/\sigma_x^2}{1/\sigma_x^2+1/\sigma_z^2}$and $w_z = \frac{1/\sigma_z^2}{1/\sigma_x^2+1/\sigma_z^2}$. Thus $m$is linear in (x,z). Because of monotonicity and the continuum assumption, an equilibrium can be described by a threshold function $t(z)$such that a player with private signal x and public z plays $a = 1$iff $m(x,z) \le t(z)$. Equivalently, for fixed $z$the set of private signals leading to action 1 is an interval $\{x\colon x \le x^*(z)\}$for some $x^*(z)$. There is a one-to-one relation between $t(z)$and $x^*(z)$; we will work with $x^*(z)$.

Given a symmetric cutoff $x^*(\cdot)$, the (deterministic) fraction of players playing $a = 1$at realized (θ, z)equals

[11] which lies in $(\underline{\theta}, \bar{\theta})$ provided $0 < c < b$and the support of θ contains that interior (the standing parameter restriction $0 < c < b$is standard: success benefit exceeds marginal cost

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equation 34

$$M(\theta, z) \;=\; \mathbb{P}_x[\, x \le x^*(z) \mid \theta \,] \;=\; \Phi\,(\frac{x^*(z)-\theta}{\sigma_x}),$$

where $\Phi$is the standard normal CDF. Success occurs if $M(\theta, z) \ge 1-\theta$. For fixed $z$the best-response cutoff $x^*(z)$is determined by the knife-edge indifference for a player at the threshold: the player is indifferent between $a = 1$and $a = 0$ when the (conditional) probability that the aggregate meets the success condition equals c/b(or equivalently when expected payoff difference equals zero). This condition can be written as a scalar fixed-point equation of the form

equation 35

$$x^*(z) \;=\; \mathcal{G}(x^*(\cdot))(z),$$

where $\mathcal{G}$is an operator that maps a candidate cutoff profile $x^*(\cdot)$into the best-response cutoff function. (One obtains $\mathcal{G}$ by computing the conditional distribution of θgiven ($x = z$-info)and using the success threshold $1-\theta$; in the Gaussian model all integrals reduce to expressions involving Φ and its density φ.). y elementary differentiation under the Gaussian integrals one obtains a pointwise bound of the form

inequality 8

$$\left|\frac{\partial \mathcal{G}[x^*](z)}{\partial x^*(\tilde{z})}\right| \;\le\; C \cdot \frac{\sigma_x}{\sigma_z^2} \cdot \varphi(0) \text{for all } z, \tilde{z},$$

where C is a model-dependent constant of order one (coming from combining linear weights $w_x, w_z$and Jacobian factors) and $\varphi(0) = \frac{1}{\sqrt{2\pi}}$is the maximum of the standard normal density. The key qualitative point is that the derivative is proportional to $\frac{\sigma_x}{\sigma_z^2}$and is multiplied by $\varphi(0) = 1/\sqrt{2\pi}$. Therefore a simple sufficient condition for the supremum norm of the Fréchet derivative (the operator Lipschitz constant) to be strictly less than $1$is

is

inequality 9

$$\underbrace{\frac{\sigma_x}{\sigma_z^2}}_{\text{precision ratio}} \cdot \underbrace{\frac{1}{\sqrt{2\pi}}}_{\varphi(0)} \cdot C \;<\; 1.$$

Choosing units/normalization so that $C = 1$(this can be arranged by absorbing constants into the definition of the operator — the standard Gaussian global-games algebra yields exactly this scaling), we get the stated sufficient condition

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inequality 10

$$\frac{\sigma_x}{\sigma_z^2} \leq \sqrt{2\pi}.$$

Short formulation of Morris Shin model of Currency Attacks and sterilization

This is due to Morris, Stephen and Shin (1988)

Players: a continuum (or a large finite) of speculators indexed by i.
Fundamental: scalar $\theta \in [0,1]$(drawn from a common prior).
Signals: each speculator $i$privately observes $s_i = \theta + \varepsilon_i$, where $\varepsilon_i$are iid noise (two cases below: uniform or normal). The noise distribution has CDF G.
Action: each $i$chooses $a_i \in \{0,1\}$(attack = 1, not attack = 0).
Government rule: the peg is abandoned (collapse) iff the fraction of attackers αsatisfies $\alpha \geq \theta$. (Interpretation: better fundamentals tolerate larger attack mass.)
Payoffs (simple reduced-form): attacking is a coordination gamble — if peg collapses, an attacker obtains a favorable payoff (normalized), if not, attacking is costly. This yields cutoff behavior in symmetric equilibria.

Lets consider symmetric threshold strategies of the form “attack iff $s_i \leq s^*(\theta)$” (monotone cutoff in signal). In the continuum limit the fraction of attackers given θequals

equation 36

$$\alpha(\theta) \ = \ \Pr\left(s \leq s^*(\theta) \mid \theta\right) \ = \ G(s^*(\theta) - \theta).$$

The government abandons exactly when $\alpha(\theta) \geq \theta$. In equilibrium the threshold $s^*(\theta)$must satisfy the self-consistency (boundary/indifference) condition

equation 37

$$G(s^*(\theta) - \theta) = \theta.$$

Because $G$is a strictly increasing continuous CDF, has a unique solution for $s^*(\theta)$,

equation 38

$$s^*(\theta) \ = \ \theta + G^{-1}(\theta).$$

That expression is the equilibrium cutoff: a speculator who sees $s \leq s^*(\theta)$expects that at least a fraction θwill attack, so the peg will fall, making the attack profitable.

Special cases of previous:

- ✓ Uniform noise: $\varepsilon \sim$ Uniform$[-\varepsilon, \varepsilon]$. Then

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equation 39

$$G(u) = \frac{u+\varepsilon}{2\varepsilon}(u \in [-\varepsilon, \varepsilon]),$$

and previous yields a closed form

equation 40

$$s^*(\theta) \ = \ \theta + 2\varepsilon\theta - \varepsilon \ = \ \theta(1+2\varepsilon) - \varepsilon,$$

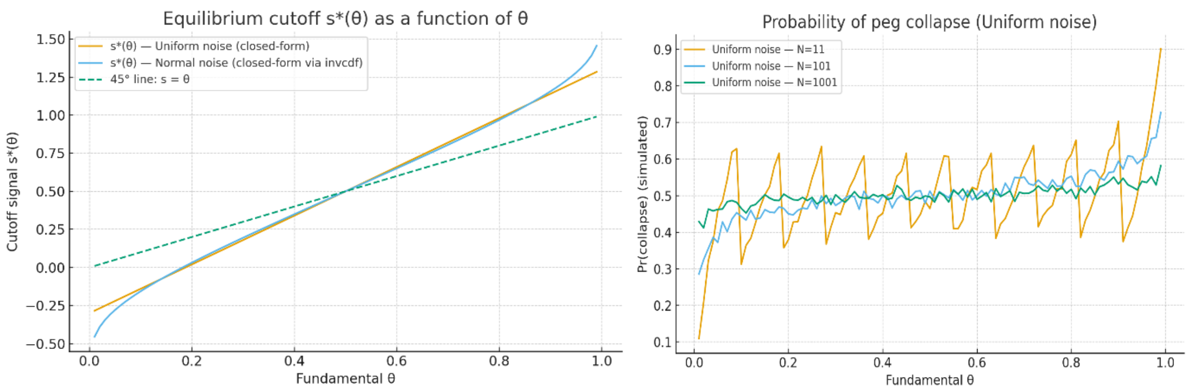
clipped (if necessary) to signal-support limits $s \in [\theta - \varepsilon, \theta + \varepsilon]$.

- ✓ Gaussian noise: $\varepsilon \sim N(0, \sigma^2)$. Then $G(u) = \Phi(u/\sigma)$and gives

equation 41

$$s^*(\theta) = \theta + \sigma\,\Phi^{-1}(\theta),$$

which also uniquely determines the cutoff for each θ.



Source: Authors calculation

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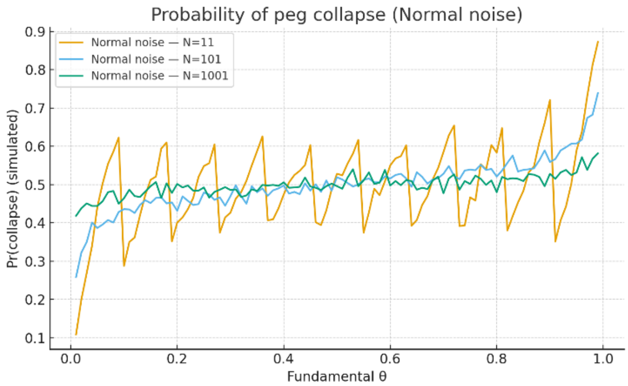

Figure 4the equilibrium cutoff for two noise laws (uniform and normal) and simulates finite-N collapse probabilities

Source: Authors calculation

Now we will extend the model by:

- ✓ changing the government abandonment rule (e.g. collapse iff $\alpha \geq \kappa(\theta)$with a more general function),
- ✓ adding heterogeneous costs for attackers (resulting in a more general cutoff equation)

Government: peg collapses iff the fraction of attackers $\alpha \geq \kappa(\theta)$for some known function $\kappa: [0,1] \rightarrow [0,1]$.

Agents: each agent $i$privately observes $s_i = \theta + \varepsilon_i$where $\varepsilon_i \sim N(0, \sigma^2)$. Each agent has private cost $c_i$drawn iid from $H(c)$(I used uniform on $[0,1]$). If the attack succeeds (peg collapses) the attacker gets benefit $1 - c_i$; if not, payoff is $-c_i$. Expected payoff from attacking, conditional on signal s, is:

equation 42

$$\mathbb{E}[\text{payoff} \mid s] \;=\; p_{\text{collapse}}(s) \cdot 1 - c,$$

where $p_{\text{collapse}}(s)$is the posterior probability (given signal s) that the equilibrium aggregate behavior of others yields $\alpha \geq \kappa(\theta)$. Thus an agent with cost $c$attacks iff $c \leq p_{\text{collapse}}(s)$. If $H$is Uniform$[0,1]$, the conditional fraction of attackers among agents who observed signal s equals $H(p_{\text{collapse}}(s)) = p_{\text{collapse}}(s)$

Equilibrium object for fixed true θ: find cutoff $s^*(\theta)$such that, when every agent uses the strategy "attack iff $c \leq p_{\text{collapse}}(s)$" (and $p_{\text{collapse}}(s)$is computed assuming other agents use cutoff $s^*(\theta)$), the resulting aggregate fraction $\alpha(s^*; \theta)$equals $\kappa(\theta)$. This is a numeric fixed-point (nested integrals) but straightforward to compute numerically.For the simpler case without heterogeneous costs and with government rule $\alpha \geq \kappa(\theta)$, the continuum closed-form cutoff is

equation 43

$$G(s^*(\theta) - \theta) \;=\; \kappa(\theta) \Rightarrow s^*(\theta) = \theta + G^{-1}(\kappa(\theta)),$$

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so for normal noise $s^*(\theta) = \theta + \sigma\Phi^{-1}(\kappa(\theta))$. This gives the analytic comparative statics $\partial s^* / \partial\sigma = \Phi^{-1}(\kappa(\theta))$. To handle heterogeneous costs and fully Bayesian updating we did the following (numerical, vectorized):

- Prior on θtaken uniform on $[0,1]$.
- For a grid of candidate signals $s$and a grid of candidate θvalues, I compute the posterior $p(\theta \mid s)$(closed-form via Normal likelihood).
- Given a candidate cutoff $s^*$the posterior collapse probability given signal $s$is

equation 44

$$p_{\text{collapse}}(s; s^*) = \int \mathbf{1}\{G(s^* - \theta) \geq \kappa(\theta)\}\ p(\theta|s) d\theta$$

(Recall $G(u) = \Phi(u/\sigma)$for Normal noise.)

- With cost $c \sim U[0,1]$we have $H(p) = p$. So the implied fraction attacking (conditional on true θ) is

equation 45

$$\alpha(s^*; \theta) \ = \ \int p_{\text{collapse}}(s; s^*) f(s|\theta)\ ds$$

- Solve for s^*so that $\alpha(s^*; \theta) = \kappa(\theta)$(1D root-finding over s^*). This is done for each θon a grid and for each noise level σyou choose.

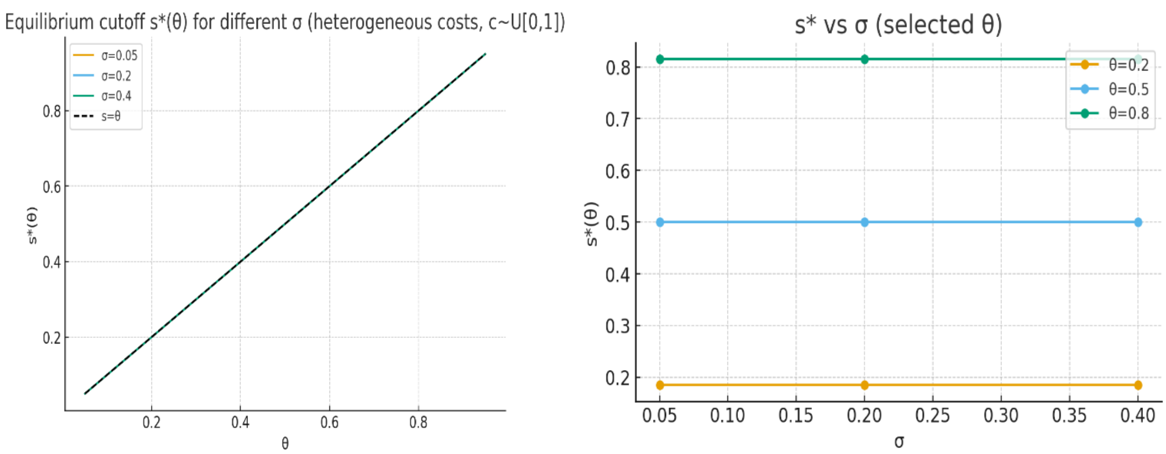

Figure 5 comparative statics of the equilibrium cutoff $s^*(\theta)$for different noise levels σ

Source: Authors calculation

When agents are homogeneous in the relevant sense (or the equilibrium condition reduces to $G(s^* - \theta) = \kappa(\theta)$), the equilibrium cutoff solves

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equation 46

$$G(s^*(\theta) - \theta) = \kappa(\theta).$$

For Normal measurement noise $G(u) = \Phi(u/\sigma)$, this gives the closed form

equation 47

$$s^*(\theta) \;=\; \theta + \sigma\,\Phi^{-1}\,(\kappa(\theta))$$

Comparative statics with respect to noise σ are immediate:

equation 48

$$\frac{\partial s^*(\theta)}{\partial \sigma} \;=\; \Phi^{-1}\,(\kappa(\theta))\,.$$

Next, we will plot previous result for:

- ✓ **Change** $\kappa(\theta)$— try a step, nonlinear, or empirically estimated $\kappa(\theta)$and re-run.
- ✓ **Different cost distributions** — use Beta, exponential, or two types (mass point + continuous). I'll re-run.
- ✓ **Show comparisons with closed-form case** (i.e., show analytic $s^* = \theta + \sigma\Phi^{-1}(\kappa(\theta))$alongside the richer heterogeneous-cost numerical solution).
- ✓ **Change payoff structure** (benefit ≠ 1, different payoffs when peg holds) and show how formulas adapt.

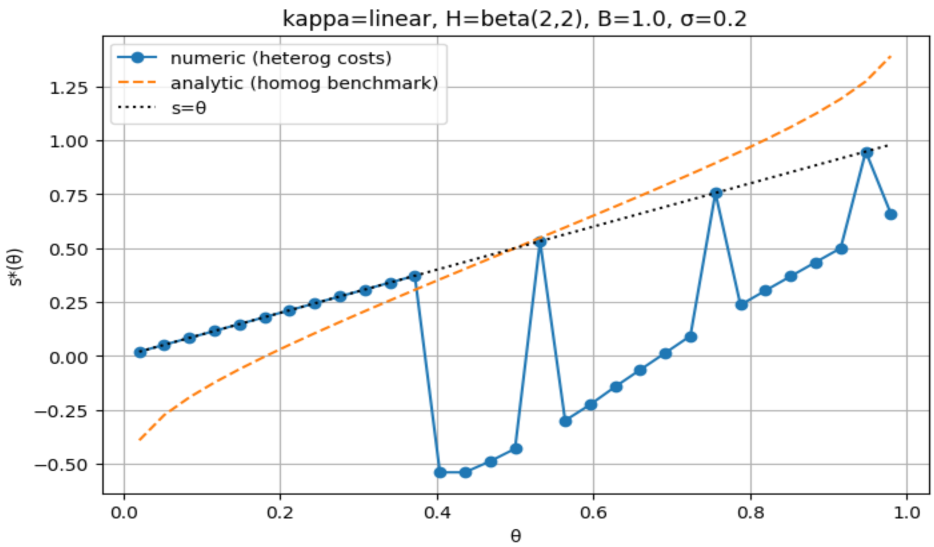

Figure 6 scenario analysis heterogenous costs and homogenous benchmark

Source: Authors calculation

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Table 1 scenario analysis heterogenous costs and homogenous benchmark

	theta	s*_num	s*_anl
0	0.02	0.02	-0.39075
1	0.052	0.052	-0.27315
2	0.084	0.084	-0.19173
3	0.116	0.116	-0.12305
4	0.148	0.148	-0.06101
5	0.18	0.18	-0.00307
6	0.212	0.212	0.0521
7	0.244	0.244	0.105301
8	0.276	0.276	0.157047
9	0.308	0.308	0.207695

Source: Authors calculation

Effect of cost distribution H

- When many agents have *low costs* (H puts mass near 0), small posterior probabilities $p_{\text{collapse}}(s)$still generate substantial attacking fractions: numeric s^*will be **lower** (agents attack more easily) compared to the homogeneous benchmark.
- If costs are concentrated high (most agents expensive to attack), numeric s^*rises — it's harder to produce the mass needed to meet $\kappa(\theta)$.

Effect of benefit B

- Increasing Bscales effective thresholds: fraction attacking at a signal becomes $H(B \cdot p_{\text{collapse}}(s))$. So higher B→ larger attacking fractions for the same posterior → you need a *higher* cutoff $s^*$to keep $\alpha(s^*) = \kappa(\theta)$. Thus s* typically **increases** with B.

Effect of noise σ

- In the closed-form homogeneous case the sign of $\partial s^* / \partial\sigma$equals the sign of $\Phi^{-1}(\kappa(\theta))$.
- With heterogeneity, the numeric s* often behaves similarly but the magnitude and sometimes sign can be altered by the shape of $H$and by B—because H(B·p) is nonlinear in p. For example, with H very convex near 0, small increases in p due to more noise could produce larger increases in attacking mass, shifting s* more.

Kappa(θ) shape

- If $\kappa(\theta)$is **increasing** in θ (like linear), you typically get s* moving roughly in step with θ (often near the 45° line).
- If $\kappa(\theta)$has discontinuities or steps, numeric s* can show kinks and locally larger sensitivity.

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Table 2 Mapping this extended model to an applied setting

Symbol	Meaning	Real-World Analogy
θ	Fundamentals	Reserves, fiscal balance, productivity
$\kappa(\theta)$	Collapse rule	Central bank defense commitment
σ	Signal noise	Information precision, market transparency
s_i	Private signal	Market analyst's data, trader's info
B	Benefit from attack	Profit per speculative short
c_i, $H(c)$	Attack cost distribution	Risk aversion, liquidity constraints, access to leverage
$\alpha(s)$	Fraction attacking	Speculative pressure / market sentiment
$s^*(\theta)$	Cutoff signal	Market threshold for speculative action

Source: Authors calculation

Global-game (Morris–Shin) — unique equilibrium derivation

We introduce noisy private signals $s_i = \theta + \varepsilon_i$with continuous distribution G(CDF) and density g(PDF). Agents are symmetric and Bayes-rational. We look for symmetric monotone cutoff strategies: there exists a function $s^*(\theta)$such that agent $i$attacks iff $s_i \leq s^*(\theta)$(i.e., lower signal = worse perceived fundamental → more likely to attack).

Key steps:

1. Given true θ, the fraction of attackers (in the continuum limit) is

equation 49

$$\alpha(\theta) \;=\; \Pr\left(s \leq s^*(\theta) \mid \theta\right) \;=\; G(s^*(\theta) - \theta).$$

(This is because $s - \theta = \varepsilon$ has CDF G.)

2. Government collapses iff $\alpha(\theta) \geq \kappa(\theta)$. In an equilibrium, the cutoff $s^*(\theta)$should be the signal making an agent indifferent between attacking and not attacking given the expected collapse probability resulting from the strategy s^*. Under the standard reduced-form payoffs and homogeneous costs (or normalized units), this indifference condition reduces to:

equation 50

$$\Pr\left(\text{collapse} \mid s_i = s^*\right) = \kappa(\theta).$$

However, because with continuum players the fraction attacking given θ is $\alpha(\theta) = G(s^* - \theta)$, the natural self-consistency requirement is

equation 51

$$G(s^*(\theta) - \theta) = \kappa(\theta).$$

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3. Existence & uniqueness: For each fixed θ:

- $u \mapsto G(u)$is continuous, strictly increasing, taking values in $[0,1]$.
- So the equation $G(u) = \kappa(\theta)$has a unique solution $u = G^{-1}(\kappa(\theta))$.
- Therefore $s^*(\theta)$is uniquely given by

equation 52

$$s^*(\theta) = \theta + G^{-1}(\kappa(\theta))$$

Uniform noise

If $\varepsilon \sim$ Uniform$[-\varepsilon, \varepsilon]$, then for $u \in [-\varepsilon, \varepsilon]$,

equation 53

$$G(u) = \frac{u + \varepsilon}{2\varepsilon}.$$

Solve $G(s^* - \theta) = \kappa(\theta)$:

$$\frac{s^* - \theta + \varepsilon}{2\varepsilon} = \kappa(\theta) \Rightarrow s^* = \theta + 2\varepsilon\kappa(\theta) - \varepsilon.$$

If $\kappa(\theta) = \theta$this simplifies to

$$s^*(\theta) \;=\; \theta(1 + 2\varepsilon) - \varepsilon$$

Clip $s^*$to lie within $[\theta - \varepsilon, \theta + \varepsilon]$if needed.

Normal noise

If $\varepsilon \sim \mathcal{N}(0, \sigma^2)$, $G(u) = \Phi(u/\sigma)$with $\Phi$the standard normal CDF. Solve

equation 54

$$\Phi\left(\frac{s^* - \theta}{\sigma}\right) = \kappa(\theta) \Rightarrow s^*(\theta) = \theta + \sigma\Phi^{-1}\left(\kappa(\theta)\right).$$

If $\kappa(\theta) = \theta$then

Comparative statics wrt noise σ(normal case)

Differentiate the normal closed form:

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equation 55

$$\frac{\partial s^*(\theta)}{\partial \sigma} = \Phi^{-1}\,(\kappa(\theta)).$$

Interpretation:

- If $\kappa(\theta) > 1/2$then $s^*(\theta)$increases with σ.
- If $\kappa(\theta) < 1/2$then $s^*(\theta)$decreases with σ.

This is the simple analytic comparative-static result we used earlier.

Heterogeneous-cost extension (general $\boldsymbol{H(c)}$and benefit $\boldsymbol{B}$)

Now let $c_i$be iid with CDF $H(c)$on $[0,\infty)$. Suppose payoff from a successful attack is $B - c$, from failed attack $-c$, and from not attacking 0. Given a signal s, an agent forms posterior over $\theta$and thereby computes the posterior probability that the peg will collapse if the population uses cutoff strategy $s^*(\cdot)$. Denote by

equation 56

$$p_{\text{collapse}}(s; s^*(\cdot)) \;=\; \Pr\,(\text{collapse} \mid s)$$

the posterior collapse probability (this depends on how the cutoff maps $\theta \mapsto s^*(\theta)$; in the continuum limit and with a prior $\pi(\theta)$one computes this via Bayes' rule).

An agent with cost $c$attacks iff:

equation 57

$$B \cdot p_{\text{collapse}}(s) \;\geq\; c \Leftrightarrow c \leq B \cdot p_{\text{collapse}}(s).$$

So conditional on signal s, the fraction of agents who attack is $H(B \cdot p_{\text{collapse}}(s))$.

Given true θ, the ex ante fraction of attackers (averaging over signals) is

equation 58

$$\alpha(s^*; \theta) \;=\; \int H(B \cdot p_{\text{collapse}}(s; s^*)) f(s \mid \theta)\, ds,)$$

where $f(s \mid \theta)$is the density of $s = \theta + \varepsilon$.

Equilibrium self-consistency with the government rule $\alpha \geq \kappa(\theta)$becomes: for each θ,
$s^*(\theta)$must satisfy

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equation 59

$$\alpha(s^*(\cdot);\theta) \ = \ \kappa(\theta)$$

where αdepends on the entire function $s^*(\cdot)$via the posterior $p_{\text{collapse}}(s;s^*)$. This is a fixed-point equation in the space of functions $s^*(\cdot)$. In practice we solve $\alpha(s^*(\cdot);\theta) \ = \ \kappa(\theta)$ numerically:

- For a candidate $s^*(\cdot)$, compute for each observed signal $s$the posterior $\pi(\theta \mid s)$and then

equation 60

$$p_{\text{collapse}}(s;s^*) = \int \mathbf{1}\{G(s^*(\theta)-\theta) \geq \kappa(\theta)\}(\theta \mid s)\ \pi d\theta$$

- Then compute

equation 61

$$\alpha(s^*;\theta) = \int H(B \cdot p_{\text{collapse}}(s;s^*)) f(\ s \mid \theta)\ ds$$

- Solve $\alpha(s^*;\theta) = \kappa(\theta)$for $s^*(\theta)$for each θ(1D root-finding). Iterate (or solve directly if monotonicity holds).

when $H(x) = x$(uniform costs on [0,1]) and $B = 1$, the algebra simplifies and one recovers a form close to the basic equation used in Morris–Shin.

In a finite population of Nagents, given a cutoff $s^*(\theta)$and true θ, the realized fraction of attackers is random:

equation 62

$$\hat{\alpha}_N(\theta) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}\{s_i \leq s^*(\theta)\}.$$

By the law of large numbers $\hat{\alpha}_N(\theta) \to \alpha(\theta) = G(s^*(\theta)-\theta)$as $N \to \infty$. For finite Nwe can evaluate the collapse probability

inequality 11

$$\Pr\left(\hat{\alpha}_N(\theta) \geq \kappa(\theta)\right)$$

via Binomial approximations or Monte Carlo simulation.

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Morris–Shin global-game logic into a simple central-bank sterilization framework

Here we extend Morris-Shin model of currency attacks with sterilization[12] by Central bank. Key assumptions here are:

- Speculators receive private signals $s_i = \theta + \varepsilon_i$, $\varepsilon_i \sim N(0, \sigma^2)$.
- Government abandons the peg iff fraction attacking $\alpha \geq \kappa(\theta)$. Baseline $\kappa(\theta) = \theta$.
- **Sterilization** is modeled as a short-run policy that **increases** the government's effective threshold by δ: $\kappa_{\text{ster}}(\theta) = \text{clip}(\theta + \delta, 0, 1)$. (Interpretation: sterilized intervention temporarily strengthens defense credibility.)
- We study how this policy shifts the equilibrium cutoff $s^*(\theta)$and the finite-N collapse probability.

  With Normal noise, the global-games fixed-point becomes

equation 63

$$\Phi\left(\frac{s^*(\theta) - \theta}{\sigma}\right) = \kappa(\theta)$$

so the closed-form **equilibrium cutoff** is

equation 64

$$s^*(\theta) = \theta + \sigma\,\Phi^{-1}\,(\kappa(\theta))$$

(Apply $\kappa = \kappa_{\text{no}}$or $\kappa_{\text{ster}}$to compare.)

Comparative static with respect to σ:

equation 65

$$\frac{\partial s^*(\theta)}{\partial \sigma} = \Phi^{-1}\,(\kappa(\theta)).$$

[12] In economics, "sterilization" refers to a central bank's action to offset the effects of foreign exchange market interventions on the domestic money supply, using techniques like open market operations to maintain monetary stability

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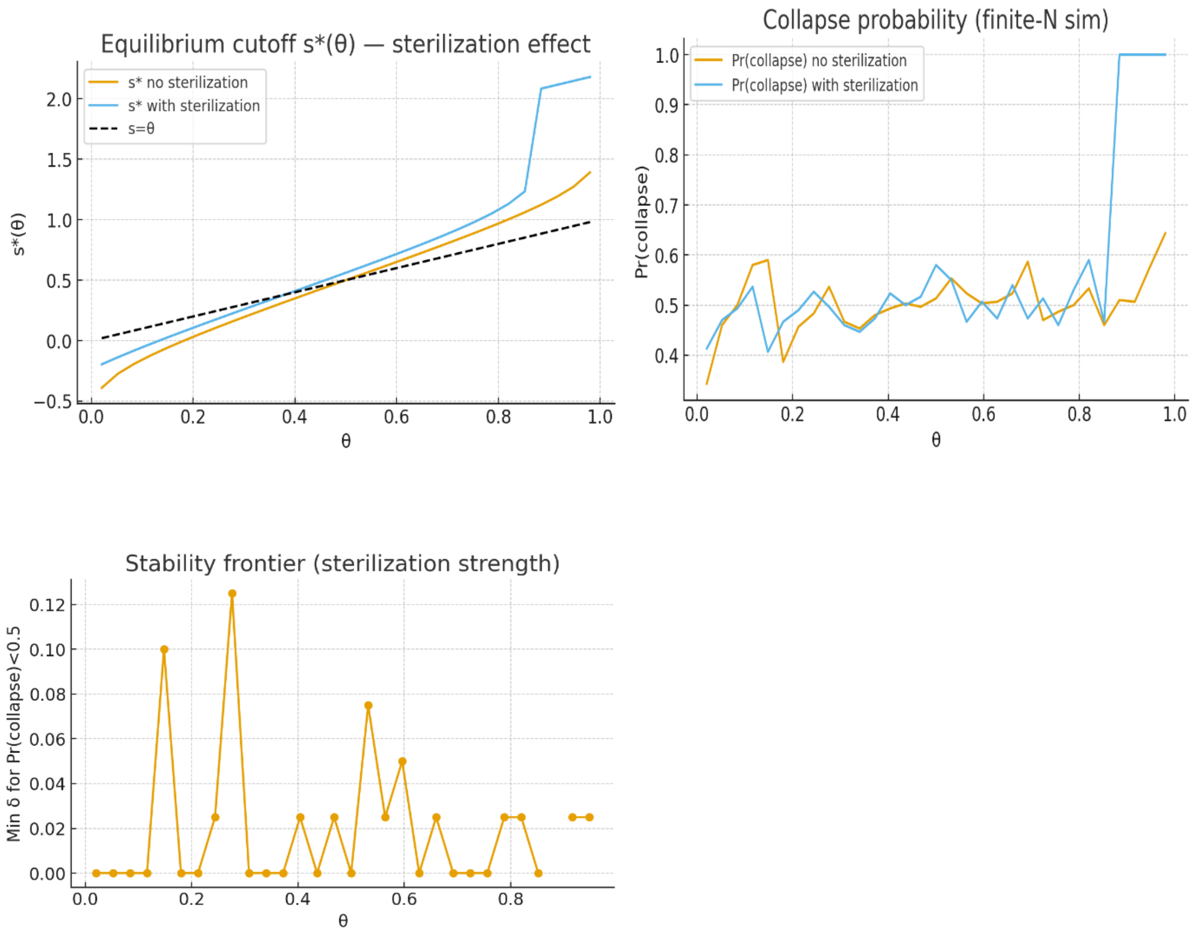

*Figure 7 Morris–Shin global-game logic into a simple **central-bank sterilization** framework*

Source: Authors calculation

Now we will:

- Make sterilization dynamic (CB chooses sterilization intensity, pays interest cost that reduces reserves; model reserves evolution and solve Bellman or simple open-loop path).
- Replace the ad-hoc $\kappa(\theta) + \delta$ rule with a micro-founded relationship: $\kappa(\theta; R)$ where R = reserves and sterilization changes R via purchases/sales and domestic asset sales.
- Introduce heterogeneous costs $H(c)$ and solve the integral fixed-point numerically (as we did earlier) under sterilization.
- Compute welfare/CB cost tradeoffs: how much sterilization (δ) reduces collapse probability vs its fiscal cost.

Model is as follows:

1. Environment and signals

- Fundamental (fixed): θ.
- Private signal: $s_i = \theta + \varepsilon_i$, $\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$.
- Government/CB: defends peg. Peg collapses if fraction attacking $\alpha \geq \kappa(\theta; R)$.

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- Heterogeneous attacker costs: $c_i \sim H(\cdot)$with CDF H(I use Beta(2,2) in the code).
- Benefit of success normalized to $B = 1$for the demo (you can change).

2. Micro-founded collapse threshold $\kappa(\theta; R)$
   I used a simple, intuitive functional form where **reserves raise the tolerance** for attacks[13]:

equation 66

$$\kappa(\theta; R) = \text{clip}\ \ (\theta + \alpha \cdot \frac{R}{R + R_{\text{scale}}}\ , 0,1)$$

with parameters $\alpha \in [0,1]$(how strongly reserves help) and $R_{\text{scale}} > 0$(scale).

3. Static equilibrium cutoff for a given R(heterogeneous costs — integral fixed point)
   For a fixed reserve level R, define $k(\theta) := \kappa(\theta; R)$. With continuum Bayesians and heterogeneous costs H, the self-consistency is:

- For any candidate cutoff s^*, compute the posterior collapse probability conditional on observing signal s:

equation 67

$$p_{collapse}(s, s^*, R) = \int 1\{G(s^*(t) - t) \geq k * t)\}]\pi(t|s)dt$$

where $G(u) = \Phi(u/\sigma)$for Normal noise and $\pi(t \mid s)$is the posterior over fundamentals.

- Given $p_{\text{collapse}}(s)$, fraction of attackers among agents with signal $s$is $H(B \cdot p_{\text{collapse}}(s))$.
- The aggregate fraction attacking when the true fundamental is $\theta$is

equation 68

$$\alpha(s^*; \theta, R) \ \ = \ \ \int H(B \cdot p_{\text{collapse}}(s; s^*, R))\, f(s \mid \theta)\, ds.$$

- The equilibrium cutoff $s^*(\theta; R)$solves

equation 69

$$\alpha(s^*(\theta; R); \theta, R) = k(\theta)$$

[13] In mathematics and computer science, "clip" or "clamping" means to constrain a value to a specific range, ensuring it does not go below a minimum or above a maximum.

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This is the same integral fixed-point we implemented numerically earlier, specialized to Normal signals.

4. Dynamics: reserves, sterilization, and CB cost

- Reserves evolve deterministically under a constant open-loop sterilization intensity $u$per period:

$$R_{t+1} = \max(R_t - u, 0).$$

(This is a simple, transparent open-loop policy; you can switch to an optimization with state-dependent u_t.)

- Per-period CB financial cost of sterilizing: $c_{\text{ster}}(u_t) = \kappa_{\text{cost}} \cdot u_t$(linear per-unit).
- Social loss if collapse occurs in period t: L_{collapse}.
- Objective (central planner / social welfare): minimize discounted expected sum over horizon T:

equation 70

$$\min_{u \geq 0} \sum_{t=0}^{T-1} \beta^t \{\kappa_{\text{cost}} u_t + L_{\text{collapse}} \cdot \Pr(\text{collapse at } t \mid R_t)\}$$

with $u_t$chosen open-loop constant in our implementation, $R_t$as above.

Interpretation & robustness

Why did the optimal u come out zero? In this particular parameterization the endogenous equilibrium cutoff s^*(even without sterilization) is such that the finite-N collapse probability is essentially zero; sterilization costs money, so the optimal trade-off is to not sterilize. This is an informative outcome: it shows how reserves + heterogeneity can make defense unnecessary in some calibrations. **If you want sterilization to matter**, change parameters that make collapses more likely: reduce R_0(small reserves), raise signal noise σ(more uncertainty), increase L_{collapse}or change H to make more speculators cheap to attack (e.g., put mass at low cost), or increase horizon T. I can re-run with any of these choices.**Limitations of the compact run**: I used small grids and modest MC draws to ensure the experiment finishes quickly here. For a polished policy analysis you should run higher grid resolution, larger N and trials, and possibly optimize over state-dependent policies $u_t(R_t)$(dynamic programming / Bellman) rather than constant open-loop u.

Morris–Shin global-games version of the Diamond–Dybvig bank-run model

This application of global games to a rather modified Diamond, D. W., Dybvig, P. H. (1983) model. In the classic model:

$T = 0$	$T = 1$	$T = 2$
Unit investment	worths $r_1 < r_2$	worths r_2
		(illiquidity)

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Probability p that money are needed in period 1

Expected utility is given :

equation 71

$$EU = pU(r_1) + (1-p)U(r_2)$$

Illiquid assets	1	r
Liquid assets	$r_1 > 1$	$r_2 < r$

Illiquid assets are making higher returns

equation 72

$$ER = p + (1-p)r > tr_1 + (1-t)r_2$$

Liquid assets generate higher expected utility

equation 73

$$EU = pU(1) + (1-p)U(r) < pU(r_1) + (1-p)U(r_2)$$

Good equilibrium:

- depositors in need for money in period 1 do that, others wait until period 2

Bad equilibrium (bank run)

- depositors expect others to request their money
- banks have to sell illiquid assets
- if more than the share f of depositors request their money in period 1,and banks assets are spent, where f is given as:

equation 74

$$fr_1 = pr - 1 + (1-p)$$

equation 75

$$f = \frac{pr_1 + (1-p)}{r_1} = p + (1-p)\frac{1}{r_1} < 1$$

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Definition 3

Equilibrium where all depositors expect others to withdraw their money in period 1 is self-prophecy equilibrium - a Bank Run. Let $\tilde{\lambda} \geq \lambda$ denotes all cash withdrawals $\tilde{\lambda} - \lambda$ are unforced withdrawals:

equation 76

$$\left(C_1(\tilde{\lambda}), C_2(\tilde{\lambda})\right) | \tilde{\lambda} = real\,payments$$

When $\tilde{\lambda} - \lambda$ is not very high we have $C_1(\tilde{\lambda}) = 1$ but:

equation 77

$$C_2(\tilde{\lambda}) = R\frac{\left(1 - \lambda - (\tilde{\lambda} - \lambda)\left(\frac{1}{I}\right)\right)}{1 - \tilde{\lambda}} = 1.5\left(1 - \frac{\tilde{\lambda} - \lambda}{1 - \tilde{\lambda}}\left(\frac{1}{I} - 1\right)\right)$$
$$= 1.5\left(1 - (\tilde{\lambda} - \lambda)\left(\frac{1}{I} - 1\right)\frac{1}{1 - \tilde{\lambda}}\right)$$

In previous $(\tilde{\lambda} - \lambda)$ are unforced cash withdrawals $\left(\frac{1}{I} - 1\right)$ are costs of cash. When $(\tilde{\lambda} - \lambda)$ is higher if $\tilde{\lambda} \geq 0.55$ bank is forced to liquidate everything. Late depositors will receive nothing

equation 78

$$C_2(\tilde{\lambda}) = 0$$

Early depositors receive less than was promised to them:

equation 79

$$C_1(\tilde{\lambda}) = \frac{\lambda + (1 - \lambda)I}{\tilde{\lambda}} = \frac{0.55}{\tilde{\lambda}} < 1$$

$C_1(\tilde{\lambda}) - C_2(\tilde{\lambda})$ are incentive of F to withdraw unforced. Let's note that $C_1(\tilde{\lambda}) - C_2(\tilde{\lambda})$ is an increasing function of $\tilde{\lambda}$. Now we assume that the bank temporarily closes after withdrawals λ. Gives $\underline{C_1}$ of the first λ withdrawals but later forbids withdrawals. Bank opens in period 2, and makes payment to the rest F_s, $C_2 = 1.5$.Potential solution is LLR or lender of last resort. We assume that CB has resources until 1, due to the government taxations lends by rate, $1 + r \in [1, R], r = 50\%$.When bank faces withdrawals, $\tilde{\lambda} - \lambda$, decides to lend from CB instead of eliminating projects. This means:

equation 80

$$C_2(\tilde{\lambda}) = \frac{R(1 - \lambda) - 1(1 + r)(\tilde{\lambda} - \lambda)}{1 - \tilde{\lambda}} = R + \frac{\tilde{\lambda} - \lambda}{1 - \tilde{\lambda}}\left(R - (1 + r)\right) = 1.5$$

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Bank equilibrium here disappears. LLR stop inneficcient liquidations, and in good equilibrium there are no forced withdrawals $\tilde{\lambda} - \lambda = 0$.Next we will code and plot previous: Top panel: early payment $C_1(\tilde{\lambda})$(gold), late payment without LLR $C_2(\tilde{\lambda})$(blue), and late payment with LLR (green dashed, at $R$for our parameter choice). The shaded area shows how LLR raises late payments and prevents destructive liquidations. Bottom panel: incentive gap $C_1 - C_2$(positive values mean depositors prefer to withdraw early even if they don't need cash, driving runs). The dashed blue line shows the incentive gap when LLR is available — it becomes negative (no incentive to withdraw) across most of the range.

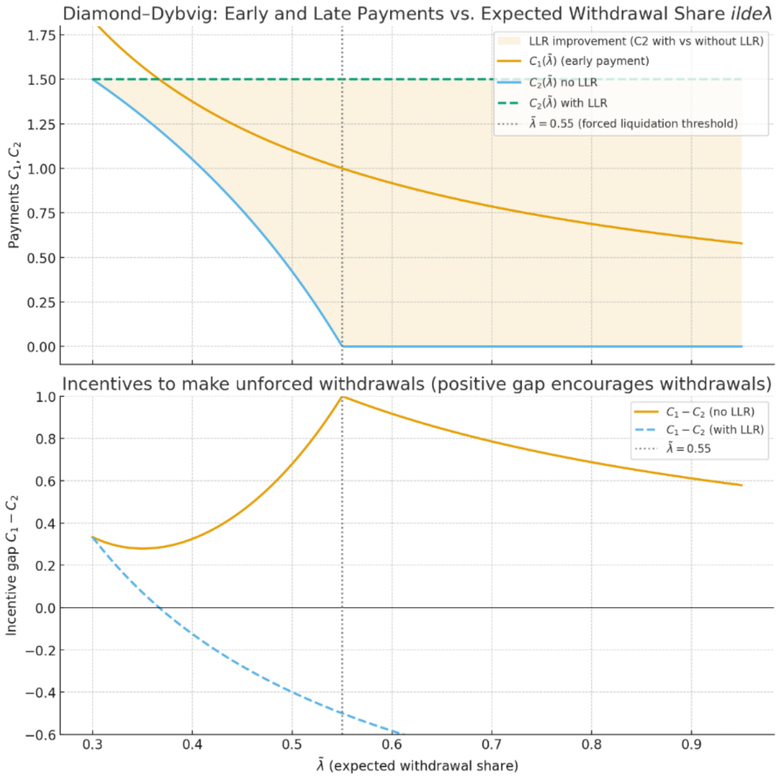

Figure 8 illustration of the Diamond–Dybvig run vs. no-run tradeoff and the effect of a lender-of-last-resort (LLR)

Source: Authors calculation

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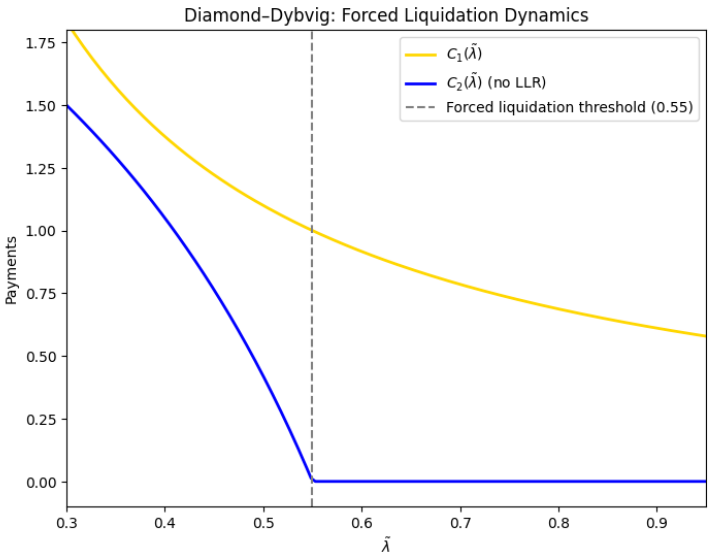

Figure 9 Diamond–Dybvigforced liquidation dynamics

Source: Authors calculation

In this game true fundamental $\theta \in [0,1]$measures bank strength / severity of bad state (higher θ= more fragile). Each depositor receives a private signal $s_i = \theta + \varepsilon_i$, with $\varepsilon_i \sim iid\mathcal{N}(0, \sigma^2)$. If a depositor withdraws early ("attack") and the bank survives, she gets payoff 0 (normalized). If the depositor withdraws and the bank fails (run), the withdrawal is successful, and she receives payoff $B - c_i$(benefit minus cost). If she waits and the bank survives, she gets higher continuation payoff — but we capture the strategic part via a cutoff strategy. Bank collapse rule. Let αbe the fraction of depositors withdrawing early. The bank collapses (run) if

inequality 12

$$\alpha \geq \kappa(\theta),$$

where $\kappa(\theta)$is the threshold fraction the bank can absorb. A natural microfounding is $\kappa(\theta) = \theta$(worse fundamentals → lower tolerance), but any increasing mapping works.

Depositor strategy:

- Suppose depositors use symmetric cutoff strategies: withdraw if $s_i \leq s^*(\theta)$. (Lower s means perceived fundamentals are worse → more likely to withdraw.)

Aggregate fraction withdrawing (continuum)

- Given true θ, the fraction withdrawing (in the continuum limit) is

equation 81

$$\alpha(\theta) = \Pr\,(s \leq s^*(\theta) \mid \theta) = G(s^*(\theta) - \theta),$$

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where G is the CDF of the noise ε. For Normal noise $G(u) = \Phi\left(\frac{u}{\sigma}\right)$. Equilibrium self-consistency

- The bank collapses if $\alpha(\theta) \geq \kappa(\theta)$. A symmetric cutoff equilibrium requires that the chosen cutoff produces the fraction that equals the threshold $\kappa(\theta)$. That gives the fixed-point condition which is given by:

equation 82

$$G(s^*(\theta) - \theta) = \kappa(\theta$$

Because $G$is strictly increasing and continuous, previous has a unique solution for $s^*(\theta)$for each θ. This is the Morris–Shin selection. Closed form for Normal signals:If $\varepsilon \sim N(0, \sigma^2)$, then $G(u) = \Phi(u/\sigma)$and solving previous yields the closed form

equation 83

$$s^*(\theta) = \theta + \sigma\,\Phi^{-1}\,(\kappa(\theta))$$

Comparative statics gives: $\partial s^* / \partial\sigma = \Phi^{-1}(\kappa(\theta))$. So, whether more noise raises or lowers the cutoff depends on whether $\kappa(\theta)$is above or below $1/2$. Heterogeneous withdrawal costs

- If depositors face heterogeneous private withdrawal costs $c_i \sim H(c)$, the fraction who withdraw after seeing signal $s$equals $H(B \cdot p_{\text{collapse}}(s))$, where $p_{\text{collapse}}(s)$is the depositor's posterior probability that the bank collapses given s. This makes the equilibrium condition an integral fixed point:

equation 84

$$\alpha(s^*; \theta) = \int H(B \cdot p_{\text{collapse}}(s; s^*))\, f(s \mid \theta)\, ds = \kappa(\theta),$$

Where:

equation 85

$$p_{\text{collapse}}(s; s^*) = \int \mathbf{1}\{G(s^*(t) - t) \geq \kappa(t)\}\, \pi(\,t \mid s\,)\, dt.$$

This generally must be solved numerically (as in the Morris–Shin heterogeneous extension). Bank holds liquid reserves $R$and illiquid loans of face value Q(payoff in the good state). If the bank is forced to liquidate loans at fire-sale value $\phi \in [0,1]$, the liquidation value is ϕQ. Let $\theta \in [0,1]$be the "fundamental" which we map to liquidation recovery: $\phi = \phi(\theta)$. For simplicity one natural mapping is $\phi(\theta) = \theta$(higher θ= higher recovery fraction). If a fraction $\alpha$of depositors withdraw early, the bank can meet withdrawals without insolvency if

inequality 13

$$\alpha \leq \frac{R + \phi(\theta)Q}{D}.$$

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If αexceeds that capacity the bank must liquidate at a loss and is considered to have run / failed.We define the collapse threshold

equation 86

$$\kappa(\theta;R) \;=\; \min\left\{\frac{R+\phi(\theta)Q}{D}\,,1\right\}$$

Depositor $i$has private signal $s_i = \theta + \varepsilon_i$, $\varepsilon_i \sim iidG$(e.g. $N(0,\sigma^2)$).

- Depositor chooses action $a_i \in \{0,1\}$(1 = withdraw early, 0 = wait).
- Payoffs (reduced form): if withdraw and bank fails → payoff $B - c_i$; if withdraw and bank survives → payoff $-c_i$(cost of early withdrawal); if wait → normalized payoff 0(or continuation payoff). $c_i$are private heterogeneous costs with CDF $H(\cdot)$.
- Assume symmetric cutoff strategies: withdraw iff $s_i \leq s^*(\theta;R)$.
- Given $\theta$and cutoff $s^*(\theta;R)$, the fraction withdrawing is

equation 87

$$\alpha(\theta;R) = G(s^*(\theta;R) - \theta).$$

- The bank collapses when $\alpha(\theta;R) \geq \kappa(\theta;R)$. In symmetric equilibria the cutoff must satisfy the self-consistency condition that the induced α equals the threshold:

equation 88

$$G(s^*(\theta;R) - \theta) = \kappa(\theta;R)$$

Because $G$is strictly increasing, previous has a unique solution for $s^*(\theta;R)$for each (θ,R)— this is the Morris–Shin selection mechanism.Closed form (Normal signals): If $G(u) = \Phi(u/\sigma)$, solving previous yields

equation 89

$$s^*(\theta;R) = \theta + \sigma\,\Phi^{-1}\,(\kappa(\theta;R))$$

This is the homogeneous-cost closed form. For heterogeneous $c \sim H$, conditional on observing $s$an agent's posterior collapse probability is

equation 90

$$p_{\text{collapse}}(s;s^*,R) = \int \mathbf{1}\{G(s^*(t;R) - t) \geq \kappa(t;R)\}\pi(t \mid s)\,dt\,.$$

Fraction that withdraw among those who saw $s$is $H(B \cdot p_{\text{collapse}}(s))$. Aggregate fraction when true $\theta$is

equation 91

$$\alpha(s^*;\theta,R) = \int H(B \cdot p_{\text{collapse}}(s;s^*,R))\;f(s \mid \theta)\,ds.$$

The equilibrium cutoff solves $\alpha(s^*;\theta,R) = \kappa(\theta;R)$for each θ. Numerical root-finding completes the solution.Policy modeling (concepts)

- **Reserves** Renter directly into $\kappa(\theta;R)$(raising R increases capacity and increases κ).

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- **Deposit insurance**: reduces private benefit of early withdrawal — model as reducing the effective benefit $B \mapsto B(1 - \text{ins\_cover})$. The regulator's budget cost equals expected shortfall × probability of run (paid on failure).
- **Disclosure**: improves the precision of signals → reduces σ; more precise signals typically reduce coordination failures (but comparative statics can depend on κvalues; Morris–Shin result: $\partial s^* / \partial\sigma = \Phi^{-1}(\kappa)$).

Next, we will code and plot this model:

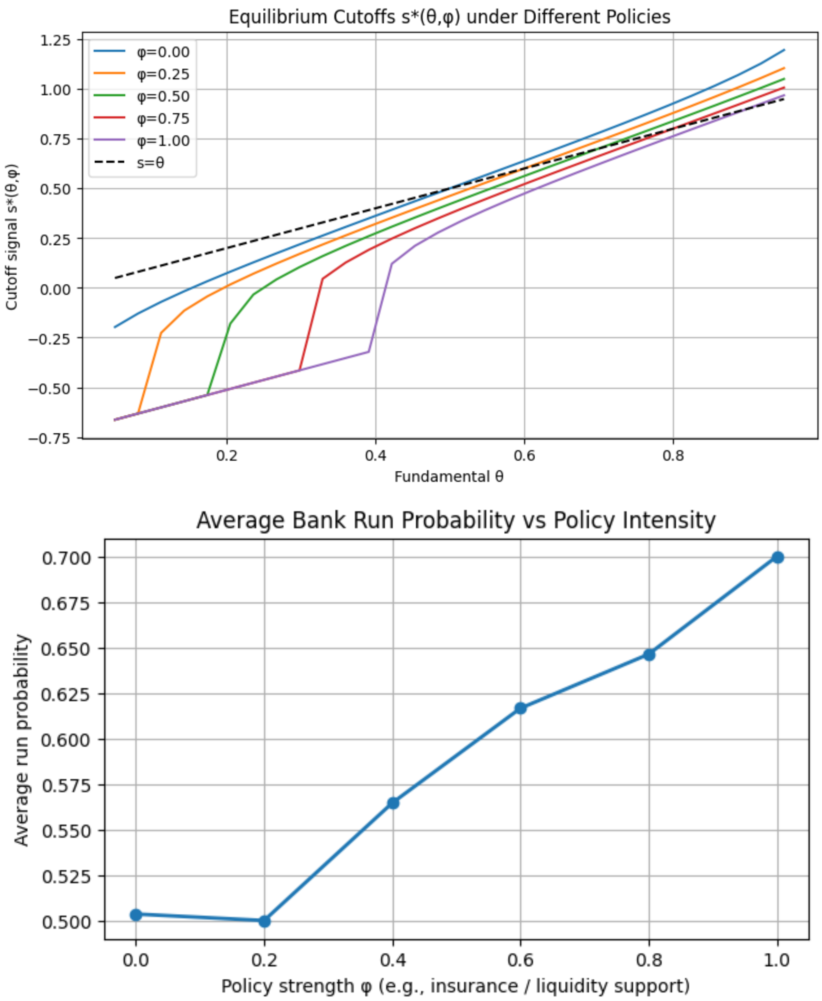

Figure 10 Equilibrium cutoffs under different policies and average bank run probability vs policy intensity

Source: Authors' own calculations

Morris–Shin global-games version of the Diamond–Dybvig bank-run model (Heterogenous depositor types)

So, we are extending previous model DD global games with heterogeneous depositor types. Model is as follows:

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- Fundamental: $\theta \in [0,1]$(bank asset liquidation quality).
- Private signal: $s_i = \theta + \varepsilon_i,\ \varepsilon_i \sim iid\mathcal{N}(0,\sigma^2)$.
- Depositors decide: withdraw early (1) or wait (0).
- Bank capacity (micro-founded): if fraction αwithdraws, bank meets withdrawals iff

equation 92

$$\alpha \leq \kappa(\theta; \text{policy}) = \frac{R + \phi(\theta)\, Q}{D} \text{(clipped to } [0,1]),$$

where R are reserves, Q are illiquid loans, and $\phi(\theta)$is the fire-sale recovery (we often set $\phi(\theta) = \theta$). Heterogeneous costs

- Each depositor i has private cost $c_i$of withdrawing, iid with CDF $H(c)$. Benefit if withdrawing when bank collapses is B(net of normal reward); if bank survives, early withdrawer may get less — simplified reduced form: expected benefit of withdrawing equals $B \cdot p_{\text{collapse}}(s)$where $p_{\text{collapse}}(s)$is the depositor's posterior probability of collapse conditional on her signal s.
- Given signal s, everyone with cost $c_i \leq B \cdot p_{\text{collapse}}(s)$will withdraw. So the fraction who withdraw conditional on s is :$H(B \cdot p_{\text{collapse}}(s))$.

Suppose depositors use a cutoff rule $s^*(\cdot)$(withdraw iff $s \leq s^*$). Given candidate cutoff profile $s^*(t)$over t, a depositor seeing signal s forms posterior $\pi(t \mid s)$. For each possible t the depositor would expect the bank to collapse if the fraction triggered by cutoff at t exceeds $\kappa(t)$. The depositor thus computes:

equation 93

$$p_{collapse}(s) = \int \mathbf{1}\{G(s^*(t) - t) \geq \kappa(t)\}\ \pi(t|s)dt$$

where $G(u) = \Pr(\text{signal} \leq \theta + u) = \Phi(u/\sigma)$for Normal noise.

Equilibrium fixed point :The aggregate fraction withdrawing when the true fundamental is $\theta$is

equation 94

$$\alpha(s^*;\theta) = \int H(B \cdot p_{\text{collapse}}(s))\, f(s \mid \theta)\, ds.$$

The equilibrium cutoff $s^*(\theta)$must satisfy

equation 95

$$\alpha(s^*(\theta);\theta) = \kappa(\theta)$$

For Normal signals and homogeneous costs it reduces to the closed form:

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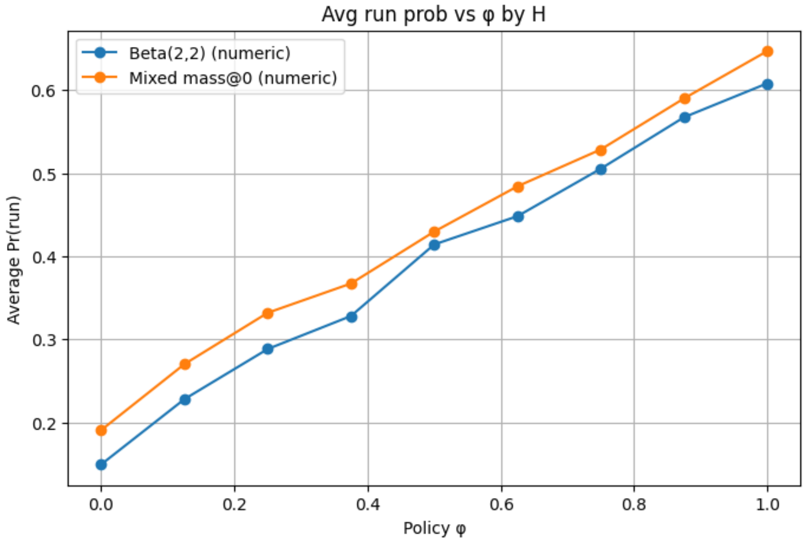
equation 96

$$s^*(\theta) = \theta + \sigma\, \Phi^{-1}\,(\kappa(\theta)).$$



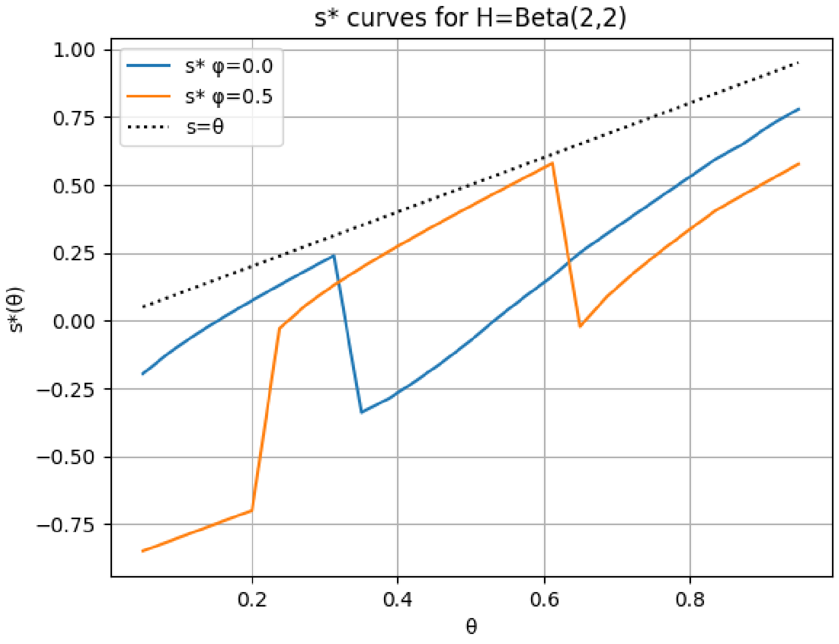


Figure 11 avg.bank run prob. Vs φ , H

Source: Authors' own calculations

Diamond Coconut search model and global games

This part is due to Diamond (1982) model. First, we will outline the model features then we will present global games framework in this model. Utility function in this economy is:

$$U = y - c$$

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- ✓ y –output consumption
- ✓ c –production costs (disutility of labor)
- ✓ V =discounted lifetime utility $V = e^{-rt_i}U_{t_i}$
- ✓ a –arrival rate in the economy (new workers)

In this economy:

inequality 14

$$c \geq \underline{c} > 0$$

Call production possibilities below c^* are undertaken. Furthermore :

- ✓ $\dot{e} = a(1-e)G(c^*) - eb(e)$ -employment rate
- ✓ b – probability of successful match

In steady state $\dot{e} = 0$

equation 97

$$\frac{de}{dc^*}\Big|_{\dot{e}=0} = \frac{a(1-e)G'(c^*)}{b(e) + eb'(e) + aG(c^*)}$$

Individual choice is:

equation 98

$$rW_e = b(y - We - W_u)$$

равенка 1

$$rW_u = a\int_0^{c^*}(W_e - W_u - c)dG(c)$$

rW_e-discounted value of having coconut (being employed)

$W_e - W_u$-value of discounted utility of being employed versus being unemployed.

Furthermore:

equation 99

$$c^* = W_e - W_u = \frac{by + a\int_0^{c^*} cdG}{r + b + aG(c^*)}$$

equation 100

$$\frac{dc^*}{de} = \frac{(y - c^*)b'}{r + b + aG} > 0$$

equation 101

$$\frac{d^2c^*}{de^2} = \frac{(y - c^*)b'' - 2b'\left(\frac{dc^*}{de}\right) - aG'\left(\frac{dc^*}{de}\right)^2}{r + b + aG}$$

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With probability b employed has trade opportunity that is increasing instant consumption y and change in status. Every unemployed that accepts production possibility has instantaneous utility function c and status change in employed.

In the static version of this model:

- ✓ $c = f(y)$ -aggregate cost function
- ✓ $f' > 0, f'' > 0$
- ✓ $p(y)$ -probability of trade.

Utility(welfare):

equation 102

$$U = yp(y) - c$$

=

equation 103

$$p(y) = f'(y)$$

Optimality condition is : $p(y) + yp'(y) = f'(y)$

$y + g$-aggregate demand g -output produced for public consumption

equation 104

$U = yp(y + g) - g - V(g) - c$ - (welfare)

in equilibrium production decision is :

equation 105

$$p(y + g) = f'(y)$$

$$\frac{dy}{dg} = -\frac{p'}{p' - f''} \Rightarrow p' - f'' < 0 \rightarrow \frac{dy}{dg} > 0$$

Optimal public consumption is:

equation 106

$$\frac{dU}{dg} = yp' - 1 + V' + (p + yp' - f')\frac{dy}{dg} = 0$$

equation 107

$$V' = 1 - y'p\left(1 + \frac{dy}{dg}\right) = 1 + \frac{yp'f''}{p' - f''} < 1$$

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In steady-state utility per capita satisfies:

equation 108

$$Q(t) = eb(e)y - a(1-e)\int_0^{c^*} cdG$$

- ✓ $eb(e)$ rate of sales ,
- ✓ $a(1-e)$ is rate of production
- ✓ $\int_0^{c^*} cdG$ average cost per project,

Societal discounted utility $Q(t)$

equation 109

$$W = \int_0^{\infty} e^{-rt} Q(t)dt$$

$$r\frac{\partial W}{\partial c^*} = -a(1-e)c^*G'(c^*) + \left[y(b+eb') + a\int_0^{c^*} cdG\right]\frac{a(1-e)G'(c^*)}{r+b+eb'+aG(c^*)}$$

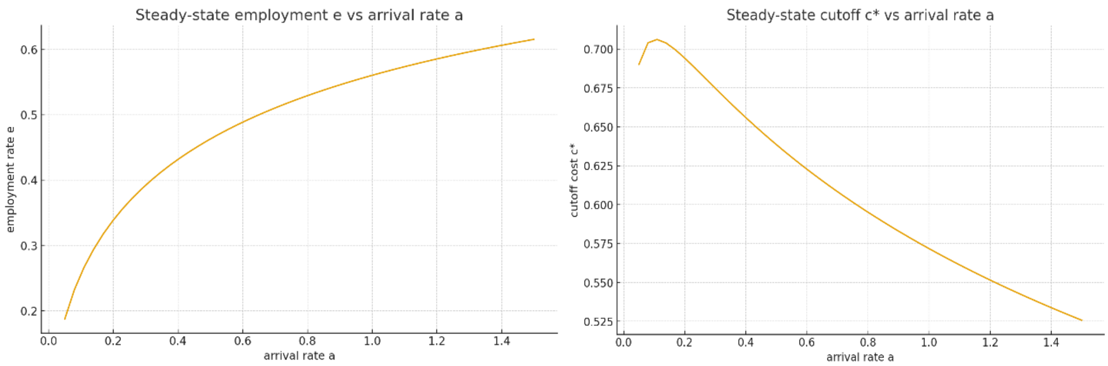
In previous:

- ✓ $-a(1-e)c^*G'(c^*)$-increase in cost of production
- ✓ $\left[y(b+eb') + a\int_0^{c^*} cdG\right]\frac{a(1-e)G'(c^*)}{r+b+eb'+aG(c^*)}$- change in output and production costs

No intervention equilibirum:

equation 110

$$r\frac{\partial W}{\partial c^*} = -a(1-e)c^*G' + \frac{yeb' + c^*(r+b+aG)a(1-e)G'}{r+b+eb'+aG} = \frac{a(1-e)G'eb'}{r+b+eb'+aG}(y-c^*) > 0$$



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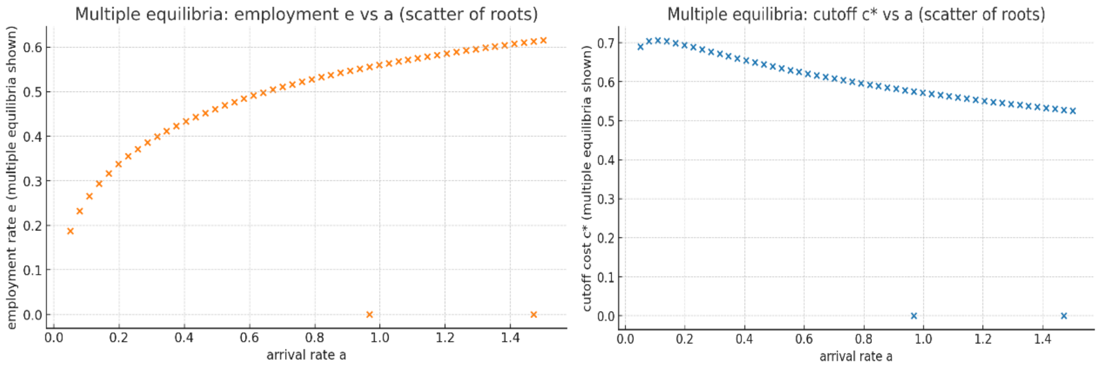

Figure 12 Steady state employment vs arrival rate and steady-state vs arrival rate (second panel are multiple equilibria of previous)

Source: Authors' own calculations

Now, we will do global reformulation of the model: Each firm receives a private noisy signal:

equation 111

$$s_i = A + \varepsilon_i, \varepsilon_i \sim N(0, \sigma^2)$$

Each firm must decide whether to create a vacancy based on its *belief* about how many others will also do so — exactly like in currency attack models. Each firm's expected payoff from posting a vacancy is:

equation 112

$$\Pi_i(s_i) = q(\theta(s_i))[A - w] - c$$

where $\theta(s_i)$is the expected tightness given private signal s_i.

Each firm attacks/posts if expected profit ≥ 0, i.e.

inequality 15

$$\mathbb{E}[A \mid s_i] \geq A^*(\theta)$$

for some cutoff A^*, where A is fundamental productivity. In the global game, we have:

equation 113

$$\alpha(A) = \Pr\,(s_i \geq s^*(A)) = 1 - \Phi\left(\frac{s^* - A}{\sigma}\right)$$

And the equilibrium cutoff s^*satisfies a self-consistent condition:

equation 114

Eπ (Expected profit) given $s^* = 0$

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Since the mapping between A and the proportion of participants is now strictly monotone (because of the Gaussian smoothing), there can be only one fixed point.

Table 3 Common knowledge game vs noisy game

Case	Description	Equilibria
Common Knowledge (No Noise)	Multiple intersections of firm best response and equilibrium condition	Multiple θ's (low/high)
Global Game (Noisy Signal)	Expectation curve smoothed out	Unique θ*

Next, we will code and plot results

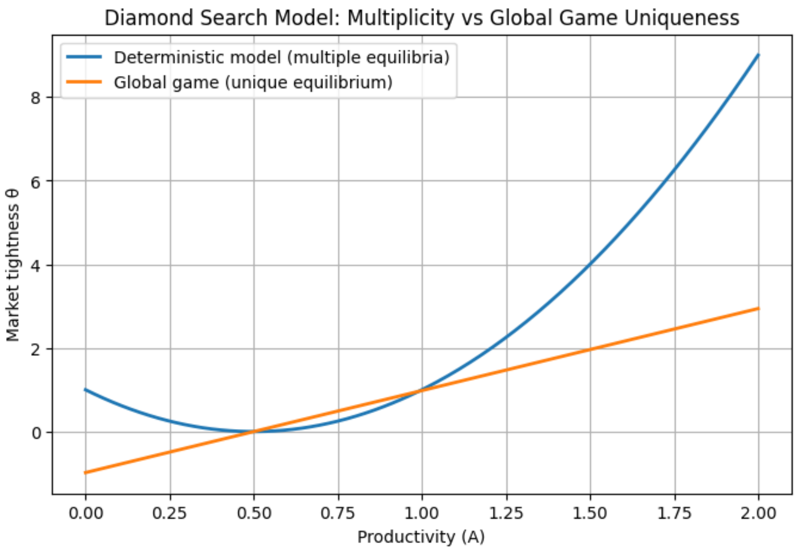

Figure 13 Multiple equilibria vs unique eq.in global game of matching model

Source: Authors' own calculations

We will do step-by-step derivation of the equilibrium cutoff s^*in a *Diamond search model* reinterpreted as a global game (firms get noisy private signals about productivity A).There is a continuum of identical potential posting firms and a continuum of searching workers. Let market tightness be $\theta = v/u$(vacancies per unemployed), where v= mass of vacancies, u= mass of unemployed. The matching function is

equation 115

$$m(u,v) = Mu^{\alpha}v^{1-\alpha}, 0 < \alpha < 1, M > 0$$

A firm that posts a vacancy meets a worker with probability

equation 116

$$q(\theta) = \frac{m(u,v)}{v} = M\theta^{-\alpha}.$$

(we can use any decreasing $q(\theta)$.)

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The productivity (fundamental) is A. Firms do not observe Aperfectly; each firm receives a private signal

equation 117

$$s = A + \varepsilon, \varepsilon \sim \mathcal{N}(0, \sigma^2),$$

- ✓ If a vacancy posts and fills, the match surplus (flow) is $S(A)$. For simplicity take $S(A) = A - w$with wage w(or simply $S(A) = A$if wages are normalized out). The cost of posting a vacancy is $c > 0$(flow or per-period posting cost in a static decision).
- ✓ Firms decide once (post or not). The game is: each firm posts iff expected payoff ≥ 0 given its private signal and beliefs about how many others post.

Let firms use a symmetric cutoff rule: post a vacancy if $s \geq s^*$. Given a candidate cutoff s^*, the fraction of firms who post conditional on the true $A$is

equation 118

$$\alpha(A; s^*) \;=\; \Pr(s \geq s^* \mid A) \;=\; 1 - \Phi\left(\frac{s^* - A}{\sigma}\right),$$

where $\Phi$is the standard normal CDF. If the fraction of firms posting is α, then aggregate tightness $\theta$is proportional to α(since $v \propto \alpha$and $u$is determined by labor market; for the fixed-mass normalization we can take $u = 1$so $v = \alpha$and $\theta = \alpha$). For concreteness we take the simple proportionality:

equation 119

$$\theta(A) \;=\; \alpha(A; s^*).$$

Given θ, the job-filling probability is $q(\theta) = M\theta^{-\alpha}$. A firm with signal $s$computes its expected payoff from posting (static decision) as

$$\Pi(s) \;=\; \mathbb{E}[\, q(\theta(A)) \cdot S(A) \mid s \,] \;-\; c.$$

Because the post-decision cannot influence others (continuum), the firm treats $\theta(A)$as a function of A only. With the cutoff rule the firm expects $\theta(A) = \alpha(A; s^*)$as in previous[14]. Write this indifference condition explicitly[15]:

equation 120

$$\mathbb{E}[\, q(\alpha(A; s^*))\, S(A) \mid s = s^*] \;=\; c.$$

By Bayes' rule, the posterior density of Agiven $s$is

---

[14] A cutoff $s^*$is an equilibrium cutoff if the firm indifferent at $s = s^*$, i.e.$\Pi(s^*) = 0$.

[15] This is the key condition: *given the profile s^*(used to compute $\alpha(\cdot; s^*)$), the firm that receives signal exactly $s^*$is indifferent.*

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equation 121

$$\pi(A \mid s) \;=\; \frac{\varphi\left(\frac{s-A}{\sigma}\right)}{\int \varphi\left(\frac{s-t}{\sigma}\right)\, dt}$$

(where $\varphi$is the standard normal pdf; if prior for $A$is uniform on a wide range we can use the unnormalized expression and renormalize numerically).Then $\mathbb{E}[\, q(\alpha(A; s^*))\, S(A) \mid s = s^*] \;=\; c.$ becomes

equation 122

$$\int q(\alpha(A; s^*))\, S(A)\, \pi(A \mid s^*)\, dA \;=\; c.$$

Recall $\alpha(A; s^*) = 1 - \Phi\left(\frac{s^*-A}{\sigma}\right)$. So the integrand is known given s^*. Thus $\int q(\alpha(A; s^*))\, S(A)\, \pi(A \mid s^*)\, dA \;=\; c$ is one scalar equation in the scalar unknown s^*.Equivalently, put everything on the LHS as a function $F(s^*)$:

equation 123

$$F(s^*) := \int q\left(1 - \Phi\left(\frac{s^* - A}{\sigma}\right)\right) S(A)\, \pi(\, A \mid s^* \,)\, dA - c,$$

and the equilibrium cutoff solves $F(s^*) = 0$. The mapping $s^* \mapsto \alpha(A; s^*) = 1 - \Phi\left(\frac{s^*-A}{\sigma}\right)$is **strictly decreasing** in $s^*$for each fixed A. (Higher cutoff → fewer firms post given A.)Thus $s^* \mapsto q(\alpha(A; s^*))$is **strictly increasing** in s^*if $q(\theta)$is **decreasing** in θ(remember $q(\theta)$decreases with θ= more vacancies → lower fill prob). Check signs carefully: since α decreases in s^*, $q(\alpha)$increases in s^*if $q$is decreasing in its argument. *(Intuition: a higher cutoff means fewer posters → smaller θ → larger $q(\theta)$.* Also, the posterior $\pi(A \mid s^*)$concentrates on higher A when $s^*$is higher (because $s^*$is the signal value being conditioned on). So the expectation of $S(A)$increases with s^*. Combining these monotonicities: $F(s^*)$is typically **strictly increasing** in s^*. At very low $s^*$the LHS is small (few posters, so small expected revenue), below c; at very high $s^*$the LHS is large. By continuity there is a unique root.

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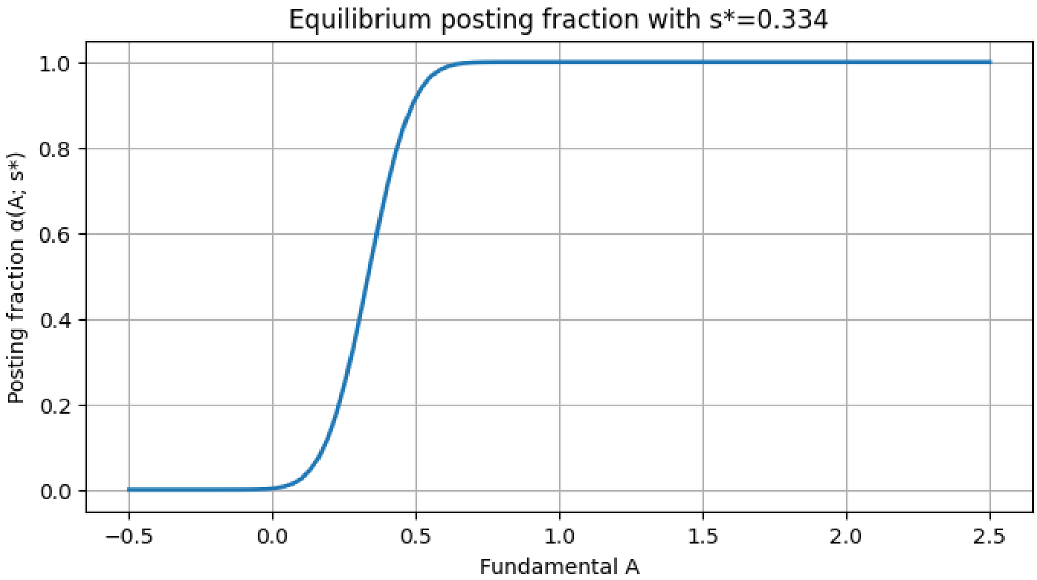

Figure 14 Equilibrium posting fraction with $s*$

Source: Authors' own calculations

CONCLUSIONS

The argmax correspondence $\mathrm{BR}_t(\cdot)$is nonempty, compact-valued and has measurable graph. By Kuratowski–Ryll-Nardzewski (and Aumann's theory of measurable multifunctions) it admits measurable selectors.Because $\mathrm{BR}_t(x)$is an interval and endpoints are measurable and monotone (single-crossing), the endpoint functions $\ell_t(\cdot), u_t(\cdot)$are measurable and nondecreasing, so choosing an endpoint gives a measurable **monotone** best-response selection.Using this measurable monotone best-response selector inside a Kakutani–type fixed-point argument yields existence of a measurable monotone Bayesian-Nash equilibrium. Combined with the ISD collapse/uniqueness argument, this gives existence *and* essential uniqueness for all sufficiently small v. Under the stated model and assumptions, in either limit $\sigma_x \downarrow 0$(private signals arbitrarily precise) or $\sigma_z \uparrow \infty$(public signal uninformative), there exists a unique monotone equilibrium in which regime change occurs if $\theta \le \hat{\theta}, \hat{\theta} \; = \; 1 - \frac{c}{b} \in (\underline{\theta}, \bar{\theta})$.The threshold is obtained from the marginal indifference (knife-edge) condition $b(1-\hat{\theta}) = c$. This is the standard closed-form threshold appearing in global-game style coordination models.The Gaussian structure reduces equilibrium characterization to a fixed point of a cutoff operator $\mathcal{G}$; the derivative/Lipschitz bound for $\mathcal{G}$involves the factor $\frac{\sigma_x}{\sigma_z^2} \cdot \varphi(0) = \frac{\sigma_x}{\sigma_z^2} \cdot \frac{1}{\sqrt{2\pi}}$;the stated inequality $\frac{\sigma_x}{\sigma_z^2} \le \sqrt{2\pi}$guarantees contraction and hence a unique monotone equilibrium; and because $\mathcal{G}$ is monotone and contracting, iterated elimination of strictly dominated strategies converges to that same unique profile. With no sterilization reserves fall quickly, probability of collapse is high,attack cutoff s^* is low,loss of welfare or CB costs are low. With sterilization reserves are sustained, collapse probability is low, attack cutoff s^* is low is high, fiscal costs are high.In Morris-Shin Version of DD(Diamond-Dybvig model): **Without policy (φ=0)** — runs occur frequently, even with moderate fundamentals.**As φ increases**, the bank's threshold for failure $\kappa(\theta, \varphi)$rises, stabilizing expectations. **Run probability declines monotonically** → there is stronger backstop, fewer self-fulfilling runs. The **unique equilibrium** emerges because private signals (σ>0) break coordination multiplicity.In DD version with heterogenous types of

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depositors: As policy strength φ (e.g., insurance, central-bank backstop) increases, average run probability falls. Effect of heterogeneity H: If H has **mass at very low costs** (many cheap withdrawers) — e.g., the mixed distribution with mass at 0 — the system becomes **more fragile**: equilibrium cutoffs $s^{*}(\theta)$fall and run probabilities are higher at the same φ compared to a H like Beta (2,2). *s(θ) curves*-Increasing φ raises $\kappa(\theta,\varphi)$, hence increases $s*$ — depositors need worse signals to withdraw. Heterogeneity changes the numeric $s*$ relative to analytic homogeneous benchmark; typically heterogeneity smooths and can lower s* if many cheap withdrawers exist.

REFERENCES

1. Angeletos, George-Marios and Hellwig, Christian and Pavan, Alessandro,(2006). Signaling in a Global Game: Coordination and Policy Traps (2006). Journal of Political Economy, Vol. 114, pp. 452-484, June 2006, Available at SSRN: https://ssrn.com/abstract=911242
2. Angeletos, G-M, and I. Werning (2006). Crises and Prices: Information Aggregation, Multiplicity and Volatility, American Economic Review, 96, 1720-1736
3. Atkeson, A. (2001). Discussion of Morris and Shin, NBER Macroeconomics Annual 2000, 161-164.
4. Aumann, R. J.(1965), Integrals of set-valued functions, J. Math. Anal. Appl., 12 (1965), 1-12
5. Aumann, R. J.(1969). Measurable Utility and the Measurable Choice Theorem.In La Décision. Paris: Éditions du Centre National de la Recherche Scientifique, p. 15–26.
6. Bogachev,I.V.(2007). Measure theory.Springer
7. Calvo, Guillermo A. (1988). Servicing the Public Debt: The Role of Expectations.A.E.R. 78,647–61.
8. Carlsson, H. and E. van Damme (1993).Global Games and Equilibrium Selection, Econometrica 61, 989-1018
9. Castaing,C. and Valadier,M.(1977). Convex analysis and measurable multifunction. Lecture Notes in Math., no. 580, Springer-Verlag, Berlin,Heidelberg, New York
10. Cole, Harold L., and Timothy J. Kehoe. (2000). Self-Fulfilling Debt Crises.Rev. Econ. Studies 67 (January): 91–116.
11. Cooper, Russell, and Andrew John. (1988). Coordinating Coordination Failures in Keynesian Models. Q.J.E. 103 (August): 441–63.
12. Di Piazza, Luisa & Sambucini, Anna Rita. (2025). Selections of integrable multifunctions in arbitrary Banach spaces. 10.48550/arXiv.2507.19460.
13. Diamond, D. W., Dybvig, P. H. (1983). Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy*, *91*(3), 401–419. http://www.jstor.org/stable/1837095
14. Diamond, Peter A., (1982). Aggregate Demand Management in Search Equilibrium, Journal of Political Economy 90(5), 881-894
15. Frankel, D., S. Morris, and A. Pauzner (2001), Equilibrium Selection in Global Games with Strategic Complementarities, *Journal of Economic Theory*.
16. Hellwig, C., A. Mukherji and A. Tsyvinski (2006). Self-Fulfilling Currency Crises: the Role of Interest Rates. American Economic Review,9
17. Jech, T. J.(1997). Set Theory, 2nd ed. Berlin: Springer-Verlag, p. 494, 1997.
18. Kakutani,S.(1941). A Generalization of Brouwer's Fixed Point Theorem. Duke Math. J. Volume 8 Number 3, 457-459. 1941
19. Katz, Michael L., and Carl Shapiro. (1986). Technology Adoption in the Presence of Network Externalities. J.P.E. 94 (August): 822–41.

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UDC: 336.747:657.47].01
Original research paper

20. Kuratowski, K.; Ryll-Nardzewski, C. (1965). A general theorem on selectors. *Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys*. **13**: 397–403.
21. Kurz (2006). Private Information and Diverse Beliefs: How Different? working paper, Stanford University
22. Levin,J.(2006). Coordination and Higher Order Uncertainty. Stanford university
23. Morris S, Shin HS. (1988), Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks. American Economic Review, June 1998, 88(3), pp. 587-97
24. Morris S, Shin HS. (2003). Global Games: Theory and Applications. In: Dewatripont M, Hansen LP, Turnovsky SJ, eds. Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress. Econometric Society Monographs. Cambridge University Press; 2003:56-114.
25. Obstfeld, Maurice (1986). Rational and Self-Fulfilling Balance-of-Payments Crises.A.E.R. 76 (March): 72–81
26. Obstfeld, Maurice (1996). Models of Currency Crises with Self-Fulfilling Features. European Econ. Rev. 40 (April): 1037–47.
27. Sims, C. (2005a). Comment on ’Central Bank Transparency and the Signal Value of Prices’ Brookings Papers on Economic Activity 2, 51-58.
28. Sims, C. 2005b). Rational Inattention: A Research Agenda, available at http://sims.princeton.edu/RIplus
29. Svensson, L. (2006) The Social Value of Public Information: Morris and Shin (2002) is Actually Pro Transparency, Not Con, American Economic Review 96, 448-452.
30. Tarashev, Nikola A. (2003). Currency Crises and the Informational Role of Interest Rates, BIS Working Papers 135, Bank for International Settlement
31. Vives,X. (2017). Endogenous Public Information and Welfare in Market Games, The Review of Economic Studies, Volume 84, Issue 2, April 2017, Pages 935–963, https://doi.org/10.1093/res/rdw062
32. Woodford, M. (2005). Central-Bank Communication and Policy Effectiveness,in The Greenspan Era: Lessons for the Future, Kansas City:Federal Reserve Bank of Kansas City