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ATKINSON-STIGLITZ THEOREM IN DIRECTED TECHNOLOGICAL CHANGE (DTC-AABH MODEL) WITH RAMSEY–MIRRLEES–PIGOU POLICY SYSTEM

Dushko Josheski[1*], Natasha Miteva[2], Tatjana Boshkov[3]

[1]*Associate professor Faculty of Tourism and Business Logistics, Goce Delcev University, Stip, North Macedonia; e-mail: dusko.josevski@ugd.edu.mk ; ORCID: 0000-0002-7771-7910*

[2]*Assistant professor, Faculty of Tourism and Business Logistics, Goce Delcev University, Stip, North Macedonia, natasa.miteva@ugd.edu.mk ; ORCID: 0009-0004-8082-6458*

[3]*Professor, Goce Delcev University of Stip, Faculty of tourism and business logistics, R.North Macedonia, e-mail: tatjana.dzaleva@ugd.edu.mk ;ORCID: 0000-0001-8821-9223*

**Corresponding author: dusko.josevski@ugd.edu.mk*

Abstract

Three forces: Externality (Pigou), Inequality (Mirrlees), Innovation direction (Acemoglu DTC-AABH) along with Ramsey (planner must raise tax revenue only through imposition of tax on commodities only) will be combined in this paper. Total optimal environmental tax includes Pigou + redistribution, Ramsey–Mirrlees term is pure redistribution correction driven by: non-separability η inequality aversion k. Ramsey-Mirrlees term shows: how dirty consumption correlates with ability and whether taxing pollution also redistributes income. Covariance term on the other hand is structural meaning:If high-weight individuals (poor) consume more dirty goods leads to positive covariance.If rich consume more dirty goods leads to negative covariance. Ramsey–Mirrlees term is: Positive if high-ability types consume more dirty goods, negative if low-ability types consume more dirty goods.

Keywords: DTC-AABH, Pigou, Ramsey, Mirrlees, optimal environmental tax equilibrium
JEL codes: H2,H23,N5, O44,

1.INTRODUCTION AND RELATED LITERATURE

This paper is working on the two-sector model (Directed Technological Change-AABH(DTC-AABH)) proposed by Acemoglu et al.(2012). We will introduce Atkinson-Stiglitz theorem in DTC-AABH model. The Atkinson-Stiglitz Theorem states that if labor is weakly separable from goods in household utility functions, differential commodity taxation should not be not part of an optimal redistributive tax system, see Boadway, R., & Pestieau, P. (2002). Paper Atkinson, Stiglitz (1976) showed that :" Even though there was a single "dimension" in which individuals differed (ability), in general, it seemed possible that one could extract information about that difference more efficiently by looking not just at the individual's labor supply, but also at his consumption patterns", see Stiglitz (2018). When, ability types are not correlated over time efficient tax systems are non-stationary and the individuals whose labor supply is distorted are those who have been low ability, see Battaglini,Coate (2003)[1].This paper also follows the Mirrlees (1971) mechanism design approach and models of information asymmetries that preclude non-distortionary taxation, see also Albanesi, Sleet (2003), Brito et al (1990), Diamond and Mirrlees (1978), Golosov, Kocherlakota and Tsyvinsky (2003),

[1] Efficient tax systems are stationary, low ability individuals face a positive marginal tax rate and high ability individuals face zero marginal tax rate, so their labor supply is undistorted.

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Golosov and Tsyvinski (2003) and Werning (2002). In the model of environment and directed technological change (DTC) that we will be investigating there are two sectors "dirty" and "clean" sector which in the original paper are combined to produce unique final good. This model will also highlight central roles played by market size and price effects on the direction of technological change, see also Acemoglu (1998), Acemoglu (2002). Induced innovation is macroeconomic hypothesis first proposed by John Hicks see Hicks (1932), where Hicks proposed: "a change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind—directed to economizing the use of a factor which has become relatively expensive[2]. Hicks (1932) also wrote:" A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind- directed to economizing the use of a factor which has become relatively expensive, see Acemoglu et al.(2015). The theorem Atkinson, Stiglitz (1976) (A-S) holds when the directed technological change does not alter the fundamental relationship between worker types and their consumption basket, or when the government can effectively tax the factor-specific returns, see Jacobs, B. and Boadway, R. (2014). The A-S theorem holds in a directed technical change model if the standard conditions for the theorem are maintained, even with endogenous technology. Those conditions are: homogenous preferences, redistribution through income tax, no other externalities, separability of future consumption, see Cremer, H., & Gahvari, F. (2015)[3].Atkinson-Stiglitz theorem in DTC model does not hold in case of: Market power and profits(DTC is modelled in monopolistic competition if government cannot tax these profits directly, the market introduces a wedge between private and social returns to innovation),externalities in research or spillovers, heterogenous preferences of households, upward sloping relative demand (If technical change is so strongly directed towards the abundant factor that the long-run relative demand curve for that factor slopes upward, the social planner's optimal strategy will differ from the competitive equilibrium, requiring intervention beyond non-linear income tax), see Acemoglu (2002) and Acemoglu et al.(2015), Møen, Jarle, (2005), Boadway, R., & Pestieau, P. (2002).Economists are seen as advocates for the use of economic instruments, such as environmental taxes to control the pollution and emission of greenhouse gasses in the atmosphere, see Stern (2007), Fullerton et al. (2010),and Jacobs, B., & de Mooij, R. A. (2015).Earlier studies on environmental taxes typically allowed only linear taxes and were adhering to representative consumer framework, see Bovenberg, A. L., & Goulder, L. H. (1996)[4].Here we will use: Ramsey (1927) taxes, Pigouvian taxation see Pigou (1920), Pigou,A.C.(1932),and Mirrlees taxation (non-linear tax) see Mirlees(1971). Mirrlees (1986), elaborates that a good way of governing is to agree upon objectives, then to discover what is possible and to optimize. The central element of the theory of optimal taxation is information. Public policies apply to the individuals on the basis of what the government knows about them. Second welfare theorem[5] states, that where a number of convexity and continuity assumptions are satisfied, an optimum is a competitive equilibrium once initial endowments have been suitably distributed. The seminal AABH model production

[2] Economists have long before recognized that the direction of innovation responds to economic incentives see Hémous, D., & Olsen, M. (2021) and Hicks J. (1932), Kennedy (1964)
[3] In this scenario, if the government can optimize non-linear income tax based on skills, the endogenous bias in technology will be incorporated into the optimal tax structure, rendering specific taxes on technology or factors (capital) superfluous, see Acemoglu, D., Atkinson, A. B., & Stiglitz, J. E. (2015).
[4] In a general-equilibrium setting, Sandmo (1975) and Bovenberg andFrederick van der Ploeg (1994) have demonstrated how the well-known "Ramsey" formula for optimal commodity taxes is altered when one of the consumption commodities generates an externality, see Bovenberg, A. L., & Goulder, L. H. (1996)
[5] Second fundamental theorem is giving conditions under which a Pareto optimal allocation can be supported as a price equilibrium with lump-sum transfers, i.e. Pareto optimal allocation as a market equilibrium can be achieved by using appropriate scheme of wealth distribution (wealth transfers) scheme (Mas-Colell, Whinston et al. 1995)

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requires dirty (polluting) and clean (non-polluting) inputs[6].This paper will first outline Atkinson-Stigilitz theorem and Mirrlees (1971) framework. Then we will investigate two cases: 1. A-S theorem holds in DTC-AABH model,2. A-S theorem fails in DTC-AABH model. Ramsey–Mirrlees–Pigou policy system in the Directed Technical Change (DTC) model of Daron Acemoglu et al. (2012) will follow in order to derive Ramsey-Mirrlees correction term and the existence of optimal environmental tax equilibrium, and the Existence of multiple equilibria in DTC with Local Weak Bias with Two Skills and Labor- Augmenting Technology (A-S holds and A-S fails). This study will draw conclusion on those equilibria and whether they are stable. DTC-AABH model provides many opportunities for introducing and investigating fiscal policy instruments and one cand derive many conclusions for optimal taxes in two-sectoral Directed technological change models.

2.ATKINSON-STIGILITZ THEOREM AND MIRRLEES (1971) FRAMEWORK

Here we are going to set standard Atkinson-Stiglitz (1976) (A-S)theorem in Mirleesian framework, see Mirlees(1971).But first, we will introduce A-S theorem and Mirlleesian framework:

Theorem 1

Commodity taxes cannot increase social welfare if utility functions are weakly separable in consumption goods versus leisure and the subutility of consumption goods is the same across individuals, i.e.,$u_i(c_1,\dots,c_k,w) = u_i(v(c_1,\dots,c_k),w)$ with the subutility function $v(c_1,\dots,c_k)$ homogenous across individuals.
Laroque (2005) and Kaplow (2006) have provided intuitive proof of this theorem as follows:

Proof:

A tax system $(\tau(\cdot),t)$that includes both nonlinear income tax and a vector of commodity taxes can be replaced by a pure income tax $(\bar{\tau}(\cdot),t=0)$ .This tax system keeps all individual utilities constant and raises at least as much tax revenue. Let $v(p+t,\gamma)=\max_c v(c_1,..c_k)$ s.t. $(p+t)\cdot c\le\gamma$ be the indirect utility of consumption goods which is common to al individuals. Now if we consider replacing $(\tau(\cdot),t)$ this tax system with $(\bar{\tau}(\cdot),t=0)$ where $\bar{\tau}(w)$ is defined such that $v\big(p+t,w-\tau(w)\big)=v\big(p,w-\bar{\tau}(w)\big)$. Here $\bar{\tau}(w)$ naturally exists a $v(p,\gamma)$ is strictly increasing in γ .Which on turn implies that $u_i(v(p+t,w-\tau(w),w)=u_i(v(p+t,w-\bar{\tau}(w),w)$,$\forall w$ .So the utility and labor supply for $\forall i$ are unchanged .Attaining utility of consumption $v\big(p,w-\bar{\tau}(w)\big)$ at price p costs at least $w-\bar{\tau}(w)$.Now, let c_i be the consumer choice of individual i under the initial tax system $(\tau(\cdot),t)$.Individual i attains utility $v\big(p,w-\bar{\tau}(w)\big)=v\big(p,w-\bar{\tau}(w)\big)$ when choosing c_i. And now $p\cdot c_i\ge w-\bar{\tau}(w)$ and we have that $\bar{\tau}(w)\ge\tau(w)+t\cdot c_i$ i.e. the government collects more taxes with $(\bar{\tau}(\cdot),t=0)$ ■

[6] There is strong literature on policy relevance of growth, DTC and environment: Gans, (2012), Golosov et al.,(2014), Van den Bijgaart, (2017), Fried, (2018), Greaker et al., (2018), Hart, (2019), Lemoine, (2024)

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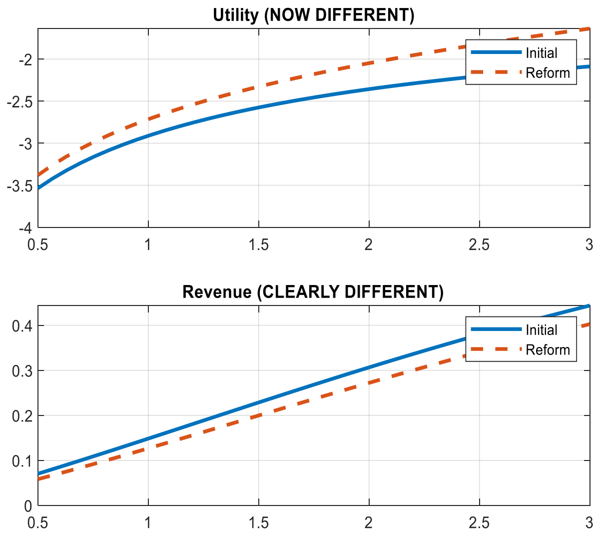
Figure 1 A-S theorem



Source: Author's own calculation

Now, for the derivation of Mirllees (1971) framework: Let's suppose the utility function g the agents in the economy Mirrlees (1971) model:

equation 1

$$\widetilde{U}(c,l) = c - \frac{l^2}{2}$$

Where $y = \theta l$ и θ represents the level of skils of the worker. Now his social welfare function SWF is :$SWF(v) = \log(v)$.Now lets find the distribution of skills when $T(y) = 0.3$ which is Pareto with $h(y) = ky^{-k-1}y^k$[7].Equation for the distribution of skills is $f(\theta) = h\big(y(\theta)\big)y'(\theta)$,from the quasi-linear utility functions : $U(c,y,\theta) = c - \frac{1}{2}\left(\frac{y}{\theta}\right)^2$.And the tax function $T(y) = \tau y$, individual with skill level θ solves :

equation 2

$$\max_y (1-\tau)y - \frac{1}{2}\left(\frac{y}{\theta}\right)^2$$

FOC is given as :$(1-\tau) - \frac{y}{\theta^2} = 0$, which implies that $y = (1-\tau)\theta^2$ and $f(\theta) = h\big(y(\theta)\big)y'(\theta) = k(\theta)^{-k-1}y^k 2(1-\tau)\theta = k\big((1-\tau)\theta^2\big)^{-k-1}y^k 2(1-\tau)\theta = 2k(1-\tau)^{-k}\theta^{-2k-1}y^k = 2k\theta^{-2k-1}\theta_l^{2k}$ .By integration one could get :

equation 3

$$F(\theta) = \int_{\theta_l}^{\theta} f(\theta)d\theta = \int_{\theta_l}^{\theta} 2k(1-\tau)^{-k}\theta^{-2k-1}y^k d\theta = [-(1-\tau)^{-k}\theta^{-2k}y^k]_{\theta_l}^{\theta}$$
$$= (1-\tau)^{-k}\theta_l^{-2k-1}\left((1-\tau)\theta_l^2\right)^k - (1-\tau)^{-k}\theta^{-2k}y^k = 1 - \theta^{-2k}\theta_l^{2k}$$

[7] This is a density of earnings function, dependent on v the skills of workers

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Now we can solve for numerical optimum. Let's use $y = 2$ and $k = 4$ and truncate the distribution[8] at the top x percentile for some small x.In this case : $\max_{v(\theta),u(\theta)} \int_{\theta_l}^{\theta^h} W[v(\theta)]f(\theta)d\theta$.Subject to :

equation 4

$$\int_{\theta_l}^{\theta^h} \big(y(\theta) - e(v(\theta), y(\theta), \theta)\big)f(\theta)d\theta \geq 0; v'(\theta) = u_\theta\,[e(v(\theta), y(\theta), \theta)]$$

$y(\theta)$ is non decreasing function. Hamiltonian is formed as $:H = W\,[v(\theta)]f(\theta) + \lambda\,(y(\theta) - e(v(\theta), y(\theta), \theta))f(\theta) + \eta\,(\theta)U_\theta\,[e(v(\theta), y(\theta), \theta), y(\theta), \theta]$.
Standard conditions are as:

equation 5

$$\frac{\partial H}{\partial y} = 0 \Rightarrow \lambda f\big(1 - e_y\big) + \eta\big[u_{\theta c}e_y + u_{\theta y}\big] = 0$$
$$\frac{\partial H}{\partial v} = \eta' \Rightarrow W'f - \lambda e_v f + \eta u_{\theta c}e_v = -\eta'$$

Transversality conditions are given as : $\eta(\theta_l) = \eta\big(\theta^h\big) = 0$ .From $W = \log(v)$ и $u(c, y, \theta) = c - \left(\frac{1}{2}\right)\left(\frac{y}{\theta}\right)^2$ will get the following derivatives :$u_\theta = \frac{y^2}{\theta^3}; u_{\theta c} = 0;\ u\theta_y = \frac{2y}{3}$; $W' = \frac{1}{v}$.Let us remember that $v = u(e(v, y, \theta), y, \theta)$,we have $1 = u_c e_v$ и $0 = u_c e_y + u_y$,therefore :$e_v = \frac{1}{u_c}; e_y = -\frac{u_y}{u_c} = \frac{y}{\theta^2}$. If we substitute in the optimality and control equations about the state variables one can get :

equation 6

$$\lambda f\ \left(1 - \frac{y}{\theta^2}\right) + \eta\left[\frac{2y}{\theta^3}\right] = 0 \text{ и } \frac{f}{v} - \lambda f = -\eta'$$

If we solve in the first equation for $y(\theta)$ we get: $y(\theta) = \frac{\lambda f\,(\theta)\theta^3}{\lambda f\,(\theta)\theta - 2\eta(\theta)}$. With the equation $\eta'\eta'(\theta) = \left(\lambda - \frac{1}{v(\theta)}\right)f(\theta)$. If we substitute for $y(\theta)$ in the constraint[9] :$v'(\theta) = u_\theta[e(v(\theta), y(\theta), \theta), y, \theta] = \frac{y^2}{\theta^3} = \left(\frac{\lambda f\,(\theta)}{\lambda f\,(\theta)\theta - 2\eta(\theta)}\right)^2\theta^3$. In the example by Diamond (1998) utility function is quasi linear:

equation 7

$$u(c, l) = c - v(l)$$

c is disposable income and the utility of suuply of labor $v(l)$ is increasing and convex in l. Earnings equal $w = nl$ where n represents innate ability. CDF of skills distribution is $F(n)$, it's PDF is $f(n)$and support range is $[0, \infty)$. Government cannot observe abilities instead it can set taxes asa function of labor income $c = w - \tau(w)$.Individual n chooses l_n to maximize:

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8 In statistics truncated distribution is a conditional distribution that comes as a result of the restriction of the domain of some other distribution or probability.

9 In Saez (2001) optimal tax formula is given as :$\tau = \frac{1-\bar{g}}{1-\bar{g}+\bar{\varepsilon}^u+\bar{\varepsilon}^c(a-1)}$

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equation 8

$$\max(nl - \tau n(l) - v(l))$$

When marginal tax rate τ is constant, the labor supply f-ction is given as: $l \to l\big(n(1-\tau)\big)$ and it is implicitly defined by the $n(1-\tau) = v'(l)$. And $\frac{dl}{d(n(1-\tau))} = \frac{1}{v''(l)}$, so the elasticity of the net-of-tax rate $1-\tau$ is:

equation 9

$$e = \frac{\left(\frac{n(1-\tau)}{l}\right) dl}{d\big(n(1-\tau)\big)} = \frac{v'(l)}{lv''(l)}$$

The government maximizes SWF :

equation 10

$$W = \int G(u_n) f(n) dn \;\; s.t. \int cnf(n)dn \le \int nlnf(n)dn - E(\lambda)$$

u_n denotes utility, $w_n = nl_n$ denotes earnings, c_n denotes consumption or disposable income, and $c_n = u_n + v(l_n)$. Now the Hamiltonian is given as:

equation 11

$$\mathcal{H} = \big[G(u_n) + \lambda \cdot \big(nl_n - u_n - v(l_n)\big)\big] f(n) + \phi(n) \cdot \frac{lnv'(ln)}{n}$$

In previous $\phi(n)$ is the multiplier of the state variable. The FOC with respect to l is given as:

equation 12

$$\lambda \cdot \big(n - v'(l_n)\big) + \frac{\phi(n)}{n} \cdot [v'(l_n) + l_n v''(l_n)] = 0$$

FOC with respect to u is given as:

equation 13

$$-\frac{d\phi(n)}{n} = [G'(u_n) - \lambda]$$

If integrated previous expression gives: $-\phi(n) = \int_n^\infty [\lambda - G'(u_m)] f(m)dm$ where the transversality condition $\phi(\infty) = 0$,and $\phi(0) = 0$, and $\lambda = \int_0^\infty G'(u_m)f(m)dm$ and social marginal welfare weights $\frac{G'(u_m)}{\lambda} = 1$.The optimal tax formula here is[10]:

equation 14

$$\frac{\tau'(w_n)}{1-\tau'(w_n)} = \frac{1}{e} \cdot \left(\frac{\int_n^\infty (1-g_m)dF(m)}{w_n h(w_n)}\right) = \frac{1}{e} \cdot \left(\frac{1-H(w_n)}{w_n h(w_n)}\right) \cdot \big(1 - G(w_n)\big)$$

In the previous expression $G(w_n) = \int_n^\infty \frac{dF(m)}{1-F(n)}$ is the average social welfare above w_n .If we change variables from $n \to w_n$, we have $G(w_n) = \int_{w_n}^\infty \frac{g_m dH(w_m)}{1-H(w_n)}$.The transversality condition implies $G(w_0 = 0) = 1$. In the Mirrlees(1971) model government , maximizes[11] :$SWF = \int_0^\infty G(u_w) f(w)dw$. The bottom tax formula in Mirrlees (1971) is derived in Piketty,

[10] FOC here was given as:$\frac{\tau'(w_n)}{1-\tau'(w_n)} = \left(1 + \frac{1}{e}\right) \cdot \left(\frac{\int_n^\infty (1-g_m)dF(m)}{nf(n)}\right)$,where $g_m = \frac{G'(u_m)}{\lambda}$ which is the social welfare on individual m

[11] Here we make assumption that wages =skill level

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Saez (2013).

equation 15

$$\frac{\tau'(0)}{1-\tau'(0)} = (g_0 - 1)\cdot\frac{F(n_0)}{n_0 f(n_0)} \Rightarrow \tau'(0) = \frac{g_0 - 1}{g_0 - 1 + \frac{n_0 f(n_0)}{F(n_0)}}$$

In previous expression $g_0 = \frac{G(u_0)u_c}{\lambda}$ is the social marginal weight of the non-worker. From previous we know that $n_0\big(1-\tau'(0)\big)u_c(c_0,0) + u_l(c_0,0) = 0$ which defines $n_0(1-\tau'(0), c_0)$.The effect of $1-\tau'(0)$ on n_0 is such that $\frac{\partial n_0}{\partial(1-\tau'(0))} = -\frac{n_0}{1-\tau'(0)}$. In Piketty, T., Saez.E., and Stantcheva,S.(2014), it is well defined aggregate elasticity of income as:

equation 16

$$\varepsilon = \frac{1-\tau}{z}\frac{dz}{d(1-\tau)} \ ,$$

where z is taxable income and $z = y - x$, where y is the real income, and x is sheltered income [12],taxable income s used in the calculation for Pareto parameter $a = \frac{z}{z-\bar{z}}$. Social marginal welfare weight[13] is given as:

equation 17

$$g_i = \frac{\omega_i G'(u^i)u_c^i}{\lambda}$$

g_i measures the dollar/euro value (in terms of public funds) of increasing consumption of individual i by \$1 or €1. Under utilitarian criterion,$g_i = \frac{u_i^c}{\lambda}$ is directly proportional to the marginal utility of consumption. Under Rawlsian criterion all the $\forall g_i = 0$ except for the most disadvantaged (poorest). Social welfare function can be :

- $SWF = \int U^i\, di$-Utilitarian or Benthamite,
- $SWF = min_i U^i$- Rawlsian $SWF = \int U^i di \to G(U) = \frac{U^{1-\gamma}}{1-\gamma}$ if $\gamma = 0$ function is utilitarian , Rawlsian if $\gamma = \infty$.
- With Pareto weights: $SWF = \int \mu_i U^i\, di$ where μ_i is exogenous.

The optimal tax government formula with Rawlsian government [14]would be :

equation 18

$$\frac{T'(w(h))}{1-T'(w(h))} = \left(\frac{1+\varepsilon}{\varepsilon}\right)\frac{1-F(w)}{wf(w)} \quad \text{or} \quad \frac{T'(w(h))}{1-T'(w(h))} = \left(\frac{1+\varepsilon}{\varepsilon}\right)\frac{\psi(w)-F(w)}{wf(w)}$$

Now if we divide and multiply by $1-F(w)$ we get :

equation 19

$$\frac{T'(w(h))}{1-T'(w(h))} = \left(\frac{1+\varepsilon}{\varepsilon}\right)\frac{\Psi(w)-F(w)}{1-F(w)}\frac{1-F(w)}{wf(w)}$$

[12] Investments or investment accounts that provide favorable tax treatment , or activities and transactions that lower taxable income.

[13] The marginal social welfare weight on a given individual measures the value that society puts on providing an additional dollar of consumption to this individual.

[14] The social welfare function that uses as its measure of social welfare the utility of the worst-off member of society. The following argument can be used to motivate the Rawlsian social welfare function.

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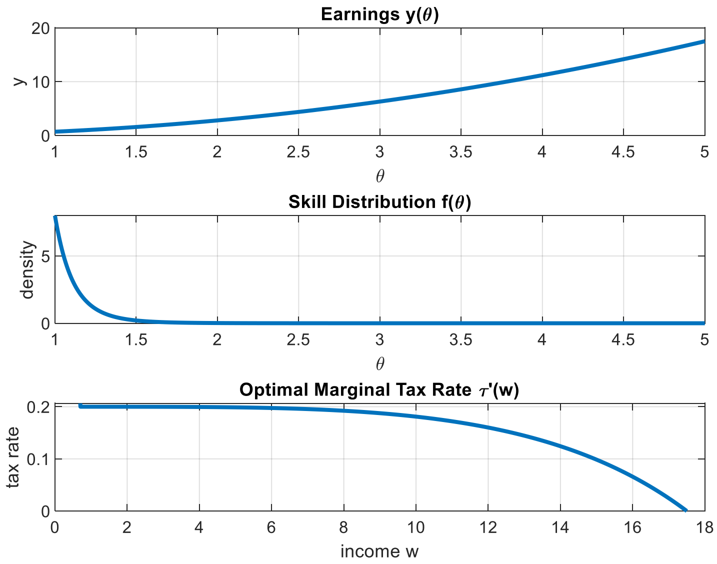
Figure 2 Mirleesian framework



Source: Author's own calculation

2.1 Planner's problem

Choose allocation $c(w), y(w)$to: $\max \int V(C(c(w)), \frac{y(w)}{w}, f(w)dw$ s.t :
Resource constraint:

inequality 1

$$\int \left[\sum_i c_i\,(w)\right] f(w)dw \le \int y(w)f(w)dw$$

Incentive compatibility (IC):

inequality 2

$$\forall w, w': \mathrm{V}\left(\mathrm{C}\big(\mathrm{c}(\mathrm{w})\big), \frac{\mathrm{y}(\mathrm{w})}{\mathrm{w}}\right) \ge \mathrm{V}\left(\mathrm{C}\big(\mathrm{c}(\mathrm{w}')\big), \frac{\mathrm{y}(\mathrm{w}')}{\mathrm{w}}\right)$$

Lemma 1

Separability: $U(c,l) = V(C(c), l)$ the IC constraint depends only on $C(c(w))$, not on the composition of c.

Proof:

$C(c)$ is a sub-utility function that aggregates the consumption vector into a single index (often viewed as "total real consumption"). For a mechanism to be IC must satisfy:

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$V\big(C\big(c(w)\big), l(w)\big) \geq V\left(C\big(c(w')\big), l(w')\right) \forall w, w'$. If the government wants to provide a "utility level" from consumption equal to K, it will do so by minimizing the cost of the bundle: $\min\limits_{c} \sum p_i c_i \ s.t. C(c) \geq K$. Because V is separable, for any given level of labor l, the agent's ranking of different bundles depends only on the value $C(c)$. If there are two bundles a, b than for any given labor: $V(C(c_a), l) = V(C(c_b), l)$.So, MRS is given by:

equation 20

$$\frac{\frac{\partial U}{\partial c_i}}{\frac{\partial U}{\partial c_j}} = \frac{V_c \cdot \frac{\partial C}{\partial c_i}}{V_c \cdot \frac{\partial C}{\partial c_j}} = \frac{\frac{\partial C}{\partial c_i}}{\frac{\partial C}{\partial c_j}}$$

Since the MRS between goods does not depend on l, all types w will choose the same relative composition of consumption goods for a given total expenditure.IC constraint can be rewritten purely in terms of aggregate C: $V(C_w, l_w) \geq V(C_{w'}, l_{w'})$■

3. OPTIMAL NONLINEAR INCOME TAX (MIRRLEES) AND THE PIGOUVIAN ENVIRONMENTAL TAX IN THE TWO-SECTOR DIRECTED TECHNICAL CHANGE (DTC) MODEL

In this section we will use two sector model proposed by Acemoglu et. al (2012) (DTC-AABH model). In this model:

equation 21

$$Y = \left[\omega Y_c^{\frac{\sigma-1}{\sigma}} + (1-\omega) Y_d^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}}$$

On the production side: $Y_c = A_c L_c, Y_d = A_d L_d$ i.e. clean and dirty output, L_c, L_d– labor allocations, σ– elasticity of substitution. Production of dirty inputs depletes environmental stock S[15]:

equation 22

$$S_{t+1} = -\xi Y_{dt} + (1+\delta) S_t, S \in (0, \bar{S})$$

The parameter ξ measures the rate of environmental degradation resulting from the production of dirty inputs, and δ is the rate of "environmental regeneration."[16].Individual productivity θ. Labor income is given as : $y = \theta l$.Utility is of type:

equation 23

$$U(c, l, S) = u(c) - v(l) - D(S)$$

Government uses

- nonlinear income tax $T(y)$

[15] S is general quality of environment, inversely related to CO2 concentration

[16] In ecology regeneration is the ability of an ecosystem – specifically, the environment and its living population – to renew and recover from damage.In this case δ measures amount of pollution that can be absorbed without extreme adverse consequences

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- Pigouvian tax $\tau_p$on dirty production.

3.1 Planner's problem

Social planner maximizes:

equation 24

$$\max \int U(c(\theta), l(\theta), S) dF(\theta)$$
$$s.t. \int c(\theta) dF(\theta) = Y - G$$

and incentive compatibility

equation 25

$$\theta l(\theta) - T(\theta l(\theta)) \geq \theta l(\tilde{\theta}) - T(\theta l(\tilde{\theta}))$$

The classical Mirrlees condition and optimal non-linear tax formula gives:

equation 26

$$\frac{T'(y)}{1 - T'(y)} = \frac{1 - F(\theta)}{f(\theta)} \frac{1}{\epsilon_l} \frac{W'(\theta)}{W(\theta)}$$

Where ϵ_l is the elasticity of labor, and $F(\theta)$ is CDF of ability. And this is non-linear income tax schedule.

3.2 Pigouvian environmental tax

Social marginal damage:$MD = D'(S)\phi$.Optimal Pigou tax:$\tau_p = MD$This internalizes the pollution externality.Here dirty output generates pollution: $S_{t+1} = (1 - \delta)S_t + \phi Y_d$. And environmental damage ϕ enters the utility function. Fundamental formula for environmental tax Pigou alike is:
equation 27

$$\tau = MEC(Y^*)$$
$$MEC = MSC - MPC$$
$$\tau_{pigou} = \frac{dD}{dQ}|_{Y^*}$$

Where $\frac{dD}{dQ}$ is the derivative of the damage function, representing the marginal damage at the socially efficient output Y^*. Taxes would be imposed on the activities that generate external diseconomies, while the sources of external economies would be subsidized, see Pigou,A.C.(1932) in Diamond, P. A., & Mirrlees, J. A. (1973).

3.3 A-S theorem holds in DTC-AABH model

Preferences here are $u(c, l) = v(c) - \psi(l)$, this ensures no commodity taxes for redistribution. Production function is:

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equation 28

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}; i \in \{c, d\}$$

Dirty externality is:$E = \phi Y_d$, and Pigouvian tax is:$\tau_d = \chi\phi$. Firm's problem i.e. decentralized equilibrium:

equation 29

$$\max p_c Y_c - wL_c - rK_c - clean\ sector$$
$$\max p_d Y_d - wL_d - rK_d - \tau_d Y_d - dirty\ sector$$

FOC's are:

$$p_c \cdot \alpha A_c k_c^{\alpha-1} = r - clean\ sector$$
$$(p_d - \tau_d) \cdot \alpha A_d k_d^{\alpha-1} = r - dirty\ sector$$

where:$k_i = \frac{K_i}{L_i}$. Since capital is mobile: $p_c \cdot \alpha A_c k_c^{\alpha-1} = (p_d - \tau_d) \cdot \alpha A_d k_d^{\alpha-1}$.Now, if we solve for capital intensity ratio:

equation 30

$$\frac{k_c}{k_d} = \left[\frac{(p_d - \tau_d)A_d}{p_c A_c}\right]^{\frac{1}{\alpha-1}}$$

Output per worker is: $y_i = A_i k_i^{\alpha}$, thus: $\frac{Y_c}{Y_d} = \frac{A_c}{A_d}\left(\frac{k_c}{k_d}\right)^{\alpha}$.Now if we substitute:

equation 31

$$\frac{Y_c}{Y_d} = \frac{A_c}{A_d}\left(\left[\frac{(p_d - \tau_d)A_d}{p_c A_c}\right]\right)^{\alpha}$$

Closed form clean-dirty technology ratio is given as:

equation 32

$$\frac{Y_c}{Y_d} = \left(\frac{A_c}{A_d}\right)^{\frac{1}{1-\alpha}} \cdot \left(\frac{p_c}{p_d - \tau_d}\right)^{\frac{\alpha}{1-\alpha}} \Rightarrow = \left(\frac{A_c}{A_d}\right)^{\frac{1}{1-\alpha}} \cdot \left(\frac{p_c}{p_d - \chi\phi}\right)^{\frac{\alpha}{1-\alpha}}$$

This $\left(\frac{A_c}{A_d}\right)^{\frac{1}{1-\alpha}}$ is the technology effect, higher clean technology leads to strong amplification[17],while $\left(\frac{p_c}{p_d-\chi\phi}\right)^{\frac{\alpha}{1-\alpha}}$ is price +tax wedge, this shifts the economy towards clean

---

[17] When clean technology becomes more advanced relative to dirty technology $\frac{A_c}{A_d}$(increases), the exponent $\frac{1}{1-\alpha}$ (which is typically greater than 1) amplifies this advantage, leading to a much higher relative output from clean technology. This effect drives the shift away from fossil fuels, as higher clean productivity means the economy can produce more final goods while emitting less, reducing the overall economic reliance on dirty inputs. This effect drives the shift away from fossil fuels, as higher clean productivity means the economy can produce more final goods while emitting less, reducing the overall economic reliance on dirty inputs.

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sector.One implication here from A-S theorem is that all distortion comes from Pigouvian correction.Log-linear form is given as:

equation 33

$$\log\frac{Y_c}{Y_d} = \frac{1}{1-\alpha}\log\frac{A_c}{A_d} + \frac{\alpha}{1-\alpha}\log\frac{p_c}{p_d-\chi\phi}$$

Key insight here is that: Under Atkinson-Stiglitz, the entire direction of structural technological change is governed by:

- Technology gap $\frac{A_c}{A_d}$
- Pigouvian wedge $\chi\phi$
- Not by redistributive taxation

Representative agents have preferences:

equation 34

$$U = \int_0^\infty e^{-\rho t}u\big(c(t), l(t)\big)dt$$

Consumption and labor are weakly separable:$u(c,l) = v(c) - \psi(l)$. Budget constraint is given as:$\dot{k} = wl + rk - c - \tau$.Dirty sector generates pollution $E = \phi Y_d$:

equation 35

$$U = \int_0^\infty e^{-\rho t}u(v(c) - \psi(l) - \chi E)dt$$

Final goods aggregation is:

equation 36

$$Y = \left[\gamma Y_c^{\frac{\sigma-1}{\sigma}} + (1-\gamma)Y_d^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

Where σ is the elasticity of substitution. Planner internalizes pollution:

equation 37

$$\max_{c,\ell,K_c,K_d,L_c,L_d}\int_0^\infty e^{-\rho t}\,[v(c) - \psi(\ell) - \chi\phi Y_d]dt$$

FOCs here are: $MPK_c = MPK_d, MPL_c = MPL_d$ but for the dirty sector we have:

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equation 38

$$MP_d^{social} = MP_d^{private} - \chi\phi$$

Firm's problems are:

Dirty firms:

equation 39

$$\max p_d Y_d - wL_d - rK_d - \tau_d Y_d$$

FOC:

equation 40

$$p_d MP_d = w, r \text{ but with} (p_d - \tau_d)$$

A-S implication is that $\tau_c = 0$ but $\tau_d = \chi\phi$. Household FOCs:

equation 41

$$v'(c) = \lambda$$
$$\psi'(\ell) = (1 - \tau_l)w\lambda$$

τ_l is possible non-linear Mirrlees tax on labor. Euler equation is given as:

equation 42

$$\frac{\dot{c}}{c} = \frac{1}{\theta}[(1 - \tau_k)r - \rho]$$

Sectoral returns are:$r = \alpha A_i \left(\frac{K_i}{L_i}\right)^{\alpha-1}$ .Dirty sector price is distorted: $p_d = MC_d + \tau_d$. Relative profitability is given as:

equation 43

$$\frac{R_c}{R_d} = \frac{A_c}{A_d} \cdot \left(\frac{p_c}{p_d - \tau_d}\right)^{\frac{1}{1-\alpha}}$$

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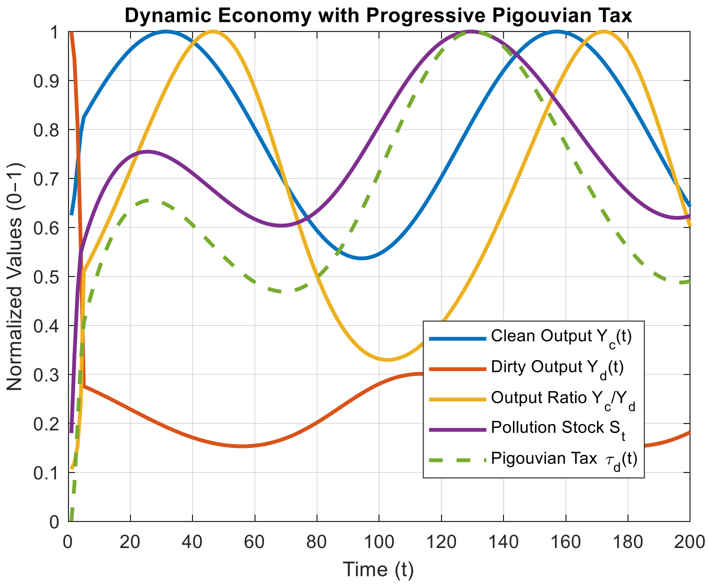
Figure 3 Dynamic economy with progressive Pigouvian tax (A-S holds)



Source: Author’s calculation

We can extend previous for Optimal Dynamic Pigou Tax (Ramsey Planner)[18]:

equation 44

$$\tau_t = \sum_{s=t}^{\infty} \beta^{s-t} D'(S_s) \frac{\partial S_s}{\partial Y_d(t)}$$

$$\Downarrow$$

$$\tau_t = \phi \sum_{k=0}^{\infty} (1-\delta)^k D'(S_{t+k})$$

If damages are local (standard in numerics):

equation 45

$$\tau_t \approx \frac{\phi}{1-\beta(1-\delta)} \cdot D'(S_t)$$

With convex damage:

$$D(S) = \frac{\eta}{2} S^2 \Rightarrow D'(S) = \eta S$$

$$\tau_t = \frac{\phi\eta}{1-\beta(1-\delta)}$$

[18] From the law of motion: $S_{t+1} = (1-\delta)S_t + \phi Y_d(t)$,forward iteration gives:$\frac{\partial S_s}{\partial Y_d(t)} = \phi(1-\delta)^{s-t}$

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This is Ramsey-Pigou[19] tax, $\tau_t = \frac{\phi\eta}{1-\beta(1-\delta)}$. Where ϕ is marginal damage, η is emission intensity, $1-\beta(1-\delta)$ is the stock multiplier,δ is the decay rate of the pollution stock, Because the pollution doesn't disappear instantly, the tax must be higher than the immediate damage ($\phi\eta$) to account for the "legacy" costs in future periods for more comprehensive details on this topic see also van der Ploeg, F & Withagen, C A A M (1991). Household problem:

equation 46

$$U = \int e^{-\rho t}[v(c) - \psi(l) - \chi D(S)]dt$$

FOCs are:

equation 47

$$v'(c) = \lambda$$
$$\psi'(l) = (1 - \tau_l(l))w\lambda$$

Mirrlees tax leads to redistribution (labor distortion). Pigou tax leads to externality correction. Under Atkinson–Stiglitz theorem:$\tau_c = 0, \tau_d =$ Pigou only. So we implement progressive labor tax:

equation 48

$$\tau_l = 1 - e^{-\kappa l}$$

We introduce nonlinear catastrophic damage:

equation 49

$$D(S) = \begin{cases} \frac{\eta}{2}S^2 & S < S^* \\ \frac{\eta}{2}S^2 + \Omega(S - S^*)^2 & S \geq S^* \end{cases}$$
$$D'(S) = \begin{cases} \eta S & S < S^* \\ \eta S + 2\Omega(S - S^*) & S \geq S^* \end{cases}$$

This is represented on this plot:

[19] In environmental macroeconomics: $\tau_t = \frac{\phi\eta}{1-\beta(1-\delta)}$ represents the optimal tax on capital (or a polluting input) when pollution is a stock externality that accumulates over time. It is called a Ramsey-Pigou tax because it combines the Pigouvian principle (internalizing an externality) with the Ramsey framework (dynamic optimization over time.

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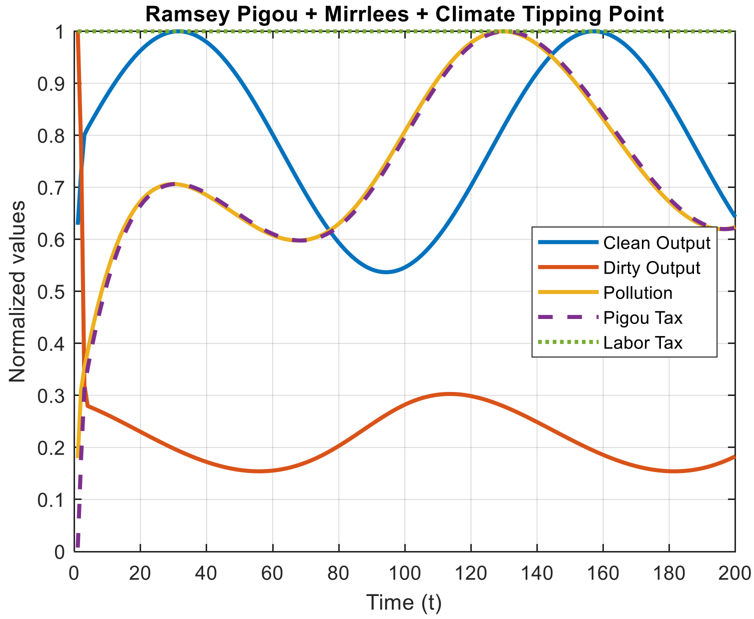
Figure 4 Ramsey-Pigou, Mirrlees, plus climate tipping point



Source: Author's calculation

3.4 A-S theorem fails in DTC-AABH model

In this case Atkinson-Stiglitz result does not hold. So, here preferences are not separable meaning:

inequality 3

$$u(c,l) \neq u(c) - \psi(l)$$

One example would be standard violation:

equation 50

$$u(c,l) = u(c) - \psi(l,c)$$

Consumption affects labor supply meaning commodity taxation becomes useful. (non A–S)

equation 51

$$\max_{T(\cdot),\tau_c(\cdot)} \int W(U(\theta)) f(\theta)\, d\theta$$

subject to:Incentive compatibility(IC) , Resource constraint(RC):

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equation 52

$$\int [T(y(\theta)) + \tau_c(\theta)\, c(\theta)] f(\theta)\, d\theta = G$$

Now, the modified marginal tax formula gets extra distortion term:

equation 53

$$T'(\theta) = \left(1+\frac{1}{\varepsilon_l}\right) \cdot \frac{1-F(\theta)}{\theta f(\theta)} \cdot (1-g(\theta)) \; + \; \underbrace{\frac{\text{Cov}_\theta(c, \; \partial U/\partial l)}{\theta f(\theta)}}_{\text{A–S breakdown term}}$$

Or extended formula:

equation 54

$$T'(\theta) = \left(1+\frac{1}{\varepsilon_l}\right) \cdot \frac{1-F(\theta)}{\theta f(\theta)} \cdot (1-g(\theta)) + \frac{\text{Cov}_\theta(c, U_l)}{\theta f(\theta)}$$

This term: $\frac{\text{Cov}_\theta(c, U_l)}{\theta f(\theta)}$ captures interaction between consumption c, labor disutility U_l.
Now Case 1: Leisure complements consumption[20]:

inequality 4

$$\text{Cov}(c, U_l) > 0$$

This means that rich consume more and value leisure more. This led to higher optimal taxes.

Case 2: Leisure substitutes consumption[21]:

inequality 5

$$\text{Cov}(c, U_l) < 0$$

This is done through two mechanisms:

[20] Leisure and consumption are often complementary, meaning leisure time increases the utility derived from consuming goods, while higher income (from less leisure) boosts consumption capacity. Economic models show that households maximize utility by balancing work, which enables consumption, with leisure, which allows them to enjoy those purchases, often treating them as normal goods.
[21] The optimal balance is reached when the Marginal Rate of Substitution (MRS)—the amount of consumption a person is willing to give up for one more hour of leisure—exactly equals their market wage.

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1. Substitution Effect:A wage increase makes leisure "more expensive" because the forgone income (opportunity cost) is higher.Individuals tend to substitute leisure with more work to increase consumption.
2. Income Effect:At higher income levels, individuals feel "wealthier" and may choose to "buy" more leisure by working fewer hours, even if wages are high.In this case, leisure acts as a normal good that people consume more of as they get richer

Without A–S result:

inequality 6

$$\tau_c(\theta) \neq 0$$

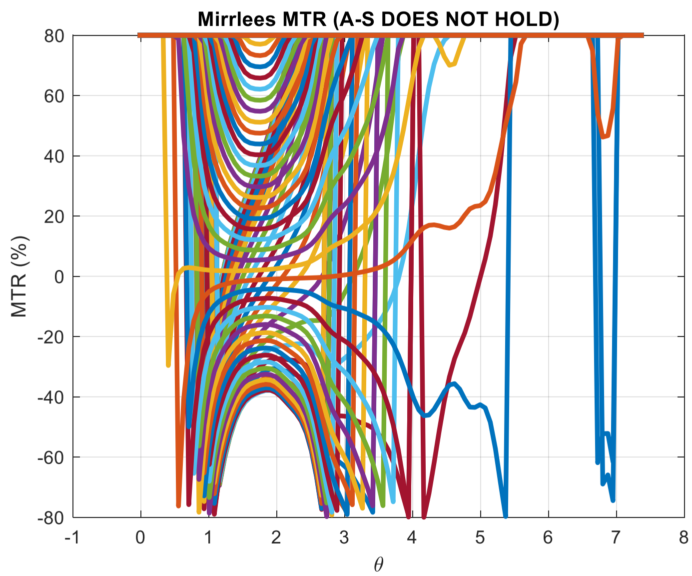
Optimal commodity tax rule:$\tau_c(\theta) \propto -\frac{\partial^2 U}{\partial c\, \partial l}$. Meaning, higher consumption reduces labor supply that leads to tax consumption.

Table 1. Key differences A-S holds and A-S fails

Component	A–S holds	A–S fails
Income tax	✓	✓
Commodity tax	x	✓
Extra term	x	✓
Formula	clean	+ covariance distortion

Source : Author's caclualtion

Figure 5 Mirrlees Marginal Tax Rates when Atkinson-Stiglitz does not hold



Source: Author's calculation

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4.RAMSEY–MIRRLEES–PIGOU POLICY SYSTEM IN THE DIRECTED TECHNICAL CHANGE (DTC) MODEL OF DARON ACEMOGLU ET AL. (2012).

When Atkinson-Stiglitz theorem fails: Optimal dirty tax becomes :$\tau_d = MD + Ramsey\ term$ or:

equation 55

$$\tau_d = D'(S)\phi + \frac{Cov(U_{c_d}, \theta)}{E[U_{c_d}]}$$

Thus:$\tau_d = Pigou + Mirrlees\ screening$, i.e. clean good tax is:

equation 56

$$\tau_c = \frac{Cov\left(U_{c_c}, \theta\right)}{E(U_{c_c})}$$

Individual productivity θ. Labor income:$y = \theta l$.Utility

equation 57

$$U(c, l, S) = u(c) - v(l) - D(S)$$

Government uses:nonlinear income tax $T(y)$,Pigouvian tax $\tau_p$on dirty production. Social planner maximizes

equation 58

$$\max \int U(c(\theta), l(\theta), S) dF(\theta)$$

subject to[22]:

equation 59

$$\int c(\theta) dF(\theta) = Y - G$$

Innovators compare returns:

equation 60

$$R_c = V_c A_c$$
$$R_d = \frac{V_d A_d}{1 + \tau_d}$$

Technology direction determined by

---

[22] And incentive compatibility:$\theta l(\theta) - T(\theta l(\theta)) \geq \theta l(\tilde{\theta}) - T(\theta l(\tilde{\theta}))$.

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inequality 7

$$R_c \geq R_d$$

Thus:With Atkinson–Stiglitz

equation 61

$$\tau_d = D'(S)\phi$$

Without Atkinson–Stiglitz:

equation 62

$$\tau_d = D'(S)\phi + screening\ tax$$

meaning that dirty innovation falls faster. Share of clean innovation and substituting taxes:

equation 63

$$x = \frac{R_c}{R_c + R_d}$$
$$x = \frac{V_c}{V_c + \frac{V_d}{1+\tau_d}}$$

Here higher τ_d leads to more clean innovation.

Theorem 2 Optimal taxes in DTC model with Mirrlees and Pigou

If preferences satisfy Atkinson-Stiglitz separability: $\tau_d = D'(S)\phi$ and optimal redistribution is implemented through the nonlinear income tax $T(y)$. If separability fails $\tau_d = (S)\phi + Ramsey - Mirrlees\ term$ and commodity taxes become part of the optimal tax system

Proof:

Following are notations: Utility: $U(c_d, c_c, l)$;Output: $Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}$, $i \in \{c, d\}$; Externality: $S = \phi Y_d$; Damage: $D(S)$[23].The social planner problem is:

equation 64

$$\max_{\{c_d, c_c, l\}} \int [U(c_d, c_c, l) - D(S)]\, f(\theta) d\theta$$

s.t.:$c_c + c_d \leq Y_c + Y_d$; $S = \phi Y_d$ and IC Mirrlees constraint. Price of clean good is :$p_c = 1$, and dirty good:$p_d = 1 + \tau_d$ .Budget constraint is given as:$c_c + (1+\tau_d)c_d = y - T(y)$.Individual optimization gives: $\frac{U_{c_d}}{U_{c_c}} = 1 + \tau_d$.Planner internalizes externality:$\frac{U_{c_d}}{U_{c_c}} = 1 + D'(S)\phi + distributional\ term$,the key object is wedge:$\tau_d = D'(S)\phi + redistributive\ correction$.In the

[23] Government instruments are: Non-linear income tax:$T(y)$ and commodity (Pigouvian) tax τ_d .

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Atkinson-Stiglitz separability case we assume: $U(c_d, c_c, l) = v(c_d, c_c) - \psi(l)$.By Atkinson-Stiglitz theorem: commodity demands do not depend on labor type θ conditional on income. Nonlinear income tax $T(y)$ is sufficient redistribution. Thus, no need for commodity taxes for redistribution exist here. Now, $\tau_d = D'(S)\phi$. In Non-separability A-S case: $U(c_d, c_c, l) = v(c_d, c_c, l)$.Now, consumption choices depend on labor supply, preferences differ across types in a way income taxation cannot fully control. Now, income tax $T(y)$ is not sufficient for redistribution. Government uses commodity taxes to:1.Relax incentive constraints,2.Improve redistribution efficiency. The planner's FOC becomes:

equation 65

$$\frac{U_{c_d}}{U_{c_c}} = 1 + D'(S)\phi + \underbrace{\frac{\text{Cov}\left(g(\theta), \frac{\partial c_d}{\partial y}\right)}{\text{efficiency term}}}_{\text{Ramsey–Mirrlees term}}$$

So now:

equation 66

$$\tau_d = D'(S)\phi + Ramsey - Mirrlees\ term$$

So, if A-S holds: $\tau_d = D'(S)\phi$. And, correction is pure Pigouvian, and redistribution is done only by $T(y)$.If Atkinson-Stiglitz fails:$\tau_d = D'(S)\phi + Ramsey - Mirrlees\ term$.So, here there is Pigouvian plus redistributive component and commodity taxation is optimal. So, in separability case: Income tax leads to redistribution and Pigou tax leads efficiency. In non-separability there is mixing of roles: commodity taxes helps efficiency and redistribution■ $Q.E.D.$

4.1 Ramsey-Mirrlees term

Now let preferences be non-separable:

equation 67

$$U(c_d, c_c, l; \theta) = u(c_c) + \gamma c_d - \frac{l^{1+\frac{1}{\varepsilon}}}{1 + \frac{1}{\varepsilon}} - \eta c_d l$$

This term $-\eta c_d l$ is non-separability.Dirty consumption correlates with labor and thus ability θ.Budget constraint is given as:$c_c + (1 + \tau_d)c_d = y - T(y)$, where $y = \theta l$. FOCs are given as:

- FOC for c_d:$\gamma - \eta l = \lambda(1 + \tau_d)$
- FOC for c_c:$u'(c_c) = \lambda$
- So:$\frac{\gamma - \eta l}{u'(c_c)} = 1 + \tau_d$

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We assume:$u(c_c) = c_c \Rightarrow u'(c_c) = 1$ i.e. quasi-linear utility in clean good[24].Now, since:$l = \frac{y}{\theta}$, we get:$1 + \tau_d = \gamma - \eta\frac{y}{\theta}$.So, $\frac{\partial c_d}{\partial y} \propto -\frac{\eta}{\theta}$.So higher types of θ consume dirty goods differently. So Mirrlees weights are: $g(\theta)$-SMW(social marginal weight)[25],and $f(\theta)$ which is probability density function. Defined covariance term here is:$Cov\left(g(\theta), \frac{\partial c_d}{\partial y}\right)$.So this term would be :

equation 68

$$\text{Ramsey} - \text{Mirrlees term} = \frac{\text{Cov}\left(g(\theta), \dfrac{\partial c_d}{\partial y}\right)}{\mathbb{E}\left[\dfrac{\partial c_d}{\partial p_d}\right]}$$

$\frac{\partial c_d}{\partial y} = -\frac{\eta}{\theta}$, this is the income sensitivity, price sensitivity (Slutsky)[26]. From demand:$c_d = \frac{\gamma - \eta l}{1+\tau_d}$.So:$\frac{\partial c_d}{\partial p_d} = -\frac{c_d}{1+\tau_d}$.Covariance term is: $\text{Cov}\left(g(\theta), \frac{\partial c_d}{\partial y}\right) = -\eta\text{Cov}\left(g(\theta), \frac{1}{\theta}\right)$.In the denominator: $\mathbb{E}\left[\frac{\partial c_d}{\partial p_d}\right] = -\mathbb{E}\left[\frac{c_d}{1+\tau_d}\right]$.So tax final closed form will look like:

equation 69

$$\tau_d = D'(S)\phi + \underbrace{\frac{\eta Cov\left(g(\theta), \dfrac{1}{\theta}\right)}{-\mathbb{E}\left[\dfrac{c_d}{1+\tau_d}\right]}}_{\textit{Ramsey–Mirrlees term}}$$

In the case of log-normal skills:$\theta \sim \log N(\mu, \sigma^2)$, and $g(\theta) = \theta^{-k}$(inequality aversion).Then:

equation 70

$$\text{cov}\left(g(\theta), \frac{1}{\theta}\right) = \mathbb{E}\left[\theta^{-(k+1)}\right] - \mathbb{E}[\theta^{-k}]\mathbb{E}[\theta^{-1}]$$
$$\mathbb{E}[\theta^{-a}] = \exp\left(-a\mu + \frac{1}{2}a^2\sigma^2\right)$$

So now:

equation 71

$$\text{Cov} = \exp\left(-(k+1)\mu + \frac{1}{2}(k+1)^2\sigma^2\right) - \exp\left(-k\mu + \frac{1}{2}k^2\sigma^2\right)\exp\left(-\mu + \frac{1}{2}\sigma^2\right)$$

So now the final formula will be:

---

[24] Then:$1 + \tau_d = \gamma - \eta l$, so c_d is implicit but it depends on l.

[25] For any individual, it is defined a generalized social marginal welfare weight g_i which measures how much society values the marginal consumption of individual i,see Saez, E., & Stantcheva, S. (2016).

[26] The Slutsky eq.decomposes the change in demand for good i in response to change in price in good j (p_j):$\frac{\partial x_i}{\partial p_j} = \frac{\partial h_i}{\partial p_j} - x_j\frac{\partial x_i}{\partial m}$,where $\frac{\partial h_i}{\partial p_j}$ is substitution effect, and $x_j\frac{\partial x_i}{\partial m}$ is the income effect.

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equation 72

$$\tau_d = D'(S)\phi + \eta \cdot \frac{\exp\left(-(k+1)\mu + \frac{1}{2}(k+1)^2\sigma^2\right) - \exp\left(-k\mu + \frac{1}{2}k^2\sigma^2\right)\exp\left(-\mu + \frac{1}{2}\sigma^2\right)}{-\mathbb{E}\left[\frac{c_d}{1+\tau_d}\right]}$$

4.2 Existence of optimal environmental tax equilibrium

Here we set up this lemma:

Lemma 2 Existence of optimal environmental tax equilibrium

∀ initial condition: $\forall(A_{c0}, A_{d0}, S_0) \in \mathbb{R}^3_{++}$,$\exists\{x_t, \tau_{dt}, A_{ct}, A_{dt}, S_t\}_{t=0}^{\infty}$ satisfying DTC-Mirrlees system.

Proof:

Equilibrium $R_c(x) = R_d(x)$ defines a continuous mapping, $R_c(x); R_d(x) \in [0,1]$. And $x \in (0,1)$ is a compact domain. We define mapping $T(x) = F\big(\tau_d(x)\big)$,tax depends on pollution and pollution depends on dirty output, so $\exists x^* \in (0,1) \to T(x^*) = x^*$ by the Brouwer fixed point[27]. Technology transitions are linear:$A_{ct+1} = A_{ct}(1 + \gamma x_t)$;$A_{dt+1} = A_{dt}(1 + \gamma(1 - x_t))$. So, now mapping system is: $(A_{ct}, A_{dt}, S_t) \mapsto (A_{ct+1}, A_{dt+1}, S_{t+1})$.The dynamic system generates equilibrium path. If research returns satisfy:$\frac{dR_c}{dx} > 0; \frac{dR_d}{dx} < 0$ then $R_c(x) = R_d(x)$ has a unique solution.■

Corollary 1 A-S case

If preferences satisfy: $U(c,l) = u(c) - v(l)$ then the optimal tax simplifies to: $\tau_d = D'(S)\phi$ and an equilibrium still exists and is unique.

This theorem shows that climate policy and innovation direction form a joint equilibrium system. It also shows that the Acemoglu policy rule (carbon tax only) is a special case of the general Mirrlees optimal taxation problem. Next, we will code and plot this theorem and corollary. In this code: $\tau_d = D'(S)\phi + \underbrace{\frac{\eta Cov\left(g(\theta), \frac{1}{\theta}\right)}{-\mathbb{E}\left[\frac{c_d}{1+\tau_d}\right]}}_{Ramsey–Mirrlees\ term}$, code implements covariance closed log-normal:$E[\theta^{-a}] = \exp\left(-a\mu + \frac{1}{2}a^2\sigma^2\right)$,so :Cov $= E[\theta^{-(k+1)}] - E[\theta^{-k}]E[\theta^{-1}]$.The result as we will see on next plot holds: If separable preferences Atkinson-Stiglitz holds then $\tau_d = D'(S)\phi$,if it does not hold: $\tau_d = \mathrm{Pigou} + \mathrm{Ramsey} - \mathrm{Mirrlees\ correction\ term}$.Next, we will plot previous in a code that finds:Covariance, Ramsey-Mirrlees term, also computes whole tax,DTC equilibrium x^*.Here τ_d line total optimal environmental tax, moves with inequality aversion k meaning environmental tax is also a redistributive tool. Non-separability is the driver:

[27] Any continuous function $G: \mathbb{B}^n \to \mathbb{B}^n$ has a fixed point where $\mathbb{B} = \{x \in \mathbb{R}^n: x_1^2 + \cdots + x_n^2 \leq 1\}$,see Samelson, H. (1963)

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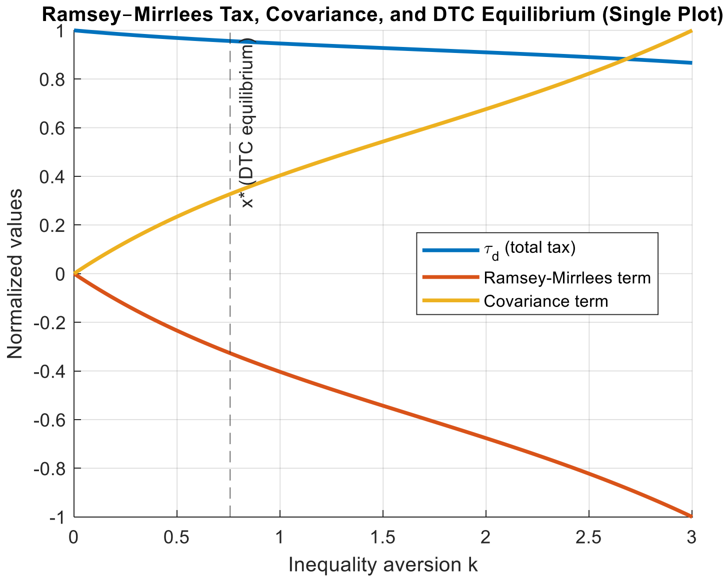
equation 73

$$U = u(c_c) + \gamma c_d - \frac{l^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} - \eta c d l$$

Optimal environmental tax is: τ_d = Pigou + Redistribution correction.

Figure 6 Ramsey–Mirrlees Tax, Covariance, and DTC Equilibrium



Source: Author's calculation

4.3 Existence of multiple equilibria in DTC with Local Weak Bias with Two Skills and Labor- Augmenting Technology (A-S holds and A-S fails)

Here we set Corollary like in Acemoglu (2007), and Loebbing,J.(2023).

Corollary 2

Local Weak Bias with Two Skills and Labor-Augmenting Technology.Suppose $N = 2, \Theta = \mathbb{R}^2_{++}$.and production F takes form:

equation 74

$$F(L,\theta) = G(\theta_1 L_1, \theta_2 L_2) - C(\theta)$$

$\Theta = \mathbb{R}^2_{++}$ is the set of feasible technologies which is a compact topological space., θ here is technology.G is twice continuously differentiable and homothetic, $N = 2$ means two skill levels.

Proposition 1

Homothetic production function see Førsund, F. R. (1975).Production function $x = F[g(v_1), \dots g(v_n)]$,where $g(\cdot)$ is homogenous of degree one, and continuously differentiable, $g(v_1, \dots v_n) \in [0, \infty]$,x is the input rate, and $v_i [i = 1, \dots\dots, n]$ is the output rate.The production

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function $x = (v_1, \dots, v_n)$is homothetic as defined by $x = F[g(v_1), \dots g(v_n)]$ if and only if the scale elasticity is constant on each isoquant i.e. the elasticity of scale is function of output.

Proof: The elasticity of the scale is given as in Frisch (1965):

equation 75

$$\varepsilon = \varepsilon(v_1, \dots, v_n) = \sum_{i=1}^{n} \frac{\partial f}{\partial v_i} \frac{v_i}{f(v_i, \dots, v_n)}$$

The scale elasticity is function of inputs. Now, inserting $x = F[g(v_1), \dots g(v_n)]$ into $\varepsilon = \varepsilon(v_1, \dots, v_n) = \sum_{i=1}^{n} \frac{\partial f}{\partial v_i} \frac{v_i}{f(v_i, \dots, v_n)}$ we get:

equation 76

$$\varepsilon = \sum_{i=1}^{n} \frac{\partial f}{\partial v_i} \frac{v_i}{f(v_i, \dots, v_n)} = \frac{F'\big(g(v_1), \dots g(v_n)\big)}{F\big(g(v_1), \dots g(v_n)\big)} \cdot \sum_{i=1}^{n} \frac{\partial g}{\partial v_i} = \frac{F'\big(G(x)\big)}{x} G(x) = \frac{G(x)}{xG'(x)}$$

i.e. the elasticity of scale is constant on each isoquant■.
Wages and wage premium in previous are given as:

equation 77

$$w_i = \frac{\partial G}{\partial(\theta_i L_i)} \cdot \theta_i$$
$$\omega \equiv \frac{w_2}{w_1}$$

Corollary 3
equation 78

$$\frac{dL_2}{L_2} \geq \frac{dL_1}{L_1} \Rightarrow d\theta^* = \nabla_L \theta^* \, (L) dL$$

Previous implies that $d\omega \geq 0$. More skilled labor means innovation biases toward skill and inequality rises.Now we introduce clean (c) and dirty (d) sectors:

equation 79

$$Y_j = G_j(\theta_{1j} L_{1j}, \theta_{2j} L_{2j}), j \in \{c, d\}$$

Dirty externality:$S = \phi Y_d$.Total output:

equation 80

$$Y = Y_c + Y_d - D(S)$$

Planners' problem is to maximize:

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$$\int W(u(\theta))f(\theta)d\theta$$

Subject to:

(i) Resource constraint

equation 81

$$c(\theta) = y(\theta) - T(y(\theta))$$

(ii) Production

equation 82

$$y = w_1 L_1 + w_2 L_2$$

(iii) Technology choice

equation 83

$$\theta^* = \arg\max_{\theta}[G(\theta_1 L_1, \theta_2 L_2) - C(\theta)]$$

(iv) Externality

equation 84

$$S = \phi Y_d$$

FOCs are : DTC

equation 85

$$\frac{\partial G}{\partial(\theta_i L_i)} L_i = \frac{\partial C}{\partial \theta_i}$$
$$\Rightarrow \theta_i^* = \theta_i^*(L_1, L_2)$$

Wage effects are: $w_i = \theta_i \cdot G_i' \Rightarrow \frac{w_2}{w_1} = \frac{\theta_2 G_2'}{\theta_1 G_1'}$. Standard Mirrlees formula:

equation 86

$$T'(y) = \frac{1 - G(\theta)}{1 - G(\theta) + \epsilon(\theta)}$$

where $G(\theta)$ is social marginal weight,and $\epsilon(\theta)$ is labor elasticity supply. Pigouvian tax is : $\tau_d = D'(S) \cdot \phi$ .Ramsey tax with with A–S and total tax are given as:

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equation 87

$$\tau_d^{Ramsey} = \frac{\lambda}{1+\lambda} \cdot \frac{1}{\epsilon_d}$$
$$\tau_d = D'(S)\phi + \frac{\lambda}{1+\lambda}\frac{1}{\epsilon_d}$$

If we combine everything, than change in skill premium:

equation 88

$$d\omega = \underbrace{\frac{\partial \omega}{\partial \theta}}_{\text{technology}} d\theta^* + \underbrace{\frac{\partial \omega}{\partial \tau}}_{\text{tax distortion}} d\tau$$

Lemma 3 DTC with A-S,Mirrlees,Pigou,Ramsey

Under labor augmenting technology, homothetic G and convex C and with A-S separability,then:$\frac{dL_2}{L_2} \geq \frac{dL_1}{L_1}$implies:

equation 89

$$d\omega = \underbrace{(+)}_{\text{DTC}} + \underbrace{(-)}_{\text{Pigou}} + \underbrace{(-)}_{\text{Mirrlees}}$$

and: sign$(d\omega)$ is ambiguous

Corollary 4

Skill-biased technical change persists iff:

inequality 8

$$\frac{\partial \theta_2^*}{\partial L_2} > \text{policy-induced dampening}$$

Now we will extend previous to: CES Production with Labor-Augmenting Technology
Let:

equation 90

$$G(\theta_1 L_1, \theta_2 L_2) = \left[(\theta_1 L_1)^{\frac{\sigma-1}{\sigma}} + (\theta_2 L_2)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

Define:$\rho = \frac{\sigma-1}{\sigma}$,For wages i.e marginal product:

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equation 91

$$w_i = \frac{\partial G}{\partial(\theta_i L_i)} \cdot \theta_i$$

CES derivative gives:

equation 92

$$w_i = \theta_i^{\rho} L_i^{\rho-1} [(\theta_1 L_1)^{\rho} + (\theta_2 L_2)^{\rho}]^{\frac{1}{\rho}-1}$$

Skill-premium ratio:

equation 93

$$\omega \equiv \frac{w_2}{w_1} = \left(\frac{\theta_2}{\theta_1}\right)^{\rho} \left(\frac{L_2}{L_1}\right)^{\rho-1}$$

We assume cost function (homothetic convex):

equation 94

$$C(\theta) = \frac{1}{2}(\theta_1^2 + \theta_2^2)$$

FOC:$\frac{\partial G}{\partial(\theta_i L_i)} L_i = \theta_i$.When we plug CES derivative:

equation 95

$$\theta_i = L_i (\theta_i L_i)^{\rho-1} [(\theta_1 L_1)^{\rho} + (\theta_2 L_2)^{\rho}]^{\frac{1}{\rho}-1}$$

Divide FOCs:

equation 96

$$\frac{\theta_2}{\theta_1} = \left(\frac{L_2}{L_1}\right)^{\frac{\rho}{1-\rho}}$$

Since:

equation 97

$$\frac{\rho}{1-\rho} = \sigma - 1$$

Closed-form result:

equation 98

$$\frac{\theta_2^*}{\theta_1^*} = \left(\frac{L_2}{L_1}\right)^{\sigma-1}$$

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For skill-premium dynamics Substitute into ω:

equation 99

$$\omega = \left(\frac{L_2}{L_1}\right)^{\rho(\sigma-1)} \cdot \left(\frac{L_2}{L_1}\right)^{\rho-1}$$

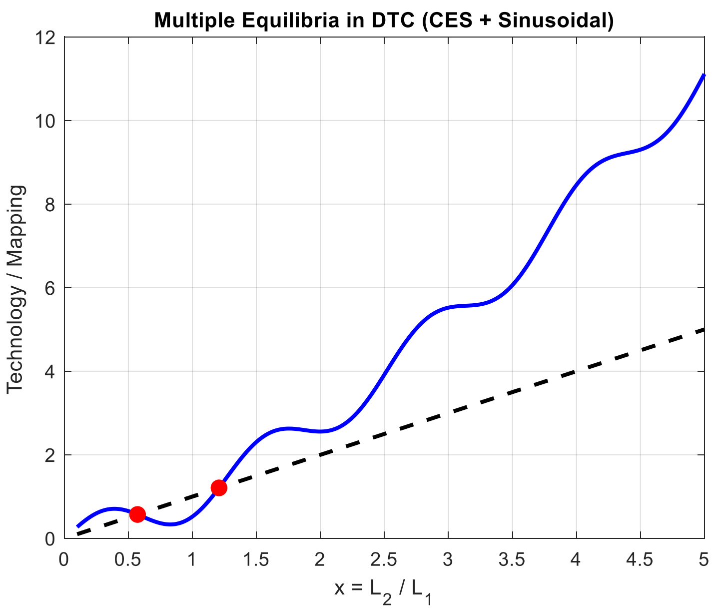
Simplify and we get: $\omega = \left(\frac{L_2}{L_1}\right)^{\sigma-1}$. This is a key result here. Dynamics of technology system is we define: $x = \frac{L_2}{L_1}, \frac{\theta_2}{\theta_1} = x^{\sigma-1}$ and dynamic adjustment: $\dot{\theta}_2 = \Phi(x, \theta_2)$.We define:$\theta_2 = x^{\sigma-1} + \alpha\sin(\beta x)$.At equilibrium:$x = \theta_2(x)$ so: $x = x^{\sigma-1} + \alpha sin(\beta x)$.For the Jacobian stability:$\mathrm{F(x)} = \mathrm{x}^{\sigma-1} + \alpha\sin(\beta\mathrm{x})$ and Jacobian:

equation 100

$$J = F'(x) = (\sigma - 1)x^{\sigma-2} + \alpha\beta cos(\beta x)$$

equilibrium is stable if $|J| < 1$.

Figure 7 Multiple equilibria in DTC-AABH-CES model



Source: Author's calculations

From now onward A-S fails.Let preferences be non-separable:

equation 101

$$u(c_d, c_c, l) = v(c_d, c_c) - \psi(l, c_d)$$

in previous dirty consumption affects labor disutility, and government uses commodity taxes for redistribution. Now modified household problems is given as:

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equation 102

$$c_c + (1+\tau_d)c_d = y - T(y)$$

FOCs are given as:

equation 103

$$\frac{\partial v/\partial c_d}{\partial v/\partial c_c} = 1 + \tau_d$$
$$w\big(1 + T'(y)\big) = \psi_l(l, c_d)$$

Production side is the same CES-DTC:

equation 104

$$G = [(\theta_1 L_1)^\rho + (\theta_2 L_2)^\rho]^{\frac{1}{\rho}}$$
$$\omega = \left(\frac{\theta_2}{\theta_1}\right)^\rho \left(\frac{L_2}{L_1}\right)^{\rho-1}$$
$$\frac{\theta_2}{\theta_1} = \left(\frac{L_2}{L_1}\right)^{\sigma-1}$$
$$L_i = L_i(\tau_d, T'(y))$$

Now taxes affect technology:

equation 105

$$\theta^* = \theta^*\big(L(\tau_d)\big)$$
$$\frac{d\theta^*}{d\tau_d} = \frac{\partial\theta^*}{\partial L}\cdot\frac{\partial L}{\partial\tau_d}$$

Labour supply distortion is given as:

equation 106

$$\begin{matrix} L_2 = L_2^0\cdot(1-\gamma\tau_d) \\ L_1 = L_1^0\cdot(1-\eta\tau_d) \end{matrix} \Rightarrow x = \frac{L_2}{L_1} = \frac{L_2^0}{L_1^0}\cdot\frac{1-\gamma\tau_d}{1-\eta\tau_d}$$

Technology equation also is distorted:

equation 107

$$\theta_2 = x^{\sigma-1}$$
$$\Downarrow$$
$$\theta_2 = \left[\frac{L_2^0}{L_1^0}\cdot\frac{1-\gamma\tau_d}{1-\eta\tau_d}\right]^{\sigma-1}$$

We endogenizes Pigouvian plus Ramsey tax:

Without A–S:

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equation 108

$$\tau_d = D'(S)\phi + \underbrace{\Lambda \cdot \frac{\partial c_d}{\partial y}}_{\text{Redistribution term}} + \frac{\lambda}{1+\lambda}\frac{1}{\epsilon_d}$$

Simplify:$\tau_d = \tau_0 + \kappa x$. Final fixed point system is given as:

equation 109

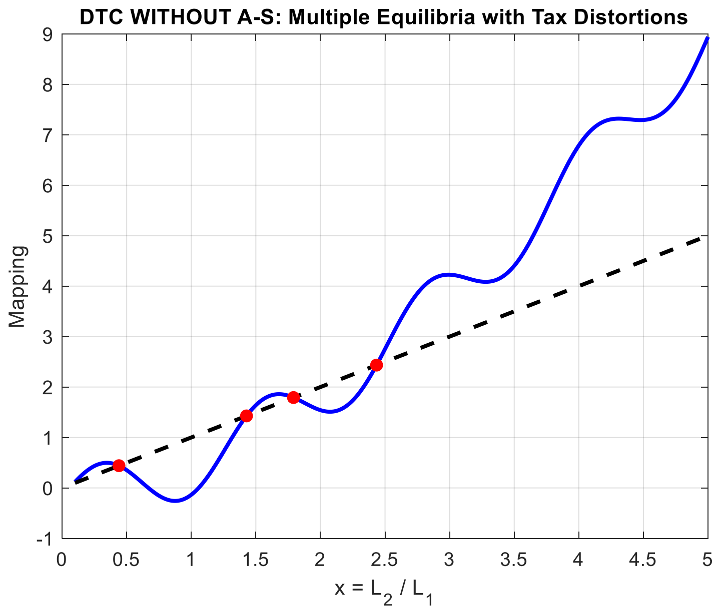
$$x = x^{\sigma-1} + \alpha \sin(\beta x) - \delta(\tau_0 + \kappa x)$$
$$F(x) = x^{\sigma-1} + \alpha \sin(\beta x) - \delta\tau_0 - \delta\kappa x$$
$$x = F(x)$$

Jacobian now includes taxes:

equation 110

$$J = (\sigma - 1)x\sigma - 2 + \alpha\beta \cos(\beta x) - \delta\kappa$$

Figure 8 DTC-without Atkinson-Stiglitz : multiple equilibria with tax distortions



Source: Author’s calculations

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Table 1 Equilibria in DTC-AABH -CES model and DTC-without A-S [28]

Equilibria/Jacobians	Multiple equilibria in DTC-AABH-CES model	Conclusion/type	DTC-without Atkinson-Stiglitz : multiple equilibria with tax distortions	Conclusion/type
Equilibrium 1	0.5718	Unstable/saddle point	0.4404	Likely Stable
Equilibrium 2	1.2082	Potential Stability	1.4277	Unstable
Equilibrium 3	/	/	1.7915	Likely Stable
Equilibrium 4	/	/	2.4345	Unstable
Jacobian 1	-1.2666	Saddle point	-1.1749	Node/spiral
Jacobian 2	4.0758	Node/spiral	3.3602	saddle point
Jacobian 3	/	/	-1.0707	Node/spiral
Jacobian 4	/	/	4.7107	saddle point

Source: Author's calculation

equation 111

$$J = \begin{pmatrix} -1.1749 & 3.3602 \\ -1.0707 & 4.7107 \end{pmatrix}$$

$$\Delta = (-1.1749 \times 4.7107) - (3.3602 \times -1.0707) = -1.9368$$

Result: The determinant is negative ($\Delta < 0$).Conclusion: Any equilibrium point with a negative Jacobian determinant is a Saddle Point.
Stability: It is unstable

equation 112

$$Trace(\tau) = -1.1749 + 4.7107 = 3.5358$$

While the trace is positive ($\tau > 0$), which often suggests instability, the determinant (Δ) is the primary indicator for this specific matrix. Conclusion: Because the determinant is negative, the equilibrium point is a saddle point and is unstable, regardless of the trace value. Next, in a table we will show what happens with: Commodity tax, labor distortion, technology distortion, multiple equilibria and policy impact on ω.

[28] A negative Jacobian determinant (Δ) always indicates a saddle point. A positive Jacobian determinant ($\Delta > 0$) means the eigenvalues have the same sign. If the Trace τ (the sum of the diagonal elements of the matrix) is negative, this point is Stable. If the Trace is positive, this point is Unstable.

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Table 2 A-S holds A-S fails in DTC-

	A–S Holds	**A–S Fails**
Commodity tax	irrelevant	essential
Labor distortion	none	yes
Technology distortion	indirect	direct
Multiple equilibria	via technology mapping	via tech.mapping + tax
Policy impact on ω	weak	strong

Source: Author's calculation

CONCLUSION

Optimal environmental tax deviates from the Pigouvian benchmark as inequality aversion increases. This deviation is driven by the covariance between social welfare weights and the income sensitivity of dirty consumption. When preferences are non-separable, environmental taxation becomes an instrument of redistribution, thereby affecting both welfare and the direction of technological change. As k increases: society cares more about low-income agents. If Covariance> 0 optimal tax is reduced since taxation hurts poor. If covariance < 0 rich people consume more dirty goods, since then taxation is progressive and optimal tax increases. Dirty consumption depends on labor and income i.e. ability θ .Here Atkinson-Stiglitz theorem does not hold so commodity taxation becomes optimal once again.Without policy, skill supply increase leads to skill biased innovation and higher inequality. Pigouvian tax here can shift innovation away from dirty sector. In return that can reduce skill bias if dirty sector is skill intensive. With Mirrlees tax government can redistribute income but it does not affect production and reduces inequality after tax. With Ramsey taxes government distorts sectoral allocation and indirectly affects innovation. Key mechanism for Acemoglu is : $d\theta^* \uparrow \Rightarrow \omega \uparrow$, and from Pigou: $\tau_d \uparrow \Rightarrow Y_d \downarrow \Rightarrow$ less demand for certain skills,and from Mirrlees : :$T'(y_2) > T'(y_1) \Rightarrow$ reduces after-tax inequality.Directed technology effect is greater than Pigou taxes plus Ramsey distortions. Skill-premium depends on relative supply but amplified by σ.So if $\sigma > 1$ there is a strong skill bias. And if $\sigma < 1$ there is compression[29].With Atkinson-Stiglitz in our system technology leads to inequality. While in the case when A-S fails: this is a fully endogenous inequality–policy–innovation system. This is a Skill-Biased Technological Change: Technology increases demand for highly skilled workers (data scientists, AI specialists), driving up their wages, while automating middle-skill jobs and reducing demand for low-skilled labor. Since tech-driven wealth often comes from stocks and assets rather than traditional wages, taxing capital gains is crucial for reducing top-tier inequality. When A-S holds: Multiple equilibria come only from innovation structure (Multiple equilibria are pure coordination/technology phenomenon), government cannot influence relative prices efficiently. When A-S fails: Government uses commodity taxes strategically, which changes: labor allocation, profitability of innovation, direction of technical change i.e. multiple equilibria become policy-dependent.

[29] Innovation compression refers to accelerating development cycles and reducing uncertainty by utilizing intuition, hands-on experience, and flexible options. It is a strategic approach to rapidly innovate in highly uncertain environments, often involving advanced technologies like AI to compress development time, or specialized engineering techniques in manufacturing.

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