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GENERAL AND PARTIAL EQUILIBRIUM IN HANK, TWO-AGENT NEW-KEYNESIAN DSGE HAND TO MOUTH PLUS OPTIMIZERS AND GE-PE IN RANK-TANK MODEL

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Abstract

In HANK model GE vs PE paths differ substantially: in GE the endogenous interest-rate and wage responses amplify the transition dynamics (GE consumption and mean assets tend to be higher during the recovery because lower early r_t or higher later returns change saving incentives). Wealth inequality (Gini on assets) rises initially in both PE and GE after the MIT shock, then declines gradually; the timing and peak differ across PE and GE because of endogenous factor-price feedbacks. PE exaggerates inequality and consumption losses, since households cannot rely on price movements.GE dampens these effects: wages and interest rates move endogenously, redistributing resources across households and time.In RANK compared to TANK: Partial Equilibrium (PE): treats policy instruments as exogenous — IRFs show the "direct" effect of shocks. General Equilibrium (GE): closes the system with a Taylor rule. Responses are typically dampened or altered because policy reacts endogenously. GE introduces stabilizing feedback; PE exaggerates shock persistence/size. Fiscal transfer is highly effective in stimulating demand because Hand to Mouth consume all of it.The central bank reacts with higher interest rates (Taylor rule), partially offsetting demand.

Keywords: HANK, RANK, TANK, general equilibrium, partial equilibrium

JEL codes:E21, E24,E52,E62

Introduction

Heterogeneous Agent New Keynesian (HANK) models are emerging as leading frameworks to study the impact of monetary and fiscal policy on the macroeconomy. The K in HANK, RANK and TANK celebrate John Maynard Keynes more specifically his general theory of unemployment Keynes, (1936). And according to Sargent (2023), Keynes wanted a theory that explains: equilibria with underemployed resources and excess supplies, reduces to a Walrasian general equilibrium theory when resources are fully employed,and theory that rationalizes light-handed fiscal-monetary interventions that depend only on aggregate data. Keynes wanted macroeconomic policies that promote aggregate efficiency while letting individuals' choices guide the allocation of resources[1]. Also, always balancing current accounts, requiring PV balance of CA not period by period, and using countercyclical capital-account deficits, but not current-account deficits, to finance public works. Keynes advocated: achieving full employment by using well timed public investment to sustain adequate demand and then relying on markets to set relative prices and allocations see Sargent (2023). Fast forward to 2018, Kaplan,Moll,Violante,(2018), compare HANK (Heterogenous agent New Keynesian) and RANK (Representative agent New Keynesian) models and find that in a environment of short term nominal interest rates[2] as the primary monetary instrument in HANK

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[1] To achieve these goals Keynes (1924) and Keynes (1925) advocated a price level target: previous two works by Keynes emphasized the priority of present value government budget balance as essential determinant of the price level.

[2] Macroeconometric analysis of aggregate time-series data finds a small sensitivity of consumption to changes in the interest rate after controlling for income Campbell and Mankiw (1989); Yogo (2004); Canzoneri, Cumby, and Diba 2007.

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framework monetary policy affects aggregate consumption through indirect effects that arise from general equilibrium increase in labor demand. This finding is in stark contrast to Representative Agent New Keynesian (RANK) framework, where intertemporal substitution drives virtually all the transmission from interest rates to consumption. In HANK model the way that fiscal policy responds to an interest rate change profoundly affects the overall effectiveness of monetary policy, a result that is also at odds with the Ricardian nature of standard RANK economies. Failure of Ricardian equivalence[3] implies that, in HANK models, the fiscal reaction to the monetary expansion is a key determinant of the overall size of the macroeconomic response.Ricardian equivalence fails in HANK models: Ricardian equivalence suggests that consumers anticipate future taxes and adjust their consumption accordingly, meaning government debt does not affect demand. However, liquidity-constrained households cannot smooth consumption in response to future income changes, challenging Ricardian predictions. So, government borrowing can influence aggregate demand, creating tension within the New Keynesian framework,see Xakousti Chrysanthopoulou, Moise Sidiropoulos, Alexandros Tsioutsios,(2024). Failure of Ricardian equivalence in HANK models is also explained by Kaplan (2025) .Because of the precautionary savings motive and heterogeneous MPCs, HANK economies violate Ricardian equivalence[4].The interconnectedness of fiscal and monetary policy has a long history and dates back to: Sargent (1981),Sargent and Wallace (1981), Sargent and Wallace (1982). The implications of government budget constraint for sticky price New-Keynesian economies have been developed by: Davig and Leeper (2007), Sims (2011) and Cochrane (2011). Transmission of MP to macroeconomic aggregates works through Households, have different levels of savings allocated to different assets, different work arrangements with different sensitivities to the business cycle, and different propensities to consume out of changes in their income and their wealth. Canonical HANK model is standard incomplete markets model in the Bewley-Huggett-Aiyagari (see Aiyagari, S Rao (1994) , Bewley (1987), Huggett (1993)) tradition with the New Keynesian paradigm, calibrated to be jointly consistent.So, in this paper we will investigate HANK GE and HANK PE or general and partial equilibrium in HANK but also in RANK Representative Agent New Keynesian (RANK) and TANK Two-Agent New Keynesian (TANK).In RANK economies where the substitution channel drives virtually all of the transmission from interest rates to consumption. TANK model feature “savers” who engage in intertemporal substitution and are highly responsive to interest rate changes, see Kaplan,Moll,Violante,(2018).Literature recognizes three main differences between HANK and RANK models: First is Heterogenous Marginal Propensities to Consume (MPCs)[5],second as previously said failure of Ricardian equivalence[6],and third Upward sloping steady-state asset supply curve in RANK models there is perfectly elastic supply curve $r^* = \rho$, in HANK there is upward slopping curve, steady-state real asset supply curve is $a(r)$[7].But as Kaplan (2025) point none of these features are unique to HANK models: TANK (Two-agent NK) models feature MPC heterogeneity see Galí, Jordi, (2008). OLG models violate Ricardian equivalence, see Aguiar et al.(2023), bonds-in-utility (BIU) models feature an upward sloping steady-state asset supply curve, see Kaplan et al.(2023).In this paper we will outline canonical HANK model, and we will mathematically derive PE and GE in HANK (canonical model).Later, we will derive PE, GE in TANK and RANK model and we will explain Hand-to-Mouth behavior, and we will derive and show Inflation and Output gap responses to permanent monetary shock AR(1).Monetary and fiscal policies are inescapably interconnected, these interconnections are especially important when households have heterogeneous MPCs and Ricardian equivalence does not hold as in HANK models.PE is “partial” because we do not enforce market-clearing or endogenous policy feedback — only the private-sector block is solved. In GE All blocks (households, firms,

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[3] The Ricardian equivalence proposition (also known as the Ricardo–de Viti–Barro equivalence theorem) is an economic hypothesis holding that consumers are forward-looking and so internalize the government's budget constraint when making their consumption decisions, see Barro (1974).

[4] In HANK models the timing and distribution of transfers do matter for spending. The timing of transfers matters because households with low wealth raise their spending in response to a lump sum transfer, even if it is compensated with an equivalent lower transfer in the future. The presence of a borrowing constraint or strong precautionary motive leads these households to act as if they had a shorter horizon than a representative household,see Kaplan (2025).

[5] Households in HANK models face uninsured idiosyncratic income risk and borrowing constraints,giving rise to a precautionary savings motive. This in turns leads to a consumption policy function that is concave in wealth. Households' holdings of liquid wealth, rather than total wealth determine MPCs.

[6] In a RANK model with infinitely lived households, the timing and distribution of lump-sum taxes or transfers does not matter for consumption.In HANK models: The timing of transfers matters because households with low wealth raise their spending in response to a lump sum transfer, even if it is compensated with an equivalent lower transfer in the future.

[7] $\lim_{r \to \rho} a(r) = \infty$, the asset market clearing condition is: $a(r^*) = \frac{\bar{s}}{r^*}$,and the level of debt in steady-state is given as:$b^* = \frac{s^*}{r^*}$,the government budget constraint is; $\dot{b}_t = r_t b_t - s_t$ where s_t is the real value of the primary surplus.

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policy) are solved simultaneously, so it's a full equilibrium system, General Equilibrium (endogenizes policy via Taylor rule etc., all markets clear).

Consumption-saving model

First, we will introduce standard consumption-savings problem. First, we begin with Bellman equation:

equation 1

$$v(A,y) = \max_c \left\{u(c) + \beta \mathbb{E}_{(y'|y)} v(A',y')\right\}; A; = R(A+y-c)$$

FOC for an interior optimum is:

equation 2

$$u'(c) = \beta R \mathbb{E}_{(y'|y)}[v_A(A',y')]$$

Where $v_A(\cdot)$ denotes the partial derivative the value function with respect to assets[8]. The solution to this problem is a policy function that relates consumption to the state vector:

equation 3

$$c = \phi(A,y)$$

the first-order condition is

equation 4

$$u'(c) = \beta R E_{(y'|y)} v_A(A',y') \; \forall (A,y)$$

Where $v_A(A',y') = \frac{\partial v(A',y')}{\partial A'}$

Euler equation is :

equation 5

$$u'(c) = \beta R E_{(y'|y)} u'(c')$$

CRRA utility : $u(c) = (c^{1-\gamma} - 1)/(1-\gamma) \quad -c''(c)/u'(c) = \gamma$. So, we can rewrite

$$v(A,y) = \max_c u(c) + \beta E_{y'|y} v(A',y')$$

As :

equation 6

$$1 = \beta R E \left(\frac{c'}{c}\right)^{-\gamma}$$

[8] $\frac{\partial v(A,y)}{\partial A} = \beta \mathbb{E}_{(y'|y)}[v_A(A',y')] \cdot \frac{\partial A'}{\partial A} = \beta R \mathbb{E}_{(y'|y)}[v_A(A',y')]$. Now take the first-order condition for the choice c,differentiate objective w.r.t. c at the optimum: $0 = u'(c^*) + \beta \mathbb{E}_{(y'|y)}[v_A(A',y')] = u'(c^*) - \beta \mathbb{E}_{(y'|y)}[v_A(A',y')]$ .Hence, $u'(c^*) = \beta R \mathbb{E}_{(y'|y)}[v_A(A',y')]$. Since both previous are equal: $v_A(A',y') = u'(c^*(A,y))$. So the marginal value of current assets equals the marginal utility of optimal consumption. (This result requires an interior optimum and usual differentiability/transversality conditions.

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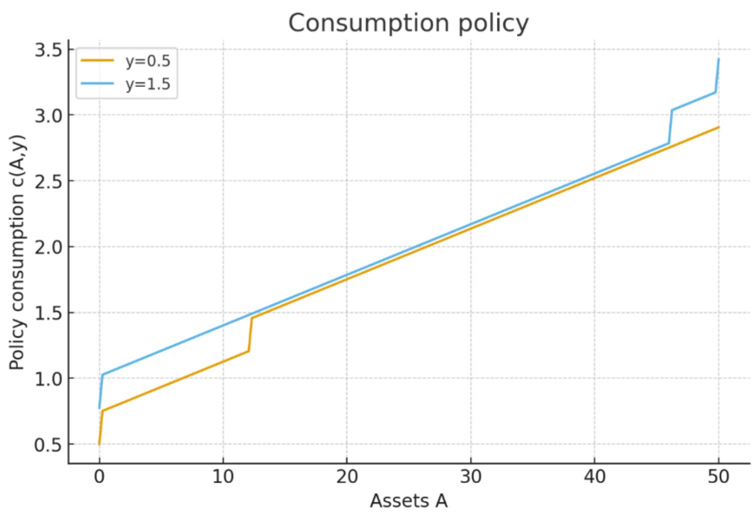

Figure 1 consumption policy

Source: Authors' calculations

Linearization of Euler equation

Let's start with:

equation 7

$$u'(c_t) = \beta R \mathbb{E}[c_{t+1}]$$

With CRRA $u'(c) = c_t^{-\gamma}$,this is:

equation 8

$$c_t^{-\gamma} = \beta R \mathbb{E}\left[c_{t+1}^{-\gamma}\right]$$

Consumption growth is : $g_{t+1} \equiv \frac{c_{t+1}}{c_t}$ then:

equation 9

$$1 = \beta R \mathbb{E}\left[g_{t+1}^{-\gamma}\right]$$

For small fluctuations of g around 1, use the first -order Taylor expansion:

equation 10

$$g^{-\gamma} \approx 1 - \gamma(g - 1)$$

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Apply this inside the expectation and plug into previous:

equation 11

$$1 = \beta R(1 - \gamma \mathbb{E}[g_{t+1} - 1])$$

Now we will solve for the expected consumption growth:

equation 12

$$\mathbb{E}[g_{t+1} - 1] \approx \frac{1 - (\beta R)^{-1}}{\gamma}$$

Equivalently, if we let $\Delta c_{t+1} \equiv c_{t+1} - c_t$ linearize and divide by c_t

equation 13

$$\mathbb{E}\left[\frac{\Delta c_{t+1}}{c_t}\right] \approx \frac{1 - (\beta R)^{-1}}{\gamma}$$

- If $\beta R = 1$ (impatient agent exactly offset by return), the right-hand side is zero and expected consumption growth is zero at first order (log consumption is martingale in that case).
- If $\beta R > 1$, expected consumption tends to grow on average; if $\beta R < 1$, expected consumption tends to decline (absent other forces).

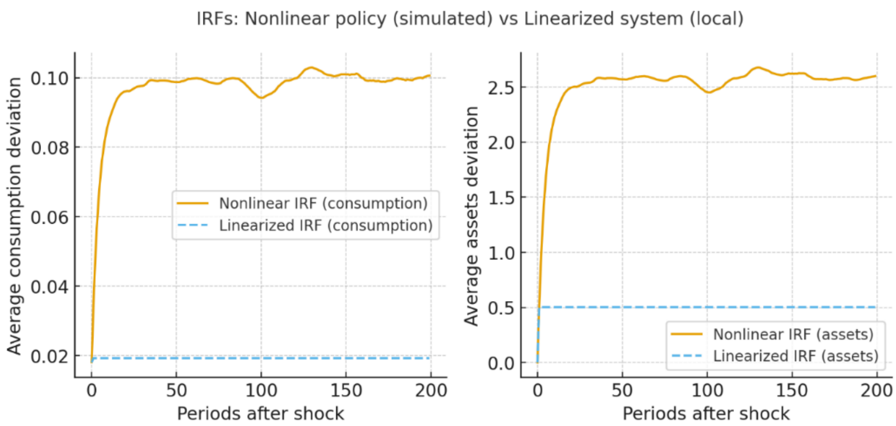

Figure 2 Non-Linear policy vs Linearized system

Source: Authors' calculations

Canonical HANK model

In this part of the paper, we will introduce canonical HANK model. We will now embed the standard incomplete markets consumption-saving behavior into a New-Keynesian model. In this model also we allow government represented through bonds, taxes, and government spending. Our reference here is Auclert et al. (2023a). We set up the model assuming perfect foresight with respect to aggregate variables "MIT shocks". Short note on MIT shocks says: Following, Boppart et al. (2018)," a simple linearization method for analyzing frameworks with consumer heterogeneity and aggregate shocks" has been applied to standard RBC model with neutral technology shocks as in Kydland,Prescott (1982),and investment specific as in investment-specific, as in Greenwood et al. (2000). In definition given by Boppart et al. (2018) "MIT shock" is defined as:

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"*An "MIT shock" is an unexpected shock that hits an economy at its steady state, leading to a transition path back towards the economy's steady state……*".

Mukoyama (2021) follows Boppart et al. (2018) definition:"…. the probability of the shock is considered zero, and no prior (contingent) arrangement is possible for the occurrence of the MIT shock"…..The dynamic analysis that was using exogenous shocks or policy changes has been used in the literature with the earlier examples including: Abel,Blanchard (1983), Auerbach, Kotlikoff (1983), and Judd (1985).And more recent examples being: Boppart et al. (2018), Kaplan et al. (2018), Boar ,Midrigan (2020), Guerrieri et al. (2020).Now, Transitory dynamics and MIT shocks (an implicit-uncertainty economy): short note will follow.In the Aiyagari version of the model[9]:

equation 14

$$r(t) = F_k(K(t),1)) - \delta$$

$$w(t) = F_l(K(t),1))$$

$$K(t) = \int a g_1(a,t)da + \int a g_2(a,t)da$$

HJB equation is given as:

equation 15

$$\rho v_j(a,t) = \max_c u(c) + \partial_a v_j(a,t)\big(w(t)z_j + r(t)a - c\big) + \lambda_j\left(v_{-j}(a,t) - v_j(a,t)\right) + \partial_t v_j(a,t)$$

Kolmogorov Forward equation is:

equation 16

$$\partial_t q_j(a,t) = -\partial_a\left[s_j(a,t)g_j(a,t)\right] - \lambda_j g_j(a,t) + \lambda_{-j}(a,t) + \lambda_{-j} g_{-j}(a,t)$$

$$s_j(a,t) = w(t)z_j + r(t)a - c_j(a,t), c_j(a,t) = (u')^{-1}\left(\partial_a v_j(a,t)\right)$$

In previous expression a represents the borrowing limit, $g_{j,0}(a)$ represents the initial condition. Now, recall discretized equations for stationary equilibrium:

equation 17

$$\rho(v) = u(v) + A(v)v$$

$$0 = A(v)^{\mathrm{T}} g$$

Transition dynamics is given as:

- First denote $v_{i,j}^n = v_j(a_i t^n)$ and stack into v^n
- Denote $g_{i,j}^n = g_j(a_i, t^n)$ and stack into g^n

Then following applies:

equation 18

$$\rho v^n = u(v^{n+1}) + A\,(v^{n+1})v^n + \frac{1}{\Delta t}(v^{n+1} - v^n)$$

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9 See lecture notes by Benjamin Moll: https://benjaminmoll.com/lectures/

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$$\frac{g^{n+1} - g^n}{\Delta t} = A(v)^{\mathrm{T}} g^{n+1}$$

Terminal condition for v is given as: $v^N = v_\infty$ which represents steady state, while initial condition is given as: $g: g^1 = g_0$. In previous HJB or Hamilton-Jacobi-Bellman equation is modeled as in Achdou et al.(2022). The deterministic optimal control problem is given as:

equation 19

$$V(x_0) = \max_{u(t)_{t=0}^{\infty}} \int_0^\infty e^{-\rho t} h(x(t), u(t) dt \text{ s.t. } \dot{x}(t) = g\big(x(t)\big), u(t), u(t) \in U \,; t \geq 0, x(0) = x_0$$

In previous expression:$\rho \geq 0$ is the discount rate, $x \in X \subseteq \mathbb{R}^m$ is a state vector; $u \in U \subseteq \mathbb{R}^n$ is a control vector, and $h: X \times U \to R$. The value function of the generic optimal control problem satisfies the Hamilton-Jacobi-Bellman equation, i.e.:

equation 20

$$\rho V(x) = \max_{u \in U} h(x, u) + V'(x) \cdot g(x, u)$$

In the case with more than one state variable $m > 1, V'(x) \in \mathbb{R}^m$ is the gradient of the value function. Now for the derivation of the discrete-time Bellman eq. we have: time periods of length Δ ,discount factor $\beta(\Delta) = e^{-\rho\Delta}$, here we can note that $\lim_{\Delta\to\infty} \beta(\Delta) = 0$ and $\lim_{\Delta\to 0} \beta(\Delta) = 1$.Now that discrete Bellman equation is given as:

equation 21

$$V(k_t) = \max_{c_t} \Delta U(c_t) + e^{-\rho\Delta} V(k_{t+\Delta}) \text{ s.t. } k_{t+\Delta} = \Delta[F(k_t) - \delta k_t - c_t] + k_t$$

For a small $\Delta = 0$ we can make: $e^{-\rho\Delta} = 1 - \rho\Delta$, so that $V(k_t) = \max_{c_t} \Delta U(c_t) + (1 - \rho\Delta,) V(k_{t+\Delta})$,if we subtract $(1 - \rho\Delta,) V(k_t)$ from both sides and divide by Δ and manipulate the last term we get : $\rho V(k_t) = \max_{c_t} \Delta U(c_t) + (1 - \rho\Delta,)[V(k_{t+\Delta}) - V(k_t)\,]$ we get :

equation 22

$$\rho V(k_t) = \max_{c_t} \Delta U(c_t) + (1 - \rho\Delta,) \frac{[V(k_{t+\Delta}) - V(k_t)\,]}{k_{t+\Delta} + k_t} \frac{k_{t+\Delta} - k_t}{\Delta}$$

If $\Delta \to 0$ then $\rho V(k_t) = \max_{c_t} \Delta U(c_t) + V'(k_t)\ \dot{k}_t$. Hamilton-Jacobi-Bellman equation in stochastic settings is given as:

equation 23

$$V(x_0)) = \max_{u(t)_{t=0}^{\infty}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} h\big(x(t), u(t)\big) dt \text{ s.t.} dx(t) = g\big(x(t), u(t)\big) dt + \sigma\big(x(t)\big) dW(t)\,, u(t) \in U \,; t \geq 0, x(0) = x_0$$

In previous expression $x \in \mathbb{R}^m; u \in \mathbb{R}^n$. HJB equation without derivation is :

equation 24

$$\rho V(x) = \max_{u \in U} h(x, u) + V'(x) g(x, u) + \frac{1}{2} V''(x) \sigma^2(x)$$

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In the multivariate case: for fixed x we define $m \times m$ covariance matrix, $\sigma^2(x) = \sigma(x)\sigma(x)'$ which is a function of $\sigma^2: \mathbb{R}^m \to \mathbb{R}^m \times \mathbb{R}^m$. HJB equation now is given as:

equation 25

$$\rho V(x) = \max_{u \in U} h(x,u) + \sum_{i=1}^{m} \frac{\partial V(x)}{\partial x_i} g_i(x,u) + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial^2 V(x)}{\partial x_i \partial x_j} \sigma_{ij}^2(x)$$

In vector notation previous is given as:

equation 26

$$\rho V(x) = \max_{u \in U} h(x,u) + \nabla_x V(x) \cdot g(x,u) + \frac{1}{2} tr\left(\Delta_x V(x) \sigma^2(x)\right)$$

Where $\nabla_x V(x)$: gradient of V (dimension $m \times 1$) ; $\Delta_x V(x)$: Hessian matrix of V (dimension $m \times m$).By Ito's lemma[10]:

equation 27

$$df(x) = \left(\sum_{i=1}^{n} \mu_i(x) \frac{\partial f(x)}{\partial x_i} + \frac{1}{2} \sum_{i=1}^{m} \sum_{i=1}^{m} \sigma_{ij}^2(x) \frac{\partial^2 f(x)}{\partial x_i \partial x_j} \right) dt + \sum_{i=1}^{m} \sigma_I(x) \frac{\partial f(x)}{\partial x_i} dW_i$$

In vector notation:

equation 28

$$df(x) = \left(\nabla_x f(x) \cdot \mu(x) + \frac{1}{2} tr\left(\Delta_x f(x) \sigma^2(x)\right) \right) dt + \nabla_x f(x) \cdot \sigma(x) dW$$

Now for the Kolmogorov Forward (Fokker-Planck[11]) equation we have folowing: let x be a scalar diffusion

equation 29

$$dx = \mu(x)dt + \sigma(x)dW, x(0) = x_0$$

Let's suppose that we are interested in the evolution of the distribution of $x, f(x,t)$ and $\lim_{t \to \infty} f(x,t)$ .So, given an initial distribution $f(x,0) = f_0(x), f(x,t)$ satisfies PDE :

equation 30

$$\frac{\partial f(x,t)}{\partial t} = -\frac{\partial}{\partial x}[\mu(x) f(x,t)] + \frac{1}{2} \frac{\partial^2}{\partial x^2}[\sigma^2(x) f(x,t)]$$

Previous PDE is called “Kolmogorov Forward Equation” or “Fokker-Planck Equation”.For this part and below see also Josheski et al. (2023a).

Corollary 1: if a stationary equilibrium exists $\lim_{t \to \infty} f(x,t) = f(x)$, it satisfies ODE

[10] Itô's lemma is an identity used in Itô calculus to find the differential of a time-dependent function of a stochastic process. It serves as the stochastic calculus counterpart of the chain rule, see Kiyosi Itô (1951).

[11] See Fokker (1914), Planck (1917), Kolmogorov (1931).

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equation 31

$$0 - \frac{d}{dx}[\mu(x)f(x)] + \frac{1}{2}\frac{d^2}{dx^2}[\sigma^2(x)f(x)]$$

In the multivariate case Kolmogorov Forward Equation is given as:

equation 32

$$\frac{\partial f(x,t)}{\partial t} = -\sum_{i=1}^{m}\frac{\partial}{\partial x_i}[\mu(x)f(x,t)] + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{\partial^2}{\partial x^2}[\sigma_{ij}^2(x)f(x,t)]$$

About Euler equation here following lemma applies see Achdou et al.(2022):

Lemma 1

The consumption and savings policy functions $c_j(a)$ and $s_j(a)$ for $j = 1,2..$ corresponding to HJB equation : $\rho v_j(a) = \max_c u(c) + v_j'(a)(y_j + ra - c) + \lambda_j\left(v_{-j}(a) - v_j(a)\right)$ which is maximized at : $0 = -\frac{d}{da}[s_j(a)g_J(a)] - \lambda_j g_j(a) + \lambda_{-jg_{-j}}(a)$ is given as:

equation 33

$$(\rho - r)u'\left(c_j(a)\right) = u''\left(c_J(a)\right)c_j'(a)s_j(a) + \lambda_j(u'\left(c_{-j}(a)\right) - u'\left(c_j(a)\right)$$
$$s_J(a) = y_J + ra - c_j(a)$$

Proof: differentiate $\rho v_j(a) = \max_c u(c) + v_j'(a)(y_j + ra - c) + \lambda_j\left(v_{-j}(a) - v_j(a)\right)$ with respect to a and use that $v_j'(a) = u'\left(c_j(a)\right)$ and hence $v_j''(a) = u''\left(c_j(a)\right)c_j'(a)$ ■

The differential equation $\begin{matrix}(\rho - r)u'\left(c_j(a)\right) = u''\left(c_J(a)\right)c_j'(a)s_j(a) + \lambda_j(u'\left(c_{-j}(a)\right) - u'\left(c_j(a)\right) \\ s_J(a) = y_J + ra - c_j(a)\end{matrix}$ is and Euler equation , the right hand side $(\rho - r)u'\left(c_j(a)\right)$ is expected change of marginal utility of consumption $\frac{\mathbb{E}_t[du\prime(c_j(a_t)]}{dt}$.This uses Ito's formula to Poisson process:

equation 34

$$\mathbb{E}_t[du'(c_j(a_t)] = \left[u''(c_j(a_t)c_j'(a_t)s_j(a_t) + \lambda_j\left(u'(c_{-j}(a_t)\right) - u'\left(c_j(a_t)\right)\right]dt$$

So, this equation $\begin{matrix}(\rho - r)u'\left(c_j(a)\right) = u''\left(c_J(a)\right)c_j'(a)s_j(a) + \lambda_j(u'\left(c_{-j}(a)\right) - u'\left(c_j(a)\right) \\ s_J(a) = y_J + ra - c_j(a)\end{matrix}$ can be written in more standard form:

equation 35

$$\frac{\mathbb{E}_t[du'(c_j(a_t)]}{dt} = (\rho - r)dt$$

Now we will plot in MATLAB Transition after unexpected decrease in aggregate productivity ("MIT shock")

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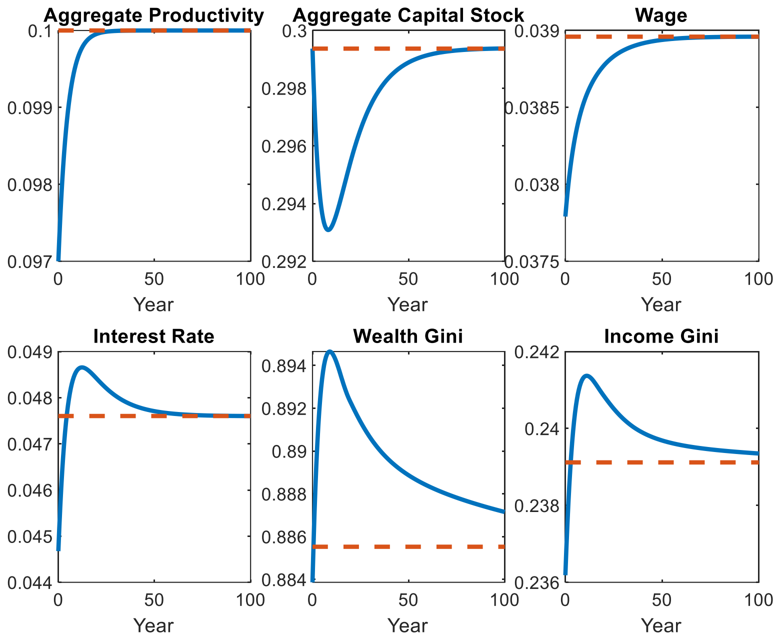

Figure 3 Transition after unexpected decrease in aggregate productivity ("MIT shock")

Source: Author’s calculations based on codes available at: https://benjaminmoll.com/codes/

Now back to our canonical HANK model[12]

Household i solves:

inequality 1

$$\max_{c_{it}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big(u(c_{it}) - v(n_{it})\big)$$
$$c_{it} + a_{it} \leq (1 + r_t)a_{it-1} + z_{it}$$
$$a_{it} \geq \underline{a}$$

This is sequence analogue to Bellman equation before in GE (general equilibrium) real after-tax income is given as:

equation 36

$$z_{it} = (1 - \tau_t)y_{it}\,;\, y_{it} = \frac{W_t}{P_t} e_{it} n_{it}$$

In previous e_{it} represents idiosyncratic productivity, so one could write $e_{it} = e(s_{it})$ this can capture progressive taxation as in Heathcote et al. (2017). In this canonical model working mechanism are sticky wages not prices. Standard NK model assumes flexible wages, but this has two big issues with HA -Heterogenous agents namely:

[12] We should always keep in mind certainty equivalence: linearizing with respect to small MIT shocks = impulse responses in stochastic model

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1. High MPCs (marginal propensities to consume) have high MPEs (marginal propensities to earning) [13]
2. profits are (almost always) countercyclical with respect to demand shocks[14].Micro found sticky wages by extending Erceg et al. (2000) means:

- every worker belongs to some union setting wages on its behalf
- agent works some fraction of total labor N_t
- all agents work same hours $n_{it} = N_t$

wage Phillips curve here is given as:

equation 37

$$\pi_t^w = \Bbbk \left(v'(N_t) - \frac{\epsilon - 1}{\epsilon}(1-\tau_t)\frac{W_t}{P_t}u'(C_t) \right) + \beta \pi_{t+1}^w$$

Where $\left(v'(N_t) - \frac{\epsilon - 1}{\epsilon}(1-\tau_t)\frac{W_t}{P_t}u'(C_t) \right)$ represents wedge in labor FOC for and "average" agent. Representative firm is with aggregate production function linear in labor:

equation 38

$$Y_t = X_t N_t$$

Where X_t is TFP. Here we assume flexible prices:

equation 39

$$P_t = \frac{W_t}{X_t} \Leftrightarrow \frac{W_t}{P_t} = X_t$$

Here goods price inflation = wage inflation minus TFP growth:

equation 40

$$1 + \pi_t = (1 + \pi_t^w)\frac{X_t}{X_{t+1}}$$

Total government revenue is:

equation 41

$$T_t = \tau_t Y_t$$

B_t are government bonds, G_t is government revenue, subject to government budget constraint:

equation 42

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t$$

[13] In data MPEs are very small. Also," Average MPEs are small in the data, around 0 to 0.04 annually "see Auclert et al. (2023b).This paper uses CI complementarity index that measures the strength of the complementarity between consumption and labor supply. When CI is positive, as households work more, they also want to consume more, and this additional consumption increases demand for output even further prompting another increase in labor effort, and so on. To cleanly isolate the importance of CI for the fiscal multiplier, we start by considering the representative-agent case. There, we show that the multiplier, if monetary policy maintains a constant real interest rate, is given by: $\frac{dY}{dG_s} = \frac{1}{1-(1-\tau)CI} \cdot \mathbf{1}_{s=t}$

[14] Depending on distribution of profits, can lead to e.g. aggregate instability, < 0 fiscal multipliers, contractions after monetary easing, backdoor "procyclical income risk", see Bilbiie, F. O. (2008)., McKay et al. (2016), Broer et al. (2020).

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Now we need to make sure that :

equation 43

$$B_{-1} = \sum_{t \geq 0} \left(\frac{1}{1+r}\right)^t (T_t - G_t)$$

Total after tax income is:

equation 44

$$Z_t \equiv Y_t - T_t = (1 - \tau_t) Y_t$$

For an individual after tax income we have:

equation 45

$$y_{it} = e_{it} Y_t \Rightarrow z_{it} = e_{it} Z_t$$

z_t is simply a share from Z_t total after tax income. Monetary authority follows an interest rate rule. Exactly, two interest rate rules:

1. Taylor rule:

equation 46

$$i_t = t + \phi_\pi \pi_t + \epsilon_t$$

2. Real rate rule

equation 47

$$r_t = r + \epsilon_t; \Leftrightarrow i_t = r + \pi_{t+1} + \epsilon_t$$

All agents optimize and markets clear:

equation 48

$$G_t + C_t = Y_t$$
$$A_t = B_t$$

Where households' aggregates are given as:

equation 49

$$C_t = \int c_t(a_-, e) dD_t(a_-, e); \; A_t = \int a_t(a_-, e) dD_t(a_-, e)$$

Next, we will plot Aiyagari/HANK household block with two idiosyncratic income types (low/high) and borrowing constraint. CRRA utility.Aiyagari (1994) model is given as:

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equation 50

$$\rho v_1(a) = \max_c u(c) + v_1'(a)(wz_1 + ra - c) + \lambda_1(v_2(a) - v_1(a))$$
$$\rho v_2(a) = \max_c u(c) + v_2'(a)(wz_2 + ra - c) + \lambda_1(v_1(a) - v_2(a))$$
$$0 = -\frac{d}{da}[s_1(a)g_1(a)] - \lambda_1 g_1(a) + \lambda_2 g_2(a)$$
$$0 = -\frac{d}{da}[s_2(a)g_2(a)] - \lambda_2 g_2(a) + \lambda_1 g_1(a)$$
$$1 = \int_{\underline{a}}^{\infty} g_1(a)da + \int_{\underline{a}}^{\infty} g_2(a)da$$
$$K = \int_{\underline{a}}^{\infty} a g_1(a)da + \int_{\underline{a}}^{\infty} a g_2(a)da$$
$$r = aK^{a-1} - \delta; w = (1-a)K^a$$

This is following Achdou et al.(2022) also written in Josheski et al.(2023).

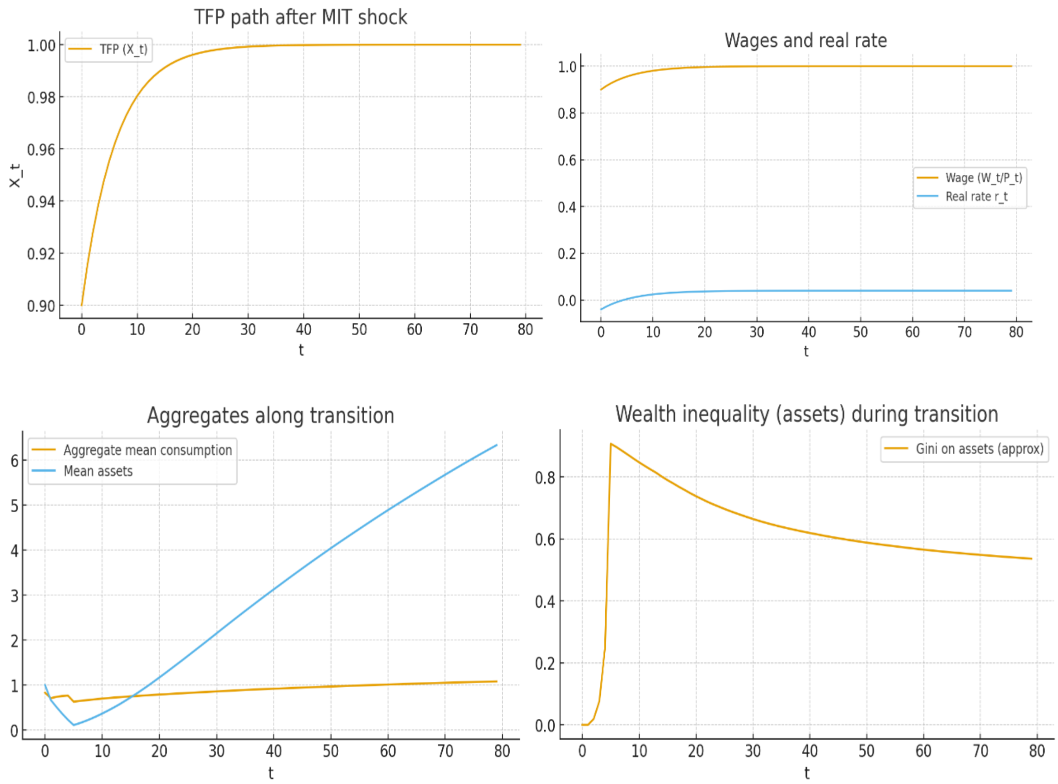

Figure 4 Aiyagari/HANK household block with two idiosyncratic income types (low/high) and borrowing constraint. CRRA utility.

Mathematical derivation of GE and PE in previous canonical HANK model

Household i chooses c_{it}, a_{it} at time t :

equation 51

$$\max_{\{c_{it}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big(u(c_{it}) - v(n_{it})\big)$$

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s.t.

inequality 2

$$a_{it} = (1 + r_t)a_{i,t-1} + z_{it} - c_{it}, a_{it} \geq \underline{a}$$

where idiosyncratic after-tax labor income $z_{it} = e_{it}w_t$ (we normalize hours or take wage income multiplicative with idiosyncratic productivity e_{it}). Utility is CRRA[15]. The coefficient of absolute risk aversion :

equation 52

$$R(c) \coloneqq -\frac{u''(c)}{u'(c)}$$

Remains finite as $a \to \underline{a}$:

inequality 3

$$-\frac{u''(y_1 + r\underline{a})}{u'(1 + r\underline{a})} < \infty$$

Borrowing constraint matters and CRRA satisfies: $-\frac{u''(0)}{u'(0)} = \infty$.Hence for standard utility functions, borrowing constraint matters $\underline{a} > -y_1$, constraint matters if it is tighter than "natural borrowing constraint", but weaker if $u'(c) = e^{-\theta c}$,constraint matters if $\underline{a} = -\frac{y_1}{r}$.

Proposition 1 Assume $r < \rho$, $y_1 < y_2$, *now assume that* $-\frac{u''(y_1+r\underline{a})}{u'(1+r\underline{a})} < \infty$ that is assumption 1 A1 holds. Then savings and consumption policy functions close to $\underline{a} = a$ satisfy:

equation 53

$$s_1(a) \sim -\sqrt{2v_1}\sqrt{a - \underline{a}}$$
$$c_1(a) \sim y_1 + ra + \sqrt{2v_1}\sqrt{a - \underline{a}}$$
$$c_1'(a) \sim r + \frac{1}{2}\sqrt{\frac{v_1}{2(a - \underline{a})}}$$

Where v_1 is a constant that depends on $r, \rho, \lambda_1, \lambda_2$ [16]

Corollary 2

The wealth of workers who keeps y_1 converges to borrowing constraint in finite time at speed governed by v_1:

$$a(t) - \underline{a} \sim \frac{v_1}{2}(T - t)^2$$

Where $T \coloneqq hitting\ time$ is:

[15] $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$

[16] "$f\ (a)\ \sim\ g(a)$" means $\lim_{a \to \underline{a}} f\ (a)/g(a)\ =\ 1$, "f behaves like g close to a"

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equation 54

$$T = \sqrt{\frac{2(a_0 - \underline{a})}{v_1}}; 0 \le t \le T$$

Proof: Suppose wealth follows deterministic drift with constant velocity v_1

equation 55

$$\dot{a}(t) = -v_1(T - t)$$

So that speed increase as $t \to T$. Now lets integrate dynamics from $t \to T$:

equation 56

$$a(T) - a(t) = \int_t^T \dot{a}\,(s)ds = -v_1 \int_t^T (T - s)ds$$

We would change variable $u = T - s$ so:

equation 57

$$a(T) - a(t) = -v_1 \int_0^{T-t} udu = -v_1 \frac{(T - t)^2}{2}$$

Thus:

equation 58

$$a(t) - a(T) = \frac{v_1}{2}(T - t)^2$$

Next, we are imposing boundary condition: $a(T) = \underline{a}$ so $a(t) - \underline{a} = \frac{v_1}{2}(T - t)^2$, this matches the claimed law of motion. Now we solve for for hitting time. At $t = 0, a(0) = a_0$,we plug that into formula:

equation 59

$$a_0 - \underline{a} = \frac{v_1}{2}T^2$$

If we rearrange :

equation 60

$$T = \sqrt{\frac{2(a_0 - \underline{a})}{v_1}}$$

Definition 1

The MPC over period of time τ is given as:

equation 61

$$MPC_{j,\tau}(a) = C'_{j,\tau}(a)$$

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Where :

equation 62

$$C_{j,\tau}(a) = \mathbb{E}\left[\int_0^\tau c_j(a_t)dt | a_0 = a, y_0 = y_j\right]$$

Lemma 2

If τ is sufficiently small so that no income switches, then:

equation 63

$$MPC_{(1,\tau)}(a) \sim \min\{\tau c_1'(a), 1 + \tau r\}$$

$MPC_{(1,\tau)}(a)$ is bounded above even though $c_1'(a) \to \infty$ as $a \downarrow \underline{a}$. ut straightforward computation using Feynman-Kac formula

Feynman-Kac formula- Suppose $\exists \mathcal{P}(t,x)$ that satisfies :$\frac{\partial \mathcal{P}}{\partial t} + f(t,x)\frac{\partial \mathcal{P}}{\partial x} + \frac{1}{2}\rho^2(t,x)\frac{\partial^2 \mathcal{P}}{\partial x^2} - R(x)\mathcal{P} + h(t,x) = 0$ s.t $\mathcal{P}(t,x) = \psi(x)$. Then $\exists \widetilde{W}(t)$ and a measure Q where solution is given as $\mathcal{P}(t,x) = E_Q[\int_t^T \mathcal{V}(t,u)h(u,x(u))du + \mathcal{V}(t,T)\psi(x(t)) | \mathcal{F}_t]; t < T$ $dx(t) = f(t,x(t))dt + \rho(t,x(t))d\widetilde{W}(t)$; $\mathcal{V}(t,u) = \exp\,(-\int_t^u R(x(s)ds)$ given that $\int_t^T E_Q\left[\left(\rho(s,x(s))\frac{\partial \mathcal{P}}{\partial x}(s,x(s))\right]^2 \middle| \mathcal{F}_t\right]$ .In previous expression $\mathcal{F}_t$ is a σalgebra[17]. So, by Feynman-Kac formula $C \coloneqq C_{1,\tau}$,satisfies backward PDE:

equation 64

$$\partial_\tau C(a,\tau) = c_1(a) + \mathcal{L}C(a,\tau), C(a,0) = 0$$

Where the generator FOC here is $\mathcal{L}f(a) = \mu(a)f'(a)$.Wi differentiate this PDE with respect to a,and put $D(a,\tau) \coloneqq \partial_a C(a,\tau) = MPC_{1,\tau}(a)$.

equation 65

$$\partial_\tau D(a,\tau) = c_a' + \mu'(a)\partial_a D(a,\tau), D(a,0) = 0$$

Solve this linear first order PDE by characteristics, along deterministic path a_s, started $a_0 = a$ so $\dot{a}_s = \mu(a_s)$.

equation 66

$$D(a,\tau) = \mathbb{E}\left[\int_0^\tau c_1'(a_s)\frac{\partial a_s}{\partial a}ds | a_0 = a\right]$$

equation 67

$$\frac{d}{ds}\partial_a a_s = \mu'(a_s)\partial a a_s; \partial_a a_0 = 1$$

equation 68

$$\partial_a a_s = \exp\left(\int_0^s \mu'(a_u)du\right) = \exp\left(\int_0^s (r - c_1'(a_u))du\right)$$

[17] Let $\mathcal{P}(x)$is a $\mathcal{P}(s)$, then a subset $\sum \subseteq \mathcal{P}(x)$ is σ-algebra if it satisfies: $x \in \sum$, and is considered to be $\cup$, and if $x \in \sum \Rightarrow \overline{x} \in \sum$; and if $x_1, x_2, ... \in \sum$ then $x = x_1 \cup x_2$ ....see Rudin (1987).

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The Feynman–Kac step gave the exact identity:

$$MPC_{1,\tau}(a) = \mathbb{E}\left[\int_0^\tau c_1'(a_s) \exp\left(\int_0^s (r - c_1'(a_u))du\right) ds\right]$$

From this identity the $\tau c_1'(a)$ estimate follows by local continuity and the $1 + r\tau$ bound follows from the budget/perturbation argument (or by noting $\int_0^s (r - c') \leq e^{rs}$ and the resource constraint bound).Next we will plot previous on four plots.

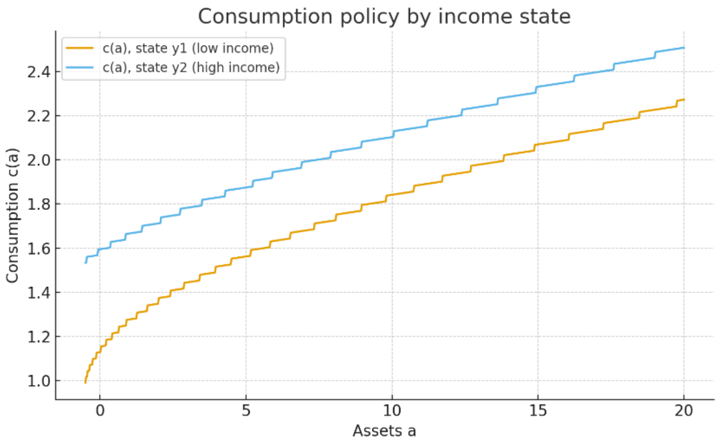

Figure 5 consumption policy by income state

Source: Author’s own calculation

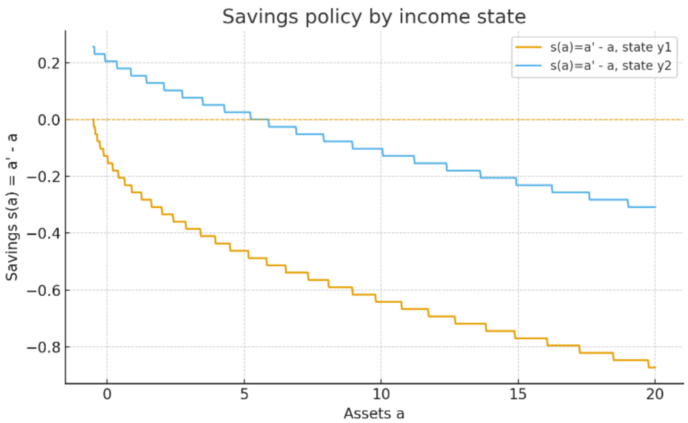

Figure 6 savings policy by income states

Source: Author’s own calculation

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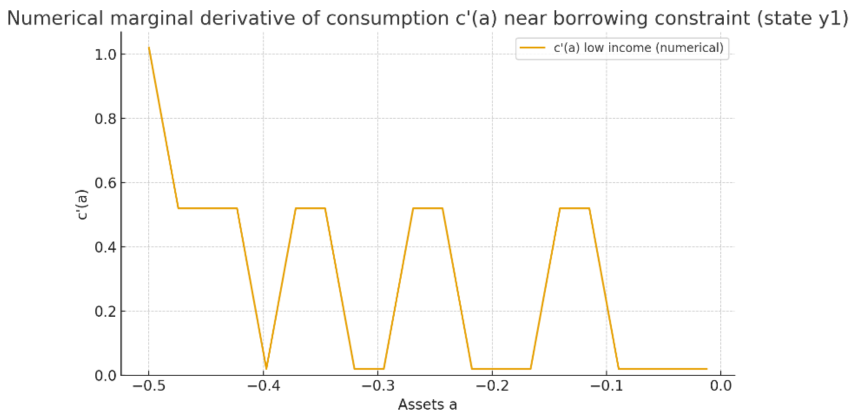

Figure 7 Marginal derivative of consumption near borrowing constraint

Source: Author’s own calculation

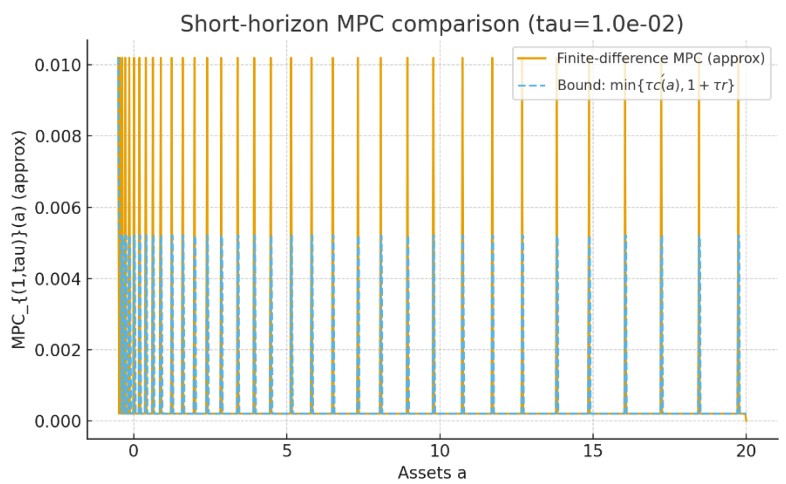

Figure 8 Short MPC comparison

Source: Author’s own calculation

From the Bellman equation finite horizon backward recursion used numerically:

equation 69

$$V_t(a,e) = \max_{a'}\{u\big((1+rt)a + ew_t - a'\big) + \beta\mathbb{E}[V_{t+1}(a',e')]\}$$

- In PE partial equilibrium PE households take the entire path of real rates r_t as given, and do not internalize that their aggregate asset choices change the interest rate. Here, we set $r_t = r_{ss}$ interest rate equals steady-state interest rate. Wages follow exogenous TFP $X_t : w_t = X_t$ .Households solve the Bellman recursion with these exogenous sequences. Then we simulate agents forward to compute aggregates A_t^{PE}, C_t^{PE} Gini and etc.
- GE requires market clearing. With production function $Y_t = X_t \cdot K_t^{\alpha}$ and labor normalized to 1, FOCs give:

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equation 70

$$r_t = \alpha X_t K_t^{\alpha-1} - \delta; w_t = (1-\alpha) X_t K_t^{\alpha}$$

- In GE the path $\{K_t\}$ aggregate assets determine $\{r_t, w_t\}$ households' optimal policies produce a distribution whose mean must equal K_t. So we search for a fixed point of this mapping.
- In the numerical method: Parameterization: two idiosyncratic income states (low/high) with a small Markov chain, CRRA utility, standard asset grid, borrowing constraint $\underline{a} = 0$.
- Terminal value function: compute steady-state value function via value-function iteration (VFI) at constant r_{ss}, w_{ss} use that as terminal condition for the finite-horizon backward recursion.
- In PE experiment:$r_{ss} = r_t \; w_t = X_t$ (TFP path with a one-off MIT shock; TFP mean reverts). Solve finite-horizon Bellman backwards to get time-varying policy functions, then forward-simulate a large population (Monte Carlo) to obtain aggregates:C_t, A_t (mean assets), and approximate Gini on assets.
- GE experiment (time-iteration): start with initial guess for K_t here are used mean assets from PE,then:

1. Compute $r_t(K_t), w_t(K_t)$ from FOC
2. solve households via backward recursion given these sequences,
3. simulate forward to get a new average asset path $\widetilde{K}_t$
4. update $K_t \leftarrow \lambda \widetilde{K}_t + (1-\lambda) K_t$

- The household solution is grid-search over a' (robust but slow). For speed/accuracy improvements one can use EGM or policy-function interpolation.
- The PE experiment completed fully: policy functions, forward simulation, and aggregates were computed.

- The GE experiment ran time-iteration. Early iterations made large corrections, then the iteration sequence improved for a few steps (convergence metric dropped), but after several iterations the iteration began to stall/oscillate. The run then hit an automatic interruption (long run) while trying to continue iterating with the grid-search solver — the slow household solver made later GE iterations time-consuming, and the process timed out in the execution environment.

The main equations used in the code are:

equation 71

$$V_t\,(a,e) = \max_{(a' \geq \underline{a})} \left\{ u\big((1+r_t)a + e w_t - a'\big) + \beta \sum_{e'} \Pi(e,e') V_{t+1}(a',e') \right\}$$

Budget is:

equation 72

$$c_t (1+r_t) a_t + e_t w_t - a_{t+1}, c_t \geq 0$$

Production used to close GE:

equation 73

$$Y_t = X_t K_t^{\alpha-1}, K_t = \int a_t d\mu_t(a,e)$$
$$r_t = \alpha X_t K_t^{\alpha-1} - \delta, w_t = (1-\alpha) X_t K_t^{\alpha}$$

GE fixed point time iteration, find Ks.t.:

equation 74

$$K_t = \mathcal{S}\big(r(\cdot;K), w(\cdot;K)\big)_t$$

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where $\mathcal{S}$ maps a path of prices into mean assets obtained by solving households and simulating.Next, we will show graphical results.

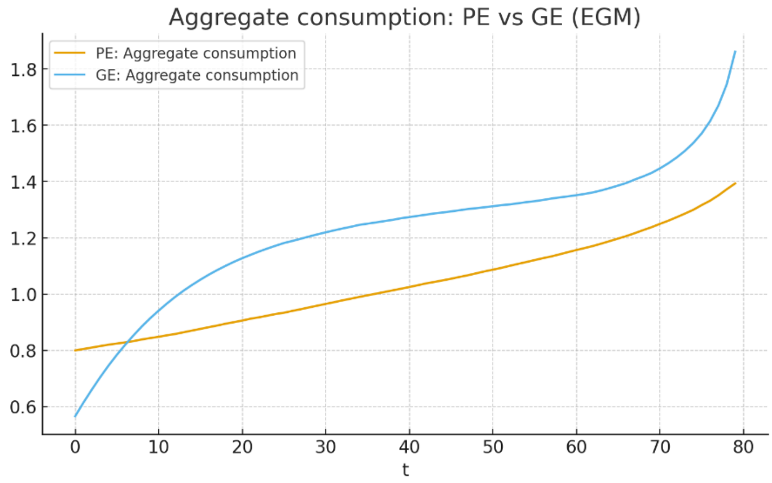

Figure 9 Aggregate consumption PE vs GE (EGM- endogenous grid method)

Source: Author’s own calculation

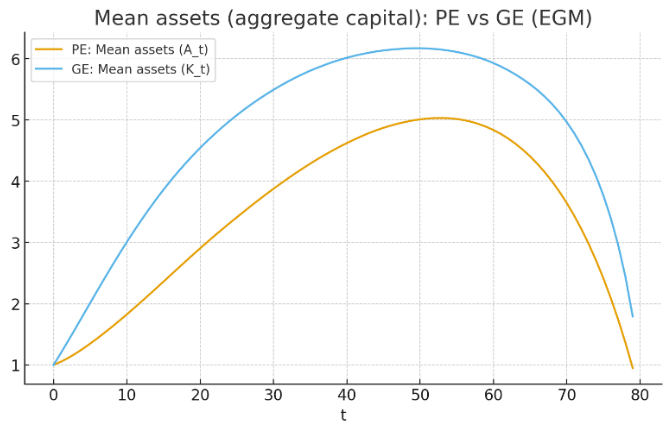

Figure 10 mean assets PE vs GE

Source: Author’s own calculation

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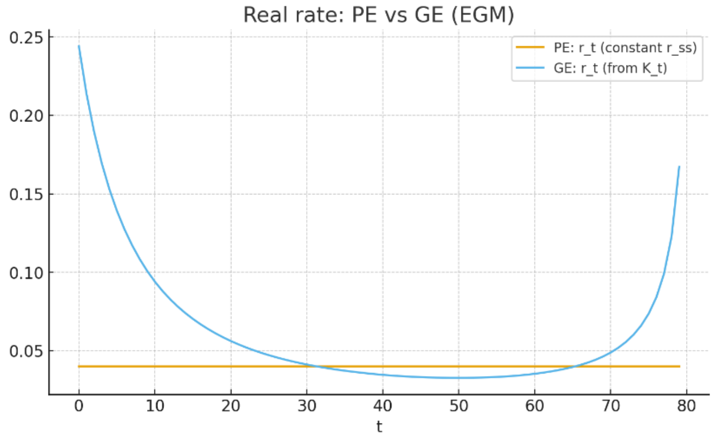

Figure 11 Real rate PE vs GE

Source: Author’s own calculation

Figure 12 Wealth inequality PE vs GE

Source: Author’s own calculation

EGM (Endogenous grid method)

EGM is a numerical method for implementing policy iteration invented by Carroll (2006). So, this is infinite horizon with CRRA utility:

equation 75

$$\max_{\{c_t, a_{9t}+1\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{ty}^{1-\gamma}}{1-\gamma}$$

subject to budget and borrowing limit:

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equation 76

$$a_{t+1} = (1+r)a_t + y_t - c_t; a_{t+1} \geq \underline{a}$$

Income y_t follows a Markov chain with states $s \in \mathcal{S}$ and transition matrix Π .Bellman equation for state s is:

equation 77

$$V(a,s) = \max_{a'>\underline{a}} u(c) + \beta \sum_{s'} \Pi_{s,s'} V(a',s')$$

With:

equation 78

$$c = (1+r)a + y_s - a'$$

FOC for Euler equation is:

equation 79

$$u'(c) = \beta(1+r)\mathbb{E}[u'(c')|s]$$

For CRRA $u(c) = c^{-\gamma}$:

equation 80

$$c^{-\gamma} = \beta(1+r)\mathbb{E}(c'^{-\gamma})$$

EGM idea (Carroll 2006): instead of iterating on value/policy on a fixed grid of current assets a, construct an endogenous grid of current assets consistent with candidate next-period policy using the Euler equation inverted. Steps per iteration:

1. Start with a guess of next-period consumption policy $c'(a',s')$ for each income state s'
2. For each current income state s and for a chosen grid of a' (the grid of end-of-period assets), compute expected marginal utility next period:

equation 81

$$\mathcal{M}(a') = \mathbb{E}[c'(a',s')^{-\gamma}|s] = \sum_{s'} \Pi_{s,s''} c'(a',s')^{-\gamma}$$

3. Use Euler inverted to get current consumption associated with that a':

equation 82

$$c(a',s) = [\beta(1+r)\,\mathcal{M}(a')]^{\frac{1}{\gamma}}$$

That $c(a',s)$ is the consumption today that is consistent with choosing a' as tomorrow's assets given the guessed future policy.)

4. Use budget constraint to compute the current (endogenous) beginning-of-period asset a that corresponds to that a' and c:

equation 83

$$a = \frac{a' - y_s + c(a',s)}{1+r}$$

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5. Thus we obtain pairs $(a, c)$for the chosen a' this is the endogenous grid. Because the computed a won't generally coincide with the exogenous grid we want, we interpolate $c$as a function of a. Also enforce the borrowing constraint: for small enough a, the borrowing constraint may bind and consumption is computed from the budget constraint (i.e. $c = (1 + r)a + y_s - \underline{a}$). Update policy $c(\cdot, s)$on the exogenous grid via interpolation from the endogenous pairs; repeat until convergence. This is the core EGM. It avoids root-finding at each grid point and is much faster.

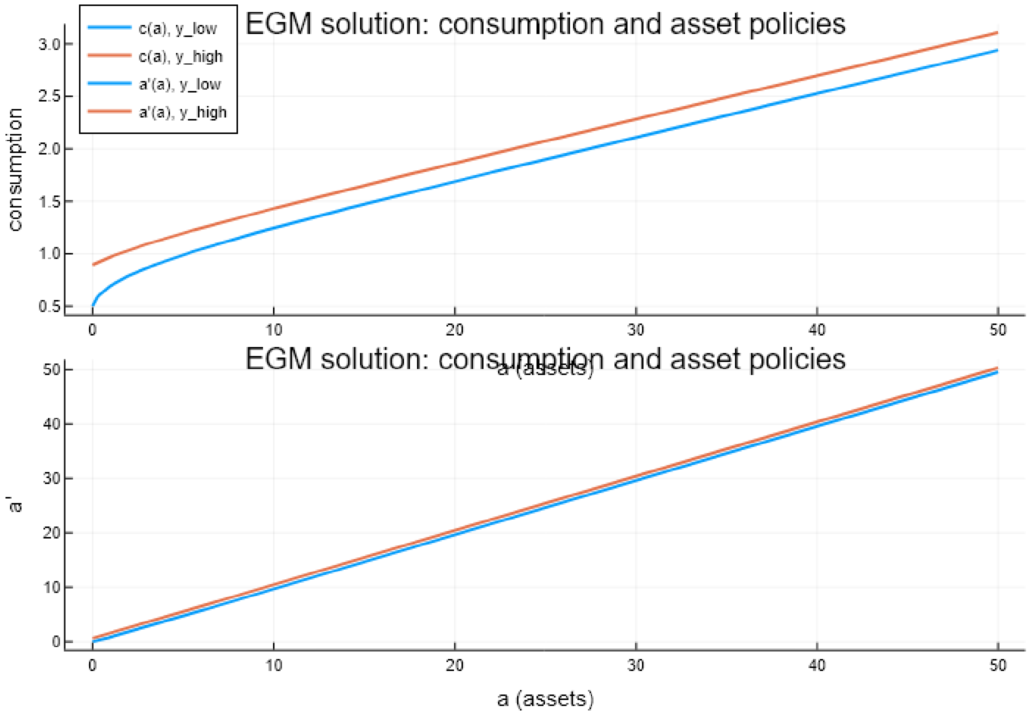

Figure 13 EGM solution: consumption and asset policies

Source: Author’s own calculation

RANK model

This is the standard NK model (with wage rigidities) like in [Woodford, M. (2003)](), [Gali (2008)]().Consumption solves:

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equation 84

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

s.t $C_t + a_t \leq (1+r_t)a_{t-1} + Z_t$, which has solution:

equation 85

$$C_t = \frac{\beta^{\frac{t}{\sigma}} q_t^{-\frac{1}{\sigma}}}{\sum_{s\geq 0} \beta^{\frac{s}{\sigma}} q_s^{1-\frac{1}{\sigma}}} \left[\sum_{s\geq 0} q_s\, Z_s + (1+r_0)a_{-1} \right]$$

Where $q \equiv (1+r_1)^{-1} \dots \dots (1+r_t)^{-1}$,with $r_t = r = \beta^{-1} - 1$ which is just:

equation 86

$$C_t = \frac{r}{1+r} \sum_{s\geq 0} (1+r)^{-s} Z_s + r a_{-1}$$

TANK model

This is same as RA model economy except that a fraction μ is hand-to-mouth (HTM)[18].Only $1-\mu$ behave according to PIH[19]. PIH agents' consumption is determined by:

$$c_t^{PIH} = \frac{\beta^{\frac{t}{\sigma}} q_t^{-\frac{1}{\sigma}}}{\sum_{s\geq 0} \beta^{\frac{s}{\sigma}} q_s^{1-\frac{1}{\sigma}}} \left[\sum_{s\geq 0} q_s\, Z_s + (1+r_0)a_{-1} \right]$$

HTM agents' consumption is determined by:

equation 87

$$C_t^{HTM} = Z_t$$

Jointly pin down aggregate consumption:

equation 88

$$C_t = (1-\mu)c_t^{PIH} + \mu c_t^{HTM}$$

If we assume $\underline{a} = 0$ Euler equations may fail

inequality 4

$$z_{it}^{-\sigma} \geq \beta\,(1+r_{t+1})\mathbb{E}_t[z_{it+1}^{-\sigma}]$$
$$z_t^{-\sigma} \geq \beta(1+r_{t+1})\mathbb{E}\left[\frac{(e')^{-\sigma}}{e^{-\sigma}}\right] z_{t+1}^{-\sigma}$$

RANK-TANK

Now we will derive PE, GE in TANK and RANK model. Notation is as follows:

[18] Hand-to-mouth (HtM) economics refers to a situation where a significant portion of the population has a high marginal propensity to consume, meaning they spend most of their income as they receive it, leading to little or no savings

[19] The Permanent Income Hypothesis (PIH), developed by Milton Friedman (1957), is an economic theory explaining that consumption is driven by a person's expected long-term average income (permanent income), not their current, temporary income.

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x_i-output gap; π_t-inflation, $s_t = \binom{x_t}{\pi_t}$; parameters are: $\sigma.\beta, \mathcal{k}, \phi_p, \phi_x$.Hand to mouth share $\lambda \in [0,1]$ RANK $=\lambda = 0$, and in TANK $\lambda > 0$. In these models there are exogenous monetary shocks: monetary i_t cost push u_t and fiscal transfer g_t.Model core is the same for RANK and TANK:

IS: $x_t = \mathbb{E}[x_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}]$

Philips curve: $\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \mathcal{k}x_t + u_t$

Under perfect foresight deterministic paths, we set:

equation 89

$$\mathbb{E}[x_{t+1}] = x_{t+1}; \mathbb{E}_t[\pi_{t+1}] = \pi_{t+1}$$

We will write this as two equation linear system:

equation 90

$$s_t = [x_t, \pi_t]^{\mathrm{T}}$$

Of the form:

equation 91

$$As_t + B_{s_{t+1}} = d_t$$

With d_t collecting exogenous shock terms (monetary, fiscal, cost-push).

Hand-to-mouth (HtM) behavior

Zeldes (1989), define low-net worth households as potentially "hand-to-mouth", an alternative subsample of interest are higher net worth individuals that have negligible or negative liquid assets, the group Kaplan, Violante and Weidner (2014) refer to as the "wealthy hand-to-mouth,"[20], see Aguiar, Mark A Bils, Mark and Boar, Corina (2020).

1. Poor HtM (classic): agents hold essentially zero liquid assets and consume their current income each period.

equation 92

$$a_t^{liq} = 0, c_t = y_t$$

2. Wealthy HtM: agents hold little liquid assets (so consume out of current income) but have large illiquid wealth (e.g. housing, pension). Economically they respond like HtM to transitory income shocks but not to permanent wealth shocks.

In the infinite horizon model:

equation 93

$$\max_{\{c_t, a_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

s.t. asset evolution:

[20] The "wealthy hand-to-mouth" are households that hold little or no liquid wealth, whether in cash or in checking or savings accounts, despite owning sizable amounts of illiquid assets (assets that carry a transaction cost,such as housing or retirement accounts).

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equation 94

$$a_{t+1} = (1 + r)a_t + y_t - c_t$$

and a borrowing constraint:

$$a_{t+1} \geq \underline{a}; \underline{a} = 0,$$

Utility is CRRA in log case: $\gamma = 1$; $u(c) = \ln(c)$.FOC Euler is:

equation 95

$$u'(c_t) = \beta(1 + r)\mathbb{E}_t[u'(c_{t+1})]$$

If $\beta(1 + r) = 1$ and with no uncertainty and CRRA, consumption is a random walk (permanent-income result). For a transitory shock to income the optimizer barely adjusts consumption — i.e. small marginal propensity to consume (MPC). In a deterministic steady state, consumption equals the annuity of total wealth:

equation 96

$$c_t = (1 - \beta)\left(a_t + \sum_{t=0}^{\infty} \beta^s y_{t+s}\right)$$

Several micro mechanisms produce HtM behaviour:

1. **Borrowing constraint + low assets + income uncertainty (buffer-stock)**
   If agents face income uncertainty and borrowing limits, they target a small buffer of liquid assets. Many agents may optimally hold near zero liquid assets — when they get income, they consume it (or save only a bit). Formally, when the optimal target $\bar{a} \approx 0$ consumption behaves like current income, i.e. HtM.
2. **Illiquid wealth (Wealthy HtM)**
   If an agent has large illiquid wealth k that cannot be used to smooth short-term consumption, but near zero liquid assets, then for transitory income shocks they consume out of liquid income: $c_t \approx y_t$ despite having large net worth.

Now we will introduce simple two-type aggregate model will split the population into fraction λ HtM agent and $1 - \lambda$ Ricardian optimizers.

- Hand-to-mouth agents consume all available (liquid) income each period:

equation 97

$$c_t^H = y_t^H$$
$$c_t^H = \alpha y_t^H$$

Optimizers (R): follow the Euler/PIH result. For a one-period transitory income shock Δy, approximate immediate consumption change:

equation 98

$$\Delta c_t^R \approx (1 - \beta)\Delta y_t^R$$

Aggregate consumption C_t and aggregate income Y_t are population weighted:

equation 99

$$\begin{cases} C_t = \lambda c_t^H + (1 - \lambda)c_t^R \\ Y_t = \lambda y_t^H + (1 - \lambda)y_t^R \end{cases}$$

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A small transitory lump-sum transfer T given top a random agent or each agent equally, per household immediate change in aggregate consumption is:

equation 100

$$\Delta C_t = \lambda \cdot \Delta Y_t + (1-\lambda) \cdot (1-\beta) \cdot \Delta Y_t$$

So aggregate MPC out of a one period transitory income shock is :

equation 101

$$MPC_{agg} = \lambda + (1-\lambda) \cdot (1-\beta)$$

If $\lambda = 0, MPC_{agg} = 1-\beta$ if $\lambda = 1$ $MPC_{agg} = 1$. From the optimizer with borrowing assets:

equation 102

$$c_t = (1-\beta)\left(a_t + \sum_{t=0}^{\infty} \beta^s y_{t+s}\right)$$

If a one-period transitory income shock $\Delta y_t = \varepsilon$ occurs in the future:

equation 103

$$\Delta c_t^R = (1-\beta)\varepsilon$$

Liquid assets a_t and illiquid assets k_t follow:

equation 104

$$a_t = (1+r)a_t + y_t - c_t - s_t$$
$$k_{t+1} = (1+R)k_t + s_t$$

Next, we will build DSGE with TANK base where optimizers are Ricardian i.e. smooth consumption via the Euler equation, sensitive to interest rates, and second HtM agents that consume their current disposable income; immediate 100% MPC out of transfers. Aggregate consumption = weighted average (with $\lambda = 0.5$ in the baseline). The New Keynesian block of equations:

equation 105

$$\pi_t = \beta E_t \pi_{t+1} + k x_t$$
$$i_t = \phi_\pi \pi_t + \phi_x x_t + \varepsilon_t^m$$

Model: small linearized system with

- Optimizers: forward-looking consumption Euler.
- HtM agents: consume current disposable income (transfer enters immediately).
- $C_{agg} = \lambda \cdot C_H + 1 - \lambda \cdot C_R$
- Phillips curve: $\pi_t = \beta E_t \pi_{\{t+1\}} + \kappa x_t$ (x = consumption-based output).
- Taylor rule: $i_t = \varphi_\pi \pi_t + \varphi_x x_t + \varepsilon_t^m$.

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Solution method is deterministic backward recursion (perfect-foresight) on a 40-period horizon (terminal variables set to zero).

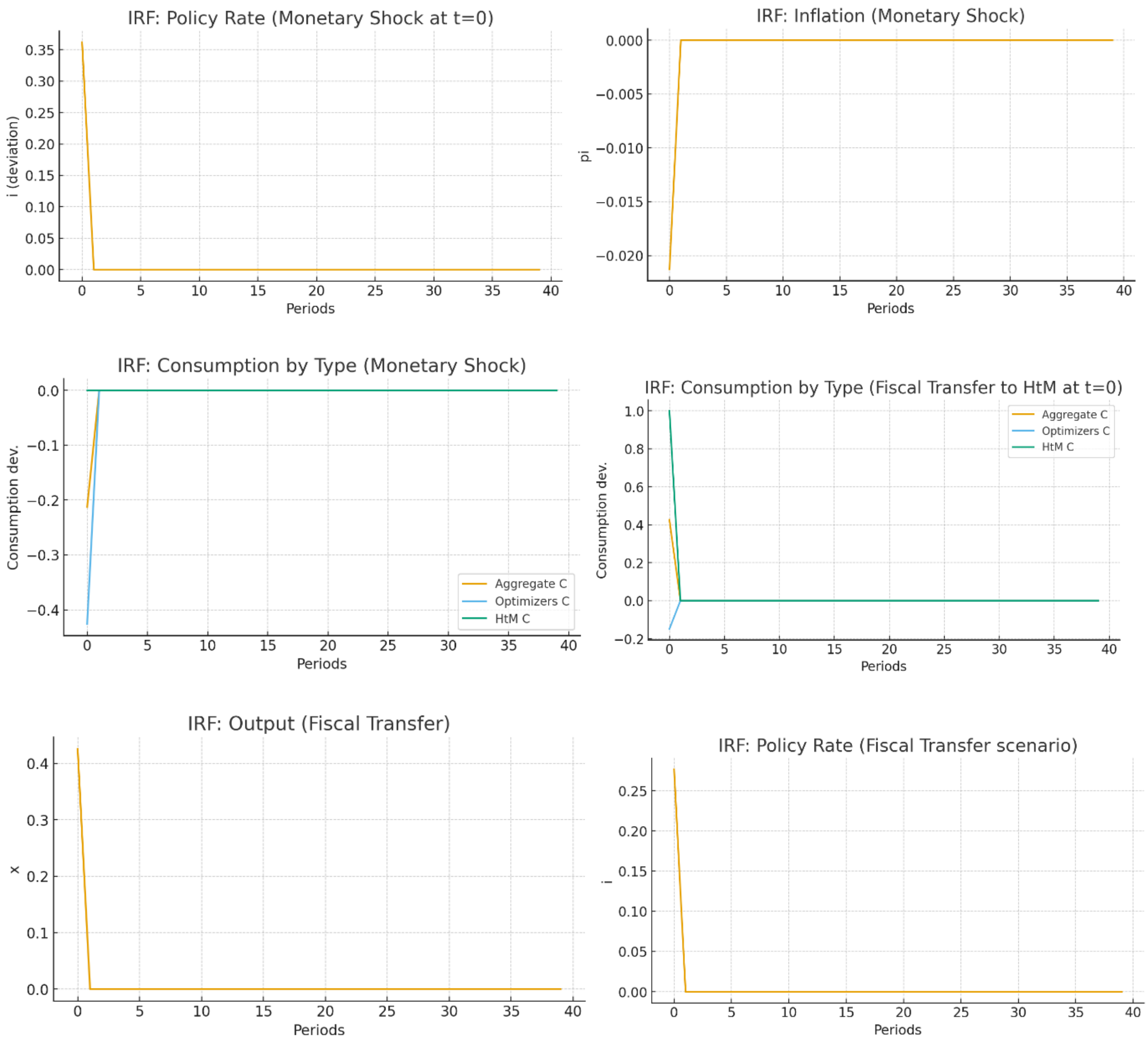

Figure 14 Two-agent New-Keynesian DSGE

Source: Author's own calculation

RANK-matrices PE-GE

Here no hand-to mouth so fiscal transfers are Ricardian and do not enter the IS,we set fiscal term to be zero:

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IS: $x_t - x_{t+1} + \frac{1}{\sigma}\pi_{t+1} = -\frac{1}{\sigma}i_t$

Philips: $\Bbbk x_t + \pi_t - \beta\pi_{t+1} = u_t$

We will stack them:

equation 106

$$\begin{pmatrix} 1 & 0 \\ -\Bbbk & 1 \end{pmatrix}\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} -1 & \frac{1}{\sigma} \\ 0 & -\beta \end{pmatrix}\begin{pmatrix} x_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sigma}i_t \\ u_t \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ -\Bbbk & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 & \frac{1}{\sigma} \\ 0 & -\beta \end{pmatrix}$$

$$d_t = \begin{pmatrix} -\frac{1}{\sigma}i_t \\ u_t \end{pmatrix}$$

For RANK PE we are solving deterministically backward on a finite horizon $t = 0, \ldots, T-1$ with terminal condition $s_T = 0$,the recursion follows from:

equation 107

$$A_{s_t} + B_{s_{t+1}} = d_t \Rightarrow s_t = A^{-1}(d_t - B_{s_{t+1}})$$

This is exactly the recursion used in the code. If $d_t = 0, \forall t \geq 1$ pure one-period pulse at $t = 0$, then except for immediate impact the recursion can drive future sts_tst to zero — that explains the zero-after-impact pathology in the first run.Now for the GE in RANK: monetary policy follows the contemporaneous Taylor rule: $i_t = \phi_\pi\pi_t + \phi_x x_t$, and we will substitute into the IS component of d_t:

equation 108

$$-\frac{1}{\sigma}i_t = -\frac{1}{\sigma}(\phi_x x_t + \phi_\pi\pi_t)$$

Move this term to the left-hand side so the left matrix becomes $A_{mod} = A + C$ where:

equation 109

$$C = \frac{1}{\sigma}\begin{pmatrix} \phi_x & \phi_\pi \\ 0 & 0 \end{pmatrix}$$

So, the GE system is:

equation 110

$$A_{mod}s_t + B_{s_{t+1}} = \begin{pmatrix} 0 \\ u_t \end{pmatrix}$$

Again, by backward recursion:

equation 111

$$s_t = A_{mod}^{-1}\left(-B_{s_{t+1}} + exog_t\right)$$

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The Taylor-rule coefficients enter A_{mod} and therefore change the contemporaneous mapping from s_{t+1} and exogenous terms to s_t. Intuitively, stronger ϕ_π or ϕ_x can damp or amplify certain modes of the system, which is why GE IRFs differ from PE.

TANK- matrices PE-GE

IS reduced form TANK:

equation 112

$$x_t - (1-\lambda)x_{t+1} + \frac{1-\lambda}{\sigma}\pi_{t+1} = -\frac{1}{\sigma}i_t + \lambda g_t$$

Phillips unchanged:

equation 113

$$-\Bbbk x_t + \pi_t - \beta\pi_{t+1} = u_t$$

So:

equation 114

$$A = \begin{pmatrix} 1 & 0 \\ -\Bbbk & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} -(1-\lambda) & \frac{1-\lambda}{\sigma} \\ 0 & -\beta \end{pmatrix}$$

$$d_t = \begin{pmatrix} -\frac{1}{\sigma}i_t + \lambda g_t \\ u_t \end{pmatrix}$$

In PE in TANK i_t, g_t are exogenous so by backward recursion:

equation 115

$$s_t = A^{-1}(d_t - B_{s_{t+1}}), d_t = \begin{pmatrix} -\frac{1}{\sigma}i_t + \lambda g_t \\ u_t \end{pmatrix}$$

now even if g_t is nonzero only at $t = 0$, the presence of the $((1-\lambda)$ coefficient in B and the nonzero Phillips coupling $\Bbbk$ can produce propagation, but if both g_t and other shocks are one-period and small λ then dynamics may still be short-lived. Persistent g_t or persistent other shocks will produce longer IRFs.In GE-TANK: we substitute Taylor rule into IS part as in RANK. Then:

equation 116

$$A_{mod} = A + \frac{1}{\sigma}\begin{pmatrix} \phi_x & \phi_\pi \\ 0 & 0 \end{pmatrix}$$

$$A_{mod}s_t + B_{s_{t+1}} = \begin{pmatrix} \lambda g_t \\ u_t \end{pmatrix}$$

By backward recursion:

equation 117

$$s_t = A_{mod}^{-1}\left(-B_{s_{t+1}} + \begin{pmatrix} \lambda g_t \\ u_t \end{pmatrix}\right)$$

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So, for PE : compute d_t from exogenous paths and set

$$s_t = A^{-1}(d_t - B_{s_{t+1}})$$

For GE form A_{mod} and endogenous RHS:

equation 118

$$s_t = A_{mod}^{-1}(-B_{s_{t+1}} + exog_t)$$

Next results will be plotted.

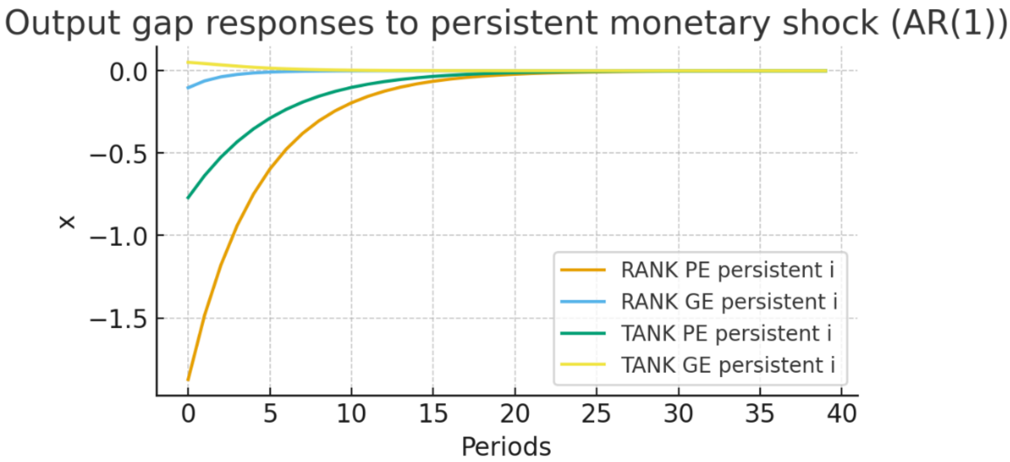

Figure 15 Output gap responses to permanent monetary shock AR(1)

Source: Author’s own calculation

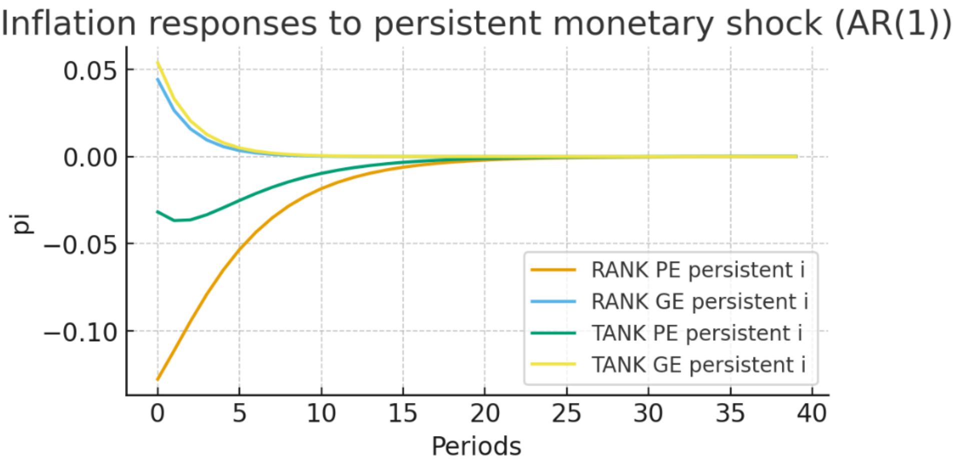

Figure 16 Inflation gap responses to permanent monetary shock AR(1)

Source: Author’s own calculation

- **Role of λ** (TANK): it replaces the forward-looking coefficients in B with $1 - \lambda$ multiples. Lower forward-looking weight (higher λ) means current-period demand is more driven by contemporaneous exogenous transfers g_t and less by x_{t+1} and π_{t+1} — this increases contemporaneous volatility (larger impact) and tends to make IRFs more immediate (less smoothing from forward-looking agents).

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- **Role of $\boldsymbol{\phi_\pi}, \boldsymbol{\phi_x}$** (GE): they enter A_{mod} and change how strongly contemporaneous inflation/output respond to future values — raising ϕ_π typically reduces inflation responses (policy stabilizes), but can amplify output fluctuations if the Taylor rule responds strongly to output
- **Role of κ** (Phillips): links x_t to π_t. Larger κ makes inflation more sensitive to output and so transmits demand shocks into price dynamics more strongly.
- **Shock persistence ($\boldsymbol{\rho}$)**: if d_t follows AR(1) with $\rho > 0$, then d_t feeds the recursion over many t and produces decaying IRFs. Algebraically, persistent shocks are nonzero terms in the RHS of the recursion at many t, hence nonzero s_t.

Next, we will show movement of aggregate capital, heuristic inflation and policy shock how do they deviate from $t = 0$.

Representative *heterogeneous* households indexed by i have state variables: liquid assets a, illiquid capital[21] k, and idiosyncratic employment state $s \in \{1, \dots, S\}$(in code $S = 2$— employed/unemployed). Let labor income given state $s$be y_s. Prices: real return on liquid asset r_a, real rental return on illiquid capital r_k, wage w.

The (recursive) Bellman equation for household $i$is

equation 119

$$V(a, k, s) \;=\; \max_{a' \geq a_{\min},\, k' \geq k_{\min}} \; u(c) + \beta\, \mathbb{E}_{s'}[V(a', k', s') \mid s],$$

subject to the budget constraint

equation 120

$$c + a' + k' \;=\; (1 + r_a)\, a + (1 + r_k)\, k + w\, y_s,$$

and nonnegativity / borrowing constraints $a' \geq a_{\min}$, $k' \geq k_{\min}$.

Utility is CRRA:

equation 121

$$u(c) = \begin{cases} \dfrac{c^{1-\gamma}}{\gamma} & ,\gamma \neq 1 \\ \ln c \;, \gamma = 1 \end{cases}$$

Idiosyncratic income follows a Markov chain $\Pi_{s,s'} = \Pr\,(s' \mid s)$. The expectation is $\mathbb{E}_{s'}[\cdot] = \sum_{s'} \Pi_{s,s'}(\cdot)$.

First-order conditions (interior) (if interior and differentiable):

equation 122

$$u'(c) = \beta \textstyle\sum_{s'} \Pi_{s,s'} \dfrac{\partial V(a', k', s')}{\partial a'}$$

[21] Illiquid capital refers to assets that are difficult to convert into cash quickly and easily without a significant loss in value. These assets include things like real estate, private company stock, and certain collectibles, which lack a ready market and have low trading activity.

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Aggregate capital is $K = \mathbb{E}_i[k_i]$. With labor normalized $L = 1$, firm production is Cobb–Douglas

equation 123

$$Y = K^{\alpha} L^{1-\alpha} = K^{\alpha}.$$

Factor prices:

equation 124

$$r_k = \alpha K^{\alpha-1} - \delta, w = (1-\alpha) K^{\alpha}.$$

In GE, r_k that households face must be consistent with the K they supply:

equation 125

$$r_k^{\star} = \alpha K^{\star\alpha-1} - \delta, K^{\star} = \int k_i^{\star} \, d\mu(i),$$

where $k_i^{\star}$ are the households' policy-induced holdings under prices $(r_k^{\star}, w^{\star})$.
Partial equilibrium vs General equilibrium (code logic)

PE: pick exogenous r (or r_a, r_k), solve households, simulate stationary distribution, compute aggregate K.

GE: find r_k such that the capital implied by household optimization equals the capital implied by firm FOC. Numerically the code does:

1. For candidate r_k, solve household problem → simulate → compute K_{supply}.
2. Compute $r_k^{\text{firm}} = \alpha K_{\text{supply}}^{\alpha-1} - \delta$.
3. Solve for root of $f(r_k) = r_k^{\text{firm}} - r_k = 0$ (brentq / coarse scan fallback).

New-Keynesian (NK) sticky-price block (heuristic in code)

The code uses a crude Phillips-like relationship to produce an inflation proxy:

equation 126

$$\pi_t = \kappa(\log Y_t - \log Y_{ss}),$$

with $Y_t = K_t^{\alpha}$. This is only a heuristic: a full NK Phillips curve would relate inflation to real marginal costs (e.g. output gap, wage markup) and forward-looking expectations:

equation 127

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa \, x_t,$$

where x_t is the output gap or real marginal cost. The code uses the simple static form above for speed/intuition.

Policy / Taylor rule (code uses liquid return as a proxy):

equation 128

$$i_t \approx r_a(t) (\text{in code}); i_t = r_{\text{ss}} + \phi_{\pi} \pi_t$$

There are two implementation approaches you saw.

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1) VFI + simulation (accurate but slow)

Discretize (a, k)on grids. For each state (a_i, k_j, s), compute:

equation 129

$$V^{(n+1)}(a_i, k_j, s) = \max_{(a_{i'}, k_{j'})} \{u(c_{i',j'}) + \beta \sum_{s'} \Pi_{s,s'} V^{(n)}(a_{i'}, k_{j'}, s')\},$$

where $c_{i',j'} = (1 + r_a)a_i + (1 + r_k)k_j + wy_s - a_{i'} - k_{j'}$, skipping infeasible choices.

Iterate until convergence to get policy functions.Simulate Nagents with Markov idiosyncratic shocks to get stationary cross-section and compute aggregates.Instead of solving the DP, assume smooth parametric rules:

equation 130

$$\text{wealth} = (1 + r_a)a + (1 + r_k)k + wy,$$
$$c = \lambda_c \text{ wealth}, k' = \text{clip } (\lambda_k \text{ wealth}, k_{\min}, k_{\max}), a' = \text{clip } (\lambda_a \text{ wealth}, a_{\min}, a_{\max}).$$

Use these closed-form policies to simulate a large cross-section forward under a time series of $r_a(t)$. This yields smooth, fast IRFs for K_t, consumption, and inflation.This is the fast script I gave later; it is not the true DP solution but captures plausible dynamics and avoids corner artifacts.

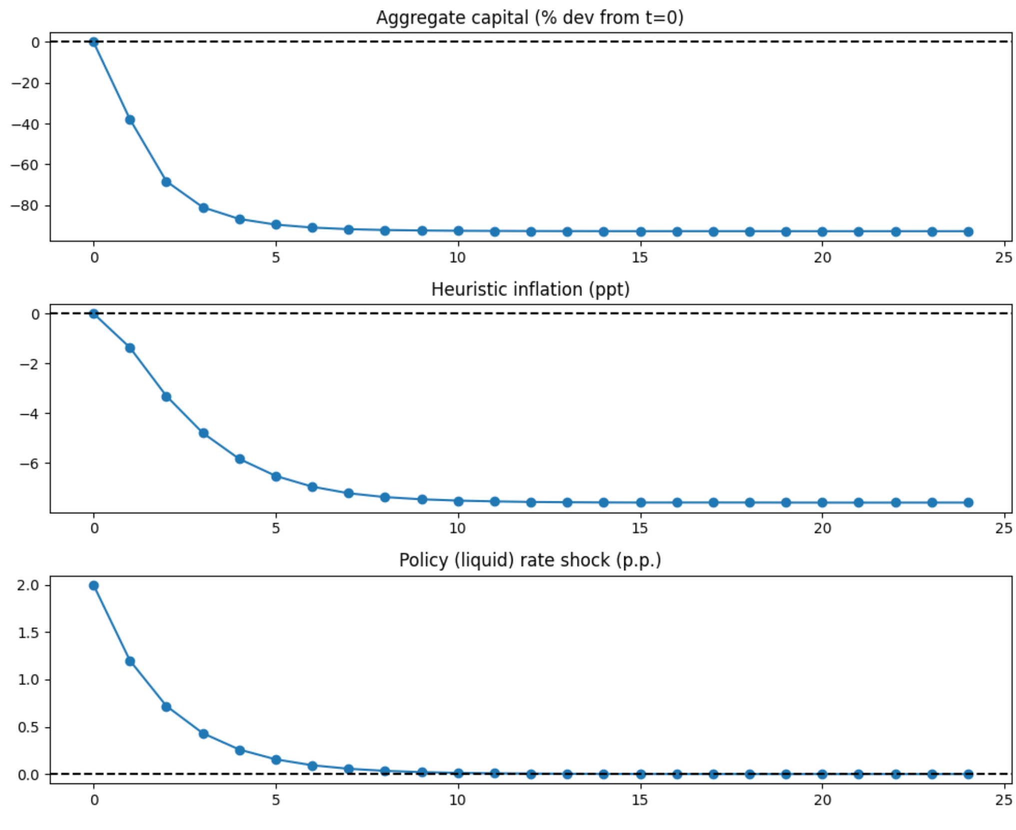

Figure 17 Movement of aggregate capital, heuristic inflation and policy shock how do they deviate from $t = 0$

Source: Author’s own calculation

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Conclusion(s)

In HANK PE solution households take interest rate $r = r_{ss}$ as fixed, wages follow exogenous productivity, so consumption falls sharply on impact of MIT shock, asset accumulation is smoother and asset Gini rises initially, over time dynamics revert as productivity recovers.in GE setup $r_t$and w_t are endogenous, so consumption falls less on impact than in PE, because wages and interest rates move to partially offset the productivity shock. Capital adjusts endogenous differently than in PE. Interest rate exhibits countercyclical pattern. EGM plus Monte Carlo gives realistic, heterogenous responses across the wealth distribution.In the EGM model: Precautionary savings, with uncertainty about income, agents in the low state save less (they can't afford to), while agents in the high state save more. This smooths consumption across states. In RANK models for Monetary shock (i_t): Without heterogeneity, the dynamics are textbook NK: clean, hump-less IRFs unless you add habit/indexation. RANK generates the classic trade-off: stabilizing inflation worsens the output gap. In RANK, fiscal transfers are neutral.In TANK when monetary shock: heterogeneity magnifies transmission mechanism, and heterogeneity worsens trade off or stabilizing inflation costs more in terms of output. Fiscal policy is effective in TANK,i.e. transfers to liquidity-constrained agents stimulate demand and inflation.

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